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The Establishment of the IHRF in Brazil: Current Situation and Future Perspectives

O Estabelecimento do IHRF no Brasil: Situação Atual e Perspectivas Futuras

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Abstract: The study and understanding of geodynamic aspects on the planet require global geodetic references with an order of accuracy better than the magnitude of the effects and reliability. The International Association of Geodesy is responsible for the geodetic infrastructure for Earth system monitoring, more specifically, for providing reliable systems for the analysis and consistent modeling of global phenomena and processes that affect the Earth's gravity field. In this sense, this paper aims to detail one of the current efforts that the geodesy community has been focusing on in recent years, which is the establishment of an international height reference system, and how these efforts bring relevant benefits to the environment, social and economic areas too. Therefore, the paper presents the definition of the system (resolutions, conventions, and constants), and its realization (infrastructure and mathematical formulation). Finally, considering the objective of this work, the paper presents the situation of the new reference system in Brazil, as well as numerical experiments aimed at illustrating the advances made in the country and pointing out future perspectives.
Keywords: International Height Reference System (IHRS). International Height Reference Frame (IHRF). Heights system. Reference System.

Resumo: O estudo e a compreensão dos aspectos geodinâmicos no planeta requerem referenciais geodésicos globais com uma ordem de acurácia melhor do que a magnitude dos efeitos e com confiabilidade. A Associação Internacional de Geodésia é a responsável pela infraestrutura geodésica para o monitoramento do sistema Terra, mais especificamente, por fornecer sistemas de referência para a análise e a modelagem consistente de fenômenos e processos globais que afetam o campo de gravidade da Terra. Nesse sentido, este trabalho tem como objetivo esmiuçar um dos atuais esforços que a comunidade geodésica tem se debruçado nos últimos anos, que é o estabelecimento de um sistema de referência internacional para altitudes e como esses esforços trazem relevantes benefícios também nas áreas ambientais, sociais e econômicas. Em vista disso, a definição do sistema (resoluções, convenções e constantes), bem como a sua realização (infraestrutura e formulação matemática) são apresentados. Finalmente, tendo em conta o objetivo deste trabalho, é apresentada a situação do novo sistema de referência no Brasil, bem como experimentos numéricos visando ilustrar os avanços realizados no país e apontar as perspectivas futuras.
Palavras-chave: IHRF. IHRS. Sistemas de Altitudes. Sistema de Referência.

1 INTRODUCTION

To determine parameters related to geodynamic processes on Earth, Geodesy can contribute with high-resolution observations over time and space, as well as with stable and consistent geodetic references. For example, the transport process and mass variations (due to geophysical signals) can be observed from measurements related to the Earth's gravity field. On the other hand, it is necessary for a highly accurate reference system to describe slight variations related to these processes. Therefore, a reference system should be of high consistency and reliability worldwide and with an order of precision more outstanding than the magnitude of the effects studied. Geodesy's contributions focus on the study of the Earth system by determining, monitoring, mapping, and understanding changes related to the shape, rotation, and mass distribution of the Earth (PLAG; PEARLMAN, 2009 e TORGE; MULLER, 2012).

Geodetic reference systems are the backbone for measurements and interpretations of the effects of global changes, sea-level variations, climate change, and natural disaster management, providing reliable information for decision-making (IAG, 2016). Regarding the vertical geodetic reference, the use of the vertical component, which is consistent and integrated, is also essential in day-to-day operations, such as the construction and monitoring of dams, highways, railways, and river crossings. For example, to work with fluid movement, and water is the most common example, it must consider heights related to the Earth's gravity field. On the other hand, heights derived from the Global Navigation Satellite System (GNSS) are not an option because they have mathematical/geometric meanings.

The scientific community has defined, materialized, and maintained consistent geocentric systems that meet the needs of Geodesy. On the other hand, the materialization and realization of a unified vertical system is still a challenge. Given the above, the International Association of Geodesy (IAG) published, in 2015, Resolution no. 1 about the definition and implementation of an International Height Reference System (IHRS) (DREWES et al., 2016). It is noteworthy that the unification of the height system is one of the three focus areas of the Global Geodetic Observing System (GGOS). GGOS (www.ggos.org) is an observation program of IAG that aims to provide the geodetic infrastructure needed to monitor the Earth's system and its global changes. This program is responsible for assisting in the monitoring of global changes, especially in the study of disaster areas management (studies of geodynamics), global warming, and climate change (atmospheric and hydrospheric variations) (PLAG; PEARLMAN, 2009).

In general terms, a reference system defines constants, conventions, models, and parameters necessary for the geometric and physical representation of quantities. Its realization is done in two ways: physically through the materialization of points on the surface and mathematically, from the determination of coordinates referred to the same reference system (IHDE et al., 2017).

The main objective of this paper is to elucidate the IHRS and IHRF, from their definition (resolutions, constants, and conventions) to their materialization (infrastructure and mathematical formulation), as well as to present the situation of this new system in the country through the existing infrastructure and mathematical computations already conducted.

2 GEODESY IN THE CONTEXT OF THE SUSTAINABLE DEVELOPMENT

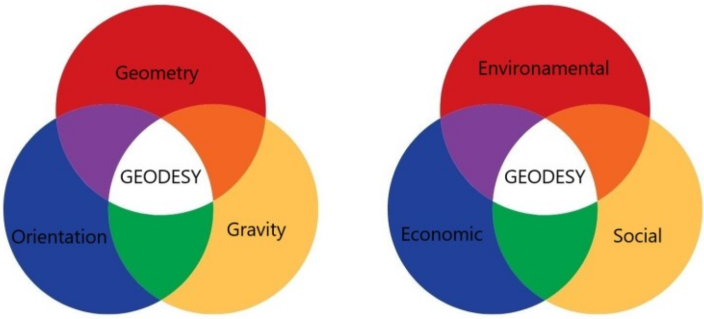
It is commonly found in the literature that geodesy is the science concerned with determining the geometry, the gravity field, and the Earth's orientation parameters in space, as well as temporal variations. An illustration based on the Venn diagram explains this definition, in which the union of the various pillars formed Geodesy (Figure 1). This science is part of Geosciences and an engineering science since it is the foundation for data collection, mapping, and navigation activities. Since the advent of the space age and considering its ability to monitor, map and understand how changes related to the shape, rotation, and mass distribution of the Earth, Geodesy has been concerned with (IAG, 2012):

- Monitoring of the solid Earth (displacement, subsidence, or deformation of the ground and st ructures due to tectonic, volcanic, and other natural phenomena, as well as human activity);
- Monitoring of variations in the liquid Earth (sea-level variations, mass, and fluids transport);
- Monitoring variations in the Earth's rotation i.e polar motion and the length of the day;
- Monitoring the atmosphere with geodetic satellite techniques (ionosphere and troposphere);
- Monitoring the temporal variations of the Earth's gravity field of the Earth;
- Determining satellite orbits; and

- Determining positions and their changes with time of points on or above the surface of the Earth.

Geodesy's ability to provide high-resolution information across time and space makes this science essential for the development and sustainability of any country. In the last 10 years, Geodesy has been included among the topics of the United Nations (UN), more specifically from Resolution 69/266, adopted at the UN General Assembly on February 26, 2015. The resolution presents the Global Geodetic Reference Frame for Sustainable Development (UN, 2015) (see section 3) and the establishment of a committee on Global Geospatial Information Management (UN-GGIM), approved in Resolution 2016/27 of the UN Social and Economic Council (UN, 2016). In this sense, a way of representing the value of Geodesy to society can be seen in Figure 1. To find the details of geodetic information's social, economic, and environmental benefits and investment in this area, access: https://ggin.un.org/.

Figure 1 – From definitions to the benefits of Geodesy.



Source: Adapted of UN-GGIM (2021).

UN-GGIM divides into regional committees. Brazil is part of the UN-GGIM Americas (see: http://www.un-ggim-americas.org/en/inicio.html) and has contributed by opening a data repository and geodetic information. The National Spatial Data Infrastructure (https://inde.gov.br/) and the Land Management Service (https://sigef.incra.gov.br/) are examples. In addition to the availability of free services; a fruitful example is the IBGE-PPP, an online service for processing GNSS data offered by the Brazilian Institute of Geography and Statistics (*Instituto Brasileiro de Geografía e Estatística* - IBGE) (https://www.ibge.gov.br/pt/inicio.html). IBGE is the Brazilian institution responsible for defining, implementing, and maintaining the geodetic infrastructure in the country and has played an important role not only on the national scene but also internationally. The institution has a close relationship with the activities developed under the Geodetic Reference System for the Americas (*Sistema de Referência Geodésico para as Américas* - SIRGAS) and the IAG. SIRGAS is also involved in the activities of the UN-GGIM Americas, as the chair and vice-chair of that organization are responsible for the Geodetic Reference Framework for the Americas (GRFA) Working Group. In addition, SIRGAS develops activities (training and support to member countries) that are aligned with the UN-GGIM Americas work plan. Finally, the professor's and researcher's involvement in Brazilian universities and research institutions should also be recognized in the development of activities and provision of geodetic information.

The accomplishment and maintenance of the tasks and activities mentioned in the previous paragraph, the entire execution of the objects of study, and the participation of Geodesy to achieve the "Sustainable Development Goals" (https://brasil.un.org/pt-br/sdgs) are only possible from GGOS. Such a system is composed of a global geodetic infrastructure that allows users to accurately determine and express their position on or near the Earth's surface, as well as quantify Earth's changes in space and time, from freely accessible geodetic products.

3 GEODETIC GLOBAL REFERENCE SYSTEM – GGRS

The geophysical and geodetic communities dedicate to several initiatives regarding the studies related to the geodynamics of the Earth System. As a result, a global geodetic infrastructure (materialized through the GGOS) is necessary to observe, understand and represent the events that occur on the planet through consistent and accurate measurement and continuous monitoring. In this way, from geodetic techniques and satellite missions, it becomes possible to observe the dynamic processes related to the solid Earth, ocean, and atmosphere.

In response to such needs and supported by governmental feasibility, the IAG, through Resolution 69/266 (UN, 2015), establishes the Global Geodetic Reference System - GGRS, materialized by the Global Geodetic Reference Frame (GGRF) (DREWES et al., 2016). From a scientific and technical point of view, it is up to IAG and GGOS to implement this geodetic reference frame in the most diverse segments of users around the planet.

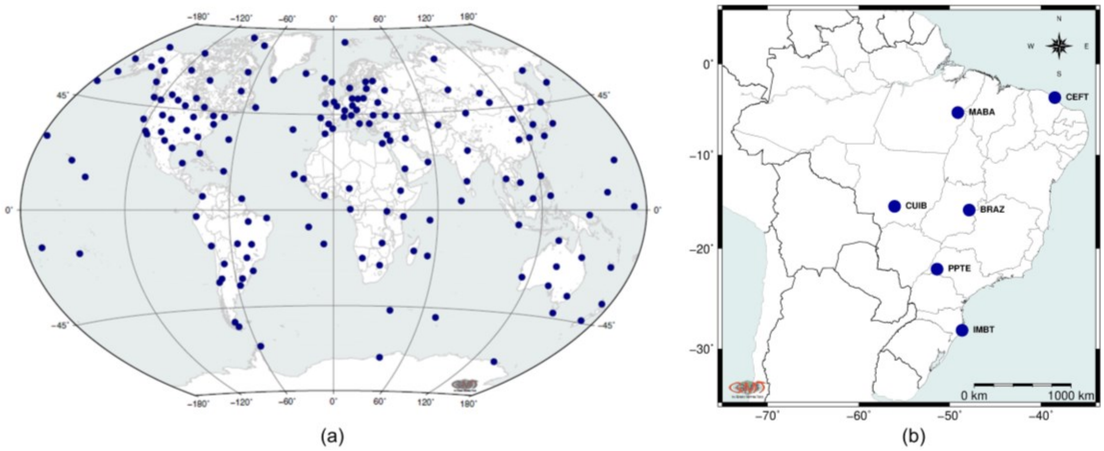
The GGRS is responsible for the definition and concept of a reference system, being translated by the theory of mathematical and physical models, constants, and numerical and physical conventions that support a reference. Thus, according to the position paper adopted by the IAG Executive Committee in April 2016, the position of a point will be represented by its three-dimensional geometric coordinates (X, Y, Z), by the gravity potential of the real Earth (W), by the physical height (H) and by the gravity vector ($\boldsymbol{g}$), and the variations over time of these quantities (see https://iag.dgfi.tum.de/fileadmin/IAG-docs/GGRF_description_by_the_IAG_V2.pdf and https://www.iag-aig.org/topic/3). The details about the specifications can be found in Dalazoana and De Freitas (2020). On the other hand, the GGRF is responsible for materializing and realizing a reference; for the practice through mathematical computations to obtain the quantities of interest and the implementation and maintenance of the infrastructure.

The set of stations installed on the earth's surface is global, with national and regional densification. GGRF stations network comprise: fundamental geodetic observatories that have the maximum of space geodetic techniques – Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS), GNSS, Satellite Laser Ranging (SLR), and Very Long Baseline Interferometry (VLBI) – and that are arranged with gravimetric instruments, allowing the connection between the position of the point (X, Y, Z), W and $\boldsymbol{g}$; other geodetic stations that include reference tide gauges; vertical reference stations and gravity stations that are, whenever possible, collocated with space geodetic instruments (SÁNCHEZ et al., 2021). In general, a fundamental geodetic observatory must include an accurate and stable timekeeping instrument and, preferably, connected to a time reference station and a gravity reference station (equipped with an absolute or superconducting gravimeter). All GGRF stations must operate continuously and on a long-term basis to ensure system stability. They must be fitted with state-of-the-art observation technology to produce high-quality geodetic quantity measurements. They must be continuously monitored to detect deformations on the earth's surface and be connected to the vertical reference to relate their potential differences accurately.

The GGRF, when fully established, will consist of four reference frames. The first, already implemented and consolidated, concerns the three-dimensional geometric component and is performed by the International Terrestrial Reference Frame (ITRF), which the last realization is ITRF2020 (https://itrf.ign.fr/en/solutions/ITRF2020). It consists of determining the position and temporal variation of a set of stations observed by spatial geodetic techniques (DORIS, GNSS, SLR, and VLBI). The International Celestial Reference System (ICRS) is the second reference system implemented and consolidated, providing the celestial foundation for the GGRS. The International Celestial Reference Frame (ICRF) is the realization of the ICRS through the observation of extragalactic objects, called quasars. The link between ICRS and the International Terrestrial Reference System (ITRS) is performed through the Earth Orientation Parameters (EOP). More details about these reference systems and their realizations are in Blitzkow et al. (2011); Dick and Altamimi (2020); Dalazoana and De Freitas (2020).

The third reference system - International Height Reference System (IHRS) -, which is the aim of this paper, is in the implementation phase. Its first realization is intended for the beginning of the current decade based on a primary network with about 170 stations (Figure 2a) that make up the International Height Reference Frame (IHRF) and are positioned with GNSS stations for continuous monitoring. In Brazil, the stations selected (Figure 2b) are: CEFT (Fortaleza), MABA (Marabá), BRAZ (Brasília), CUIB (Cuiabá), PPTE (President Prudente) and IMBT (Imbituba). Experts chose the stations from each country/region based on some requirements that will be addressed in section 3.1.1. The quantities computed at each station will be expressed in terms of gravity potential (W_P) and geopotential number (C_P) values, allowing to obtain physical heights (IHDE et al., 2017).

Figure 2 – a) IHRF network stations and b) stations in Brazil.

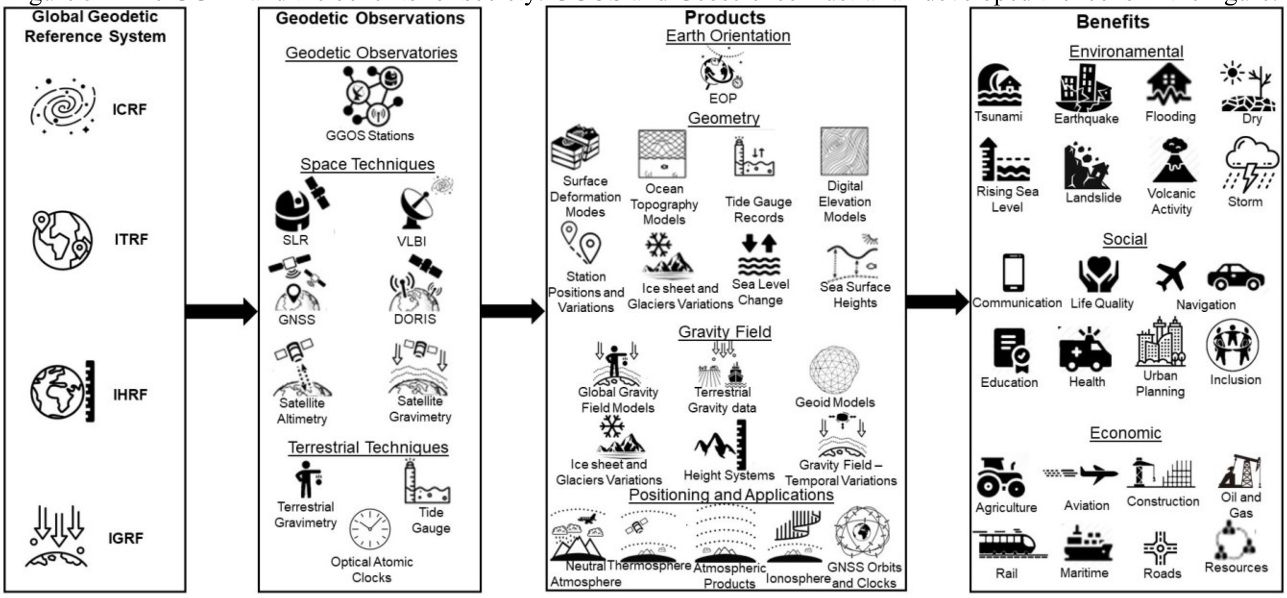


Source: Adapted from Sánchez et al. (2021).

Finally, the fourth reference system - International Gravity Reference System (IGRS) - is also in the implementation phase. Its realization will be conducted by the International Gravity Reference Frame (IGRF) (WZIONTEK et al., 2021). The IGRF aims to replace two infrastructures that are not in line with the current requirements of contemporary Geodesy: the IGSN71 and the International Absolute Gravity Base Station Network (IAGBN) (DREWES et al., 2016). The first step towards the establishment of a global gravimetric system was taken with the publication of resolution nº 2/2015 adopted by the IAG at the IUGG General Assembly in July 2015, entitled “Establishment of a global absolute gravity reference system”. This Resolution is available in Drewes et al. (2016). The document “Strategy for Metrology in Absolute Gravimetry” (MARTI et al., 2014), developed jointly between the IAG and the Consultative Committee for Mass and Related Quantities (CMM), contributed to joining gravity measurements at the highest level for Metrology and Geodesy and should be mentioned in this paper.

The GGRF is moving towards supporting the collection and management of nationally integrated geospatial information to monitor the dynamics of the planet. Once IHRF and IGRF conclude, in association with the ICRF and ITRF (Figure 3 - first column), geodetic observations can be acquired (at a global level that follows the same collection policies, standards, and conventions) by using geodetic observatories (GGOS stations), space techniques (SLR, VLBI, GNSS, DORIS, altimetry, and satellite gravimetry) and/or terrestrial techniques (gravimetry, tide gauges, and atomic clocks). It is worth highlighting the importance of countries' engagement in multilateral cooperation to address issues related to infrastructure gaps and duplications to develop a more sustainable global geodetic reference. The information obtained through geodetic observations to observe and model the dynamics of the planet (Figure 3 - second column) is only possible from a geodetic infrastructure that includes equipment, technology, geospatial data, and opening repository, human resources, products, and services. Several products (Figure 3 – third column) are derived from geodetic measurements and observations and are related to planet orientation, geometry, gravity field, and positioning activities and their applications. In this sense, the products and solutions obtained from geodetic observations bring relevant environmental, social, and economic benefits (Figure 3 – fourth column).

Figure 3 – The GGRF and the benefits for society. GGOS and Geoscience Australian developed the icons in the figure.



Source: The authors (2022).

As an example of some environmental benefits, radar altimetry missions aboard satellites (Jason-1, Jason-2, and Topex/Poseidon) (MEYSSIGNAC; CAZENAVE, 2012) generated products related to sea-level change, allowing the monitoring and decision making. The ERS-1/2, Jason-3, and SWOT satellites are currently responsible for developing the information mentioned above. Gravity Recovery and Climate Experiment (GRACE) satellite provide products related to the temporal variation of the gravity field, making it possible to follow, over time, regions affected by drought or floods, in addition to allowing studies related to groundwater (RICHEY et al., 2015). In terms of social benefits, a tracking system using the Global Positioning System (GPS) was developed to help save the lives of women and their babies who move with the herd of cattle in rural Kenya and do not always have access to maternal care (see the report at: https://www.bbc.com/news/av/world-africa-49120067). Another example is the development of GPS apps for people with visual impairments (MORAD, 2010; VELÁZQUEZ et al., 2018). In terms of economic benefits, a project developed by the Australian government in partnership with the New Zealand government showed that an investment of US$ 14 million in Satellite-Based Augmentation System (SBAS) technology can generate savings of US$ 7 .4 billion for the next 30 years in areas such as agriculture, rail, highway, aviation, maritime, construction, and human resources (see the report at: https://www.ga.gov.au/news-events/news/latest-news/trial-of-accurate-positioning-technology-wins-asia-pacific-spatial-excellence-ward).

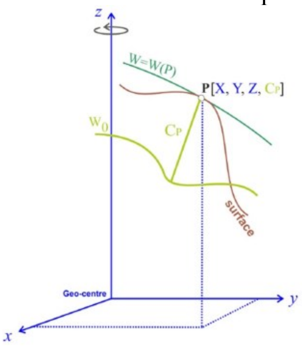
3.1 International Height Reference System – IHRS

The IHRS is one of the realization targets in Geodesy, conducted by the IAG and present as a focus area in the GGOS. It is a system that aims to standardize the vertical reference surface, as well as the way to determine the vertical component consistently. Currently, most countries use a local or regional height system (commonly called "Vertical Datum"), which has been individually established by applying different procedures. Individual efforts culminated in dozens of physical heights connected to local observations of mean sea level. Some countries have more than one vertical reference, as is the case of Brazil, which has its heights linked to the tide gauge of Imbituba and Santana (IBGE, 2019). Furthermore, such references are stationary and do not consider variations in time.

Different vertical systems between countries make it impossible to exchange geospatial data and make it difficult to carry out projects in border regions. Furthermore, geodetic quantities (physical heights, gravity anomalies, geoidal models, etc.) are restricted for use in limited geographic areas. Their global combination or associated with satellite information (such as GNSS) is incompatible because the discrepancies obtained are more significant than the accuracy currently required. Aware of the need for a univocal and global vertical reference, the IAG presented the IHRS in 2015 as the conventional and global vertical system related to the gravity field (Resolution No. 1 of 2015 can be consulted in Drewes et al. (2016)).

According to Ihde et al. (2017), the global vertical reference system is a set of physical points materialized by coordinates ($\boldsymbol{X_P}$) and distributed over the earth's surface. It considers gravity potential (W_P) and geopotential number (C_P) values, referred to the geopotential (W_0) (Figure 4), with the position *P* expressed in terms of its coordinates in the ITRS. In this way, the vertical components are determined from the differences (ΔW_P) between the surface level potential of the Earth's gravity field that touches the point *P* on the Earth's surface and the reference potential of the vertical system (W_0) associated with a gravity acceleration value. The potential difference $-\Delta W_P$ is designated as C_P (Eq. (1)).

Figure 4 – IHRF station components.



Source: Adapted from Sánchez et al. (2017).

The IHRS definition is based on the conventions described in Resolution No. 1 of 2015 (DREWES et al., 2016) and presented by Ihde et al. (2017): **1)** the vertical reference surface is an equipotential surface of the Earth's gravity field whose potential W_0 was defined as a constant of value equal 62,636,853.4 m^2s^{-2}; **2)** the vertical coordinate is the potential difference ($-\Delta W_P$) between the potential W_P at point *P* on the earth's surface and the reference geopotential W_0. The potential difference is called the geopotential number (C_P); **3)** the position of a given point *P* for the potential W_P is defined based on ITRS; **4)** the parameters, observations, and data must be related to the mean tide concept and; **5)** the units of measurement are the meter and the second, following the International System. Furthermore, in that resolution, the determination of $\boldsymbol{X_P}$, W_P (or C_P) includes the variations overt time $\dot{\boldsymbol{X}}_{\boldsymbol{P}}$, $\dot{W}_P$ (or $\dot{C}_P$).

The challenge of the geodetic community is to determine the W_P value as accurately as possible. According to Ihde et al. (2017), the accuracy of the potential values must follow the same level as the $\boldsymbol{X_P}$ coordinates. The accuracy within the current requirements is ± 3 mm in the position of the vertical component and 0.3 mm/y (year) in the vertical speed of the station. It corresponds to $\pm 3 \times 10^{-2} m^2s^{-2}$ and to $\pm 3 \times 10^{-3} m^2s^{-2}y^{-1}$ for the potential values W_P and $\dot{W}_P$, respectively. According to Sánchez et al. (2021), the accuracies, as mentioned earlier, are not realistic at this time, based on available sources of information. Thus, for the first realization of the IHRF, efforts are being concentrated on determining static potential values (without considering variations in time), as well as on the possibility of reaching an accuracy of $\pm 1 \times 10^{-1} m^2s^{-2}$ (around 1 cm in height). In addition, three options for the determination of potential differences referred to the IHRS are considered by the specialist group: **1)** use of high-resolution global geopotential models; **2)** use of regional gravity field models (geoid and quasi-geoid), and; **3)** unification (transformation) of the existing vertical system to the IHRS. The most practical option to obtain Eq. (1) is to use high-resolution Global Geopotential Models (GGMs).

$$C_P = -\Delta W_P = W_0 - W_P \tag{1}$$

The direct computation of W_P is possible by inserting the ITRF coordinates of an IHRF station in Eq. (2) (BARTHELMES, 2013) which represents the spherical harmonic coefficients representing in the spectral domain the global structure and irregularities of the gravity field of the Earth:

$$W_P(r,\lambda,\varphi) = \frac{GM}{R}\sum_{l=0}^{l_{max}}\sum_{m=0}^{l}\left(\frac{R}{r}\right)^{l+1} P_{lm}(sen\varphi)\left(\bar{C}_{lm}^{W}\cos m\lambda + \bar{S}_{lm}^{W} sen\, m\lambda\right) \tag{2}$$

where (r, λ, φ) are the spherical geocentric coordinates of computation point (radius, longitude, latitude); R reference radius; GM product of gravitational constant and mass of the Earth; l, m degree, order of spherical harmonic; P_{lm} is fully normalized Legendre functions and; $\bar{C}_{lm}^{W}$, $\bar{S}_{lm}^{W}$ are Stokes' coefficients (fully normalized).

The advancement of satellite gravimetry, especially with the GRACE and GOCE satellites, combined with satellite-based radar altimetry information, terrestrial gravity, and topographic models, allowed an improvement in GGMs and the possibility of extending them to 5540 degree and order (~3.6 km resolution). EGM2008 (PAVLIS et al., 2012) was the first model to have spherical harmonic coefficients with high degree and order (degree 2190 and order 2159), being the reference for the computation of recent models as EIGEN-6C4 (FÖRSTE et al., 2014), GECO (GILARDONI; REGUZZONI; SAMPIETRO, 2016), XGM2019 (ZINGERLE et al., 2019) and SGG-UGM-2 (LIANG et al., 2020). The reliability of the models is commonly evaluated by comparing the geodetic functionals (geoid undulation or height anomaly) with GNSS stations on leveling (see: GUIMARÃES et al., 2012; NICACIO; DALAZOANA; FREITAS; 2018). Gruber and Willberg (2019) demonstrated that recent GGMs have an accuracy of the order of 2 cm for regions with smooth topography and well-developed in terms of gravity densification. In the areas with gravity voids or where the topography is accentuated, uncertainties can vary from ±20 to ±40 cm. In the study carried out by the authors, 683 GNSS stations on leveling located in Brazil were used in comparison with six GGMs; the discrepancies ranged from 25 to 33 cm. The differences reflect a combination of systematic and random errors arising from the geometric leveling procedure, the geodetic height obtained through GNSS, and the commission and omission errors of the GGMs. Therefore, separating the magnitude of each error component to get the absolute uncertainty of the global models is not a viable task. Thus, using only GGMs to estimate potential values is not appropriate now, as the available models do not reach the precision required by the IHRS (σ_{W_P}= ± 0.01 m^2s^{-2}).

The second option is more appropriate within the context. The potential values are obtained from the modeling of the regional gravity field. Therefore, the approach is given by the Geodetic Boundary Value Problem (GBVP) solution, which can formulate in a few ways depending on the known geodetic quantities and the boundary surface to be considered. The most used formulations are the free scalar GBVP (potential and the vertical position on the boundary surface are unknown and the horizontal position (φ, λ) of the gravity observations are known) and the fixed GBVP (unknown potential, boundary surface, and coordinates (φ, λ, h) from known gravity observations). More details can be found at Priest; Sansò (1986), Heck (1989), and Guimarães; Blitzkow (2011). Concerning the free scalar GBVP, a reference surface must be adopted. In this case, there are two possibilities: assume the physical surface of the Earth and the telluroid as a reference surface (Molodensky's theory approach); or assume as the contour surface an equipotential surface (eg the geoidal surface (W_0)), whose reference surface is the ellipsoidal surface (U_0). The unknown potential is represented in terms of disturbing potential ($T = W - U$). The resolution of the GBVP to model the regional gravity field is usually treated from the application of the integral. The integration considers the values of gravity acceleration, which are expressed in terms of gravity anomaly (Δg) or gravity disturbance (δg). For the free scalar GBVP, the modified Stokes' integral is applied (Eq. (3)) and for the fixed GBVP, the Hotine-Koch's function is selected (Eq. (4)) (HOFMANN-WELLENHOF; MORITZ, 2006). Eqs. (3) and (4) are presented in modified form using the reduced surface gravity anomaly and gravity disturbance and expressed in terms of Δg_{res} and δg_{res}, respectively.

$$T_{g_{res}} = \frac{R}{4\pi}\iint_{\sigma} \Delta g_{res}\, S(\psi) d\sigma \tag{3}$$

$$T_{g_{res}} = \frac{R}{4\pi}\iint_{\sigma} \delta g_{res} H\,(\psi) d\sigma \tag{4}$$

where σ is the unit sphere, R mean radius, Δg_{res} residual gravity anomaly, δg_{res} residual gravity disturbance, $S(\psi)$ Stokes' function, and $H(\psi)$ Hotine-Koch's function.

Classic Stokes and Hotine-Koch integrations require global coverage of gravity data, which is impossible since many regions do not have gravity measurements, especially in the oceans. In practice, such integrals are solved by truncating the computation from an integration radius around the point of interest. So, these integrals can be solved by numerical integration, Fast Fourier Transform, Least Squares Collocation (LSC) (FORSBERG; TSCHERNING, 1981; TSCHERNING, 1985), and spherical radial basis function (BENTEL; SCHMIDT; GERLACH, 2013; LIEB et al., 2015). Regardless of the methodology chosen to obtain the potential values, a commonly used strategy is the application of the procedure called Remove-Compute-Restore (SCHWARZ; SIDERIS; FORSBERG, 1990). This process combines information that expresses the terrestrial gravity field in different wavelengths (gravity data, GGM, and Digital Terrain Model - DTM). In this case, the residual disturbing potential ($T_{g_{res}}$) obtained by Eqs. (3) or (4) is added to the contributions of the DTM (T_{DTM}) and GGM (T_{GGM}), resulting in the disturbing potential:

$$T_P = T_{g_{res}} + T_{DTM} + T_{GGM} \tag{5}$$

In this sense, the W_P value in an IHRS station can be expressed by:

$$W_P = U_P + T_P \tag{6}$$

where U_P is the normal potential of a reference ellipsoid computed for a position P.

Sánchez et al. (2021) point out that the challenge for the first solution of the IHRF is to assess the consistency between the different varieties of strategies, methods, and approaches used to compute regional gravity field models. Besides that, the standardizing and unifying processes may not be so simple since regions on the planet have different characteristics and require approaches (modifications of kernel functions in integrals, size of the integration radius, and geophysical reductions). An alternative is to standardize as many processes as possible to ensure similar and compatible results from different methodologies. In this sense, a first step towards the realization of the IHRF was taken from the so-called "Colorado Experiment" organized by the focus area "Unified Height System" of the GGOS (WANG et al., 2021). Some specialists computed gravity potential values using their methodologies and computer programs for a test area in the Colorado state region, in the United States, from the same input data and pre-established conventions and standards (details at https://www.isgeoid.polimi.it/Projects/colorado_experiment.html).

The third option is not exactly to compute potential values but to allow the unification and densification of the IHRF concerning the existing vertical systems. The geopotential number (C_P^{DVL}) connected to the Local Vertical Datum (LVD) is expressed by the difference between the gravity potential (W_0^{DVL}) of the LVD and the gravity potential of the point of interest (W_p) (SÁNCHEZ et al., 2021):

$$C_P^{DVL} = W_0^{DVL} - W_P \tag{7}$$

and can be converted into a geopotential number linked to the IHRF (C_P^{IHRF}) from the knowledge of the difference (δW_0^{DVL}) between W_0 and W_0^{DVL}:

$$C_P^{IHRF} = (W_0 - W_0^{DVL}) + C_P^{DVL} \tag{8}$$

More details about how to estimate δW_0^{DVL} can be found in Sánchez and Sideris (2017).

3.1.1 THE CURRENT IHRF STATIONS INFRASTRUCTURE IN BRAZIL

The IBGE, the institution responsible for Geodesy in the country, selected the stations to compose the

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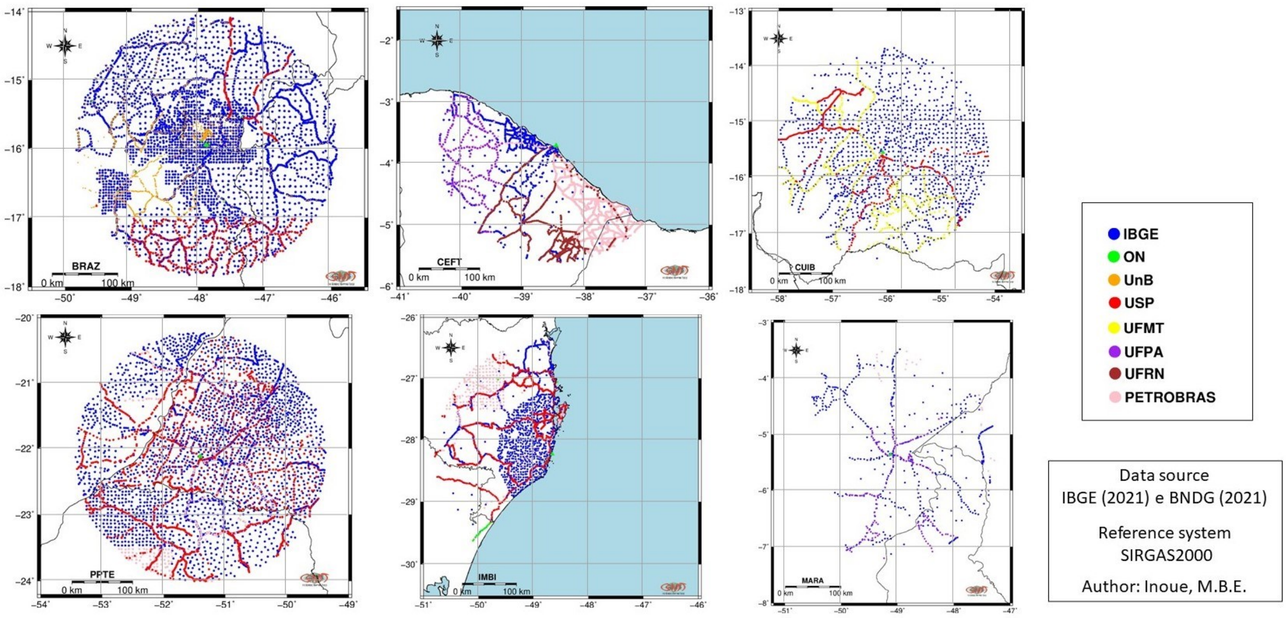
first realization of the IHRF. The stations belong to the Brazilian Network for Continuous Monitoring of GNSS Signals (*Rede Brasileira de Monitoramento Contínuo dos Sistemas* GNSS – RBMC) and are connected to the regional network, as recommended by Sánchez et al. (2021). In this case, the SIRGAS network. Furthermore, CEFT and BRAZ stations are placed together with other geodetic techniques, the first with a VLBI station and the second with an SLR station. Sánchez et al. (2021), also recommend that IHRF stations are collocated with gravity measurements, allowing to support the connection between the geometric reference (geocentric cartesian coordinates $\boldsymbol{X_{(P)}}$) and the physical reference (values of potential (W and C) and gravity value (g). Besides that, it recommends that stations are connected to the local vertical datum facilitating the unification of the IHRF. If not, the connection must be made through geometric leveling. According to the IBGE database (http://www.bdg.ibge.gov.br/appbdg/), BRAZ, CUIB, PPTE, and MABA stations are connected to the national vertical network and the last station has a relative gravity value. It is worth noting that the first three stations already have absolute gravity measurements.

To achieve the desired accuracy for potential values (W_p) at IHRF stations, the distribution and quality of gravity measurements are essential. According to the guidelines and recommendations of Sánchez et al. (2017); Sánchez et al. (2021), the filling of the gravimetric voids around the IHRF reference station is the first fundamental point to be highlighted. The distribution of gravity points should be as homogeneous as possible, with measurements up to 100 km (~1°) or 200 km (~2°) radius from the IHRF station, depending on the topography. For the punctual computation of potential values, the gravity information (contained in the mentioned radius) is used to solve the GBVP. Outside the radius of the IHRF station, high-resolution GGMs are employed to minimize the truncation error caused by the omission of the gravity field signal. The combination of gravity measurements and satellite information improves the accuracy of potential values at IHRF stations.

In this sense, Figure 5 shows the terrestrial gravity distribution around the IHRF station considering a 2º radius and the institutions producing gravity data. Data were acquired from the IBGE database (http://www.bdg.ibge.gov.br/appbdg/) and the National Gravimetric Database (*Banco Nacional de Dados Gravimétricos* - BNDG) of the National Agency for Petroleum, Natural Gas and Biofuels (*Agência Nacional do Petróleo* - ANP) (https://www.gov.br/anp/pt-br/assuntos/exploracao-e-producao-de-oleo-e-gas/dados-tecnicos/legislacao-aplicavel/bndg-banco-nacional-de-dados-gravimetricos).

Figure 5 – Gravity distribution around the IHRF stations



Source: The authors (2022).

As shown in Figure 5, BRAZ and PPTE stations have a homogeneous distribution, while CUIB, CEFT, and IMBT stations have an intermediate scenario containing “gravimetric voids” that need densification. On the other hand, MABA station deserves special attention due to the scarcity of information in the study region.

It notes that CEFT and IMBT stations are on the coast. In this case, marine gravity information should be considered. Ribeiro, Guimaraes and Camargo (2022) have been dedicated to the possibility of using the available gravity data in the oceans to compute the potential values in the referred stations.

Sánchez et al. (2021) still recommend a spatial resolution for gravity measurements from 1' to 3' (~2 to 4 km), depending on the relief (in mountainous areas, gravity observations should have a better resolution than in flat areas). It is well known that terrestrial relative gravity measurements are costly and time-consuming. Brazil has been working with a spatial resolution of 5' (~9 km) (resolution of the latest national geoid model – MAPGEO2015). Therefore, an analysis was performed considering the spatial resolution recommended by Sánchez et al. (2021) (in this case, 3 km was adopted) and the current resolution used in national gravity densification (Table 1). For CEFT and IMBT stations, it was considered only the continental area.

Table 1 – Gravity stations available and necessary to achieve 3 and 9 km resolutions.

Station	Total of points available at BDG and BNDG	Amount needed to complete 3 km resolution	Amount needed to complete 9 km resolution
CEFT	4.003	3.054	0
MABA	702	15.912	1.045
BRAZ	5.799	10.815	0
PPTE	7.742	8.872	0
CUIB	2.209	14.405	0
IMBT	3.159	3.831	0

Source: The authors (2022).

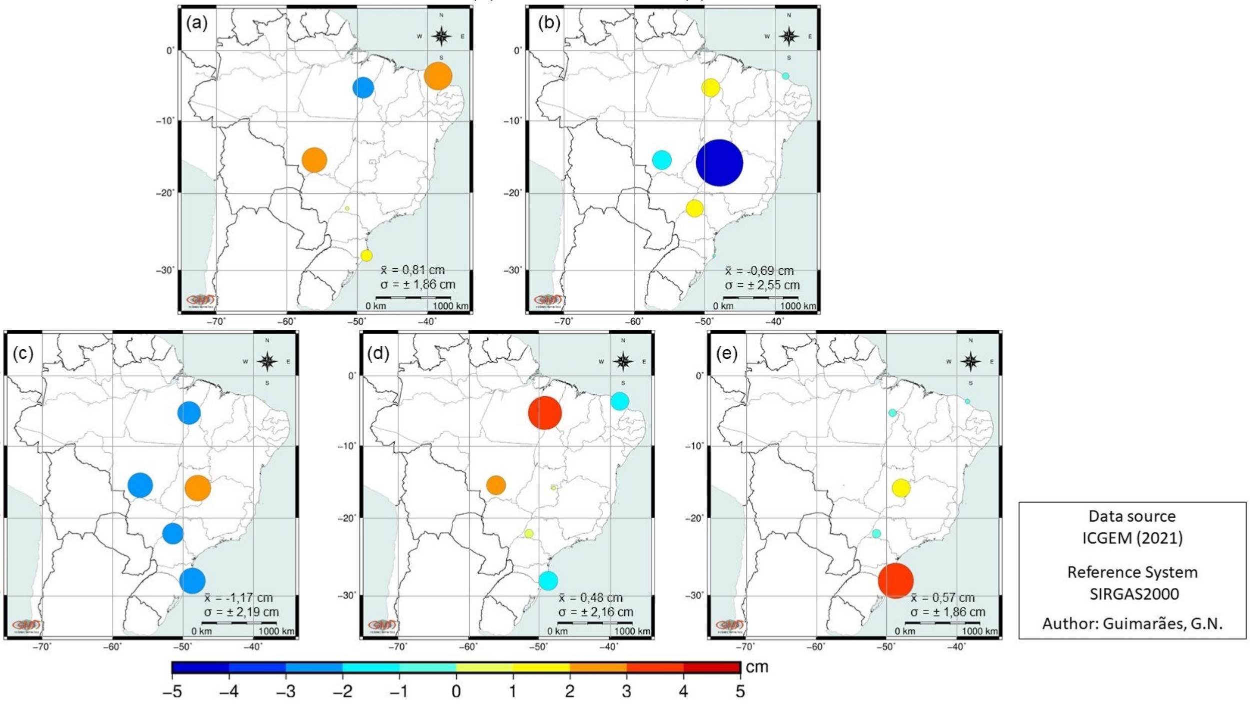
The second column of Table 1 shows the number of gravity stations available in the BDG and BNDG, already considering duplicate points since the BNDG also concentrates on information acquired by the IBGE. The third column of the respective table elucidates the number of gravity stations needed to reach 3 km resolution. A considerable effort is necessary to achieve such a resolution. Due to the lack of access and paths, combined with a high investment, the adoption of such a resolution is unlikely. In remote areas with an absence of roads and trails, such as forests and deserts, as well as on top of mountains, airborne intended for geodetic purposes is an option, although the investment required is high. On the other hand, considering the 9 km resolution (fourth column of Table 3) only MABA station needs more gravimetric measurements.

3.1.2 IHRF STATUS IN BRAZIL

Brazil has participated in activities related to the realization of the IHRF. Brazilian researchers are involved in working groups within the scope of SIRGAS and IAG to address the issue. It is worth noting that master's and doctorate degrees have been defended on the subject, papers have been presented at national and international conferences and published in journals (further details in Albarici et al. (2019), Albarici et al. (2021) and Ribeiro, Guimaraes and Camargo (2022).

The potential values and, consequently, the normal heights were computed based on the methodologies presented in section 3.1. At first, the W_P values were calculated directly in the International Center for Global Gravity Field Models (ICGEM) service by Eq. (2). The geodetic functional "gravity potential" was determined from five high-resolution GGM (EIGEN-6C4, GECO, XGM2019, and SGG-UGM-2 up to 2190 and SGG-UGM-1 up to 2159). The selected reference system was the Global Reference System 1980 (GRS80), adopting the concept of zero-tide and considering the zero-order term. Then, the C_P values were computed by Eq. (1) and the respective normal height at each station was obtained from the division between the C_P and the average value of the normal gravity. At last, the average of the normal height of the five models was computed and the result was compared with the normal height obtained in each model (Figure 6).

Figure 6 – Comparison of the GGMs normal height average related each GGM. (a) EIGEN-6C4, (b) GECO, (c) SGG-UGM-1, (d) XGM2019, and (e) SGG-UGM-2



Source: The authors (2022).

Figure 6 shows that the discrepancies ranged from -4.9 cm (Figure 6(b) BRAZ station – GGM GECO) to 3.7 cm (Figure 6(e) IMBT station – GGM SGG-UGM-2). Note that there was not a GGM that was better adjusted to the mean of the GGMs, and the standard deviation ranged from ± 1.86 to ± 2.55 cm. PPTE station was the one that best adjusted with all GGMs, which can be explained by the significant amount of gravity information available in the region, which was used in the computation of each model associated with a flat relief region.

The W_P values were also computed from the regional gravity field modeling. In this scenario, two geoid models were used: MAPGEO2015 (BLITZKOW et al., 2016) and GEOID2021 (MATOS et al., 2021), and the potential values can be inferred using Eq. (9). For a quasi-geoidal model, the formulation adopted must be different (SÁNCHEZ et al., 2021).

$$W_P = W_0 - (h_P - N_P)\bar{g}_P \qquad [m^2 s^{-2}] \tag{9}$$

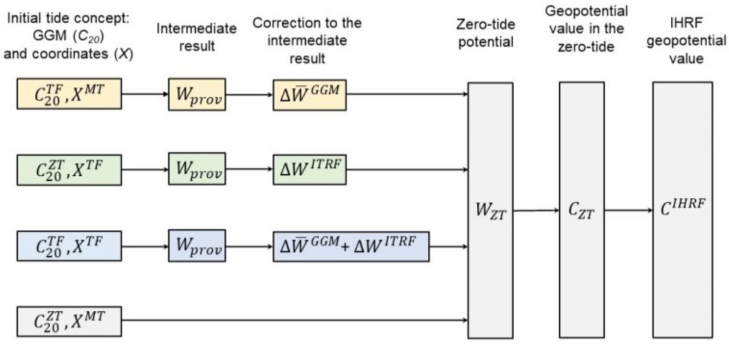
with

$$\bar{g}_P = g_P + 0{,}424 * 10^{-6}(h_P - N_P) + TC_P \qquad [m^2 s^{-2}] \tag{10}$$

where $\bar{g}_P$ is the mean gravity along the plumb line between P_0 located at the geoidal surface and P on the physical surface, h_P is the ellipsoidal height of the IHRF station, N_P is the geoid undulation interpolated from the geoid model at the IHRF station and g_P is the real gravity value at the station (if it was not observed directly, it may be interpolated from the terrestrial gravity data set used for the geoid determination). The factor 0,424 x 10^{-6} refers to an average density of topographic masses of ρ = 2,670 km m^{-3} and TC_P is the Terrain Correction (MATOS, 2005; GEMAEL, 2012).

The importance of using a geoid model called "pure gravimetric" that is, without any combination or adjustment with GNSS data on leveling (called hybrid geoid) is highlighted. In addition, special attention to standardization and model conventions must be considered. In this case, the correction of the zero-degree term between the GGM constants used in the geoid model and the GRS80 must be considered. For MAPGEO2015, this correction was added, while the GEOID2021 was computed already considering the correction, which in this case is -0.17 m. For the formulation of the zero-degree term, see Sánchez et al. (2021). The correct treatment of the permanent tidal concepts must be considered: **1)** to observe in which tidal concept the input data are (GNSS coordinates and GGM used in the computation of the geoid or quasi-geoidal model); **2)** to do the corrections in the intermediate results, for the geopotential number should be expressed in the concept of zero-tide (C^{MZ}) and, **3)** to determine C^{IHRF} in the mean-tide concept. In terms of GGM, the coefficient of second-degree C_{20} (degree 2 and order 0) is related to the tidal concept and defines in which tidal concept the potential value will be obtained. Figure 7 illustrates four different possibilities relating to input data and tidal concepts. In the first situation (Figure 7 in orange), if the GGM (C_{20}^{TF}) is in the tide-free concept and the GNSS coordinates in the mean-tide concept (X^{MT}), the provisional potential value (W_{prov}) is computed and the correction ($\Delta\overline{W}^{GGM}$) used to obtain the potential at zero-tide (W^{ZT}). In the second situation (in green), the GGM (C_{20}^{ZT}) is in the concept of zero-tide, while the GNSS coordinates in the concept of tide-free (X^{TF}). In this case, the correction applied is for the GNSS coordinates to be corrected for the concept of zero-tide (ΔW^{ITRF}). In the third case, both the GGM (C_{20}^{TF}) and the coordinates (X^{TF}), which are in the tide-free concept, must be corrected to the zero-tide concept (($\Delta\overline{W}^{GGM} + \Delta W^{ITRF}$). Finally, the last case does not need to correct the intermediate result, since the input data (C_{20}^{ZT} and X^{MT}) is in the concept of zero-tide and mean-tide, respectively. The mathematical formulation regarding the corrections can be found in Sánchez et al. (2021).

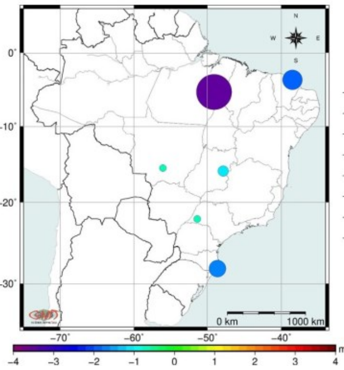
Figure 7 – Scheme of the corrections applied for the computation of the potential value according to the tide concept in the input data.



Source: Adapted from Sánchez et al. (2021).

The geopotential number of each of the six national IHRF stations was computed using the last two geoid models available for the study region and compared with each other (Figure 8). The difference involving the MABA station (-3.51 m^2s^{-2}) can be explained by the influence of the GGMs used in the models since, in that region, gravity measurements were not included from MAPGEO2015 to GEOID2021. MAPGEO2015 uses EIGEN-6C4 and GEOID2021 uses XGM2019 (both with degree and order 200). The area around the MABA station has little terrestrial gravity information. So, the data from the models (GGM and/or DTM) influence the results more significantly than in regions with a dense and homogeneous gravity distribution (as was the case of the BRAZ, CUIB, and PPTE stations, where the discrepancies were minor). For coastal stations (CEFT and IMBT), the reason for the differences (1.96 and 1.68 m^2s^{-2}, respectively) may be the models used for the oceanic area. While MAPGEO2015 uses DTU10 (ANDERSEN, 2010), GEOID2021 uses DTU17 (ANDERSEN; KNUDSEN, 2019).

Figure 8 – Comparison between geopotential values computed at IHRF station using MAPGEO2015 and GEOID2021 models.



Station	Discrepancy (m^2s^{-2})	Discrepancy (m)
CEFT	-1.96	-0.20
MABA	-3.51	-0.36
BRAZ	-1.05	-0.11
CUIB	-0.68	-0.07
PPTE	-0.70	-0.07
IMBT	-1.68	-0.17

Data source
MAPGEO2015 and GEOID2021

Reference System
SIRGAS2000

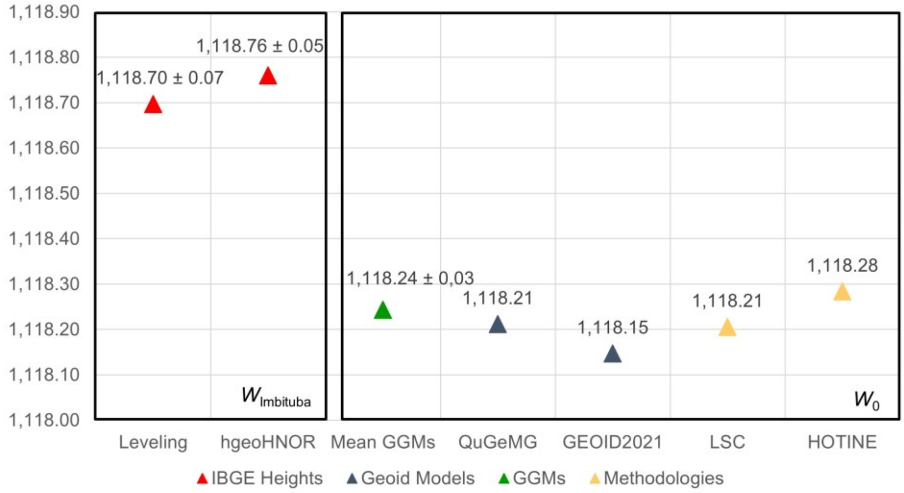
Author: Guimarães, G.N.

Source: The authors (2022).

Potential values were still computed from the GBVP approach using Hotine's integral and the Least Squares Collocation. In each methodology What is obtained at the end of the process is the disturbing potential (Eq. (5)) at the station of interest. Details on the mathematical formulation can be found in Işık et al. (2021) and Ribeiro, Guimarães and Camargo (2022).

As a way of inferring the results arising from the possible ways of obtaining the potential values and, consequently, heights linked to the IHRF, an investigation was carried out at BRAZ station. Since it has a homogeneous distribution of gravity stations, a connection with leveling (Level Reference - RN 2369V), i. e. BRAZ station is connected to the Brazilian Geodetic System. Thus, normal heights linked to W_0 were estimated in different ways: from the average of recent GGMs (as illustrated in Figure 6), using current quasi-geoidal models - in addition to GEOID2021, the model of the state of Minas Gerais (QuGeMG) (GUIMARÃES et al., 2022) and through the methodologies of Hotine and LSC. The results, in terms of normal heights, are shown in Figure 9, which also illustrates IBGE normal heights, linked to the local Datum ($W_{Imbituba}$) and obtained from leveling process (RN 2369V) and using *hgeoHNOR* conversion program (IBGE, 2021). This tool estimates a conversion factor that is added to the value of the ellipsoidal height to obtain the normal height.

Figure 9 – Normal Heights from BRAZ station and RN 2369V station obtained from different models and methodologies and referred to W_0 and $W_{Imbituba}$, respectively (in meters).

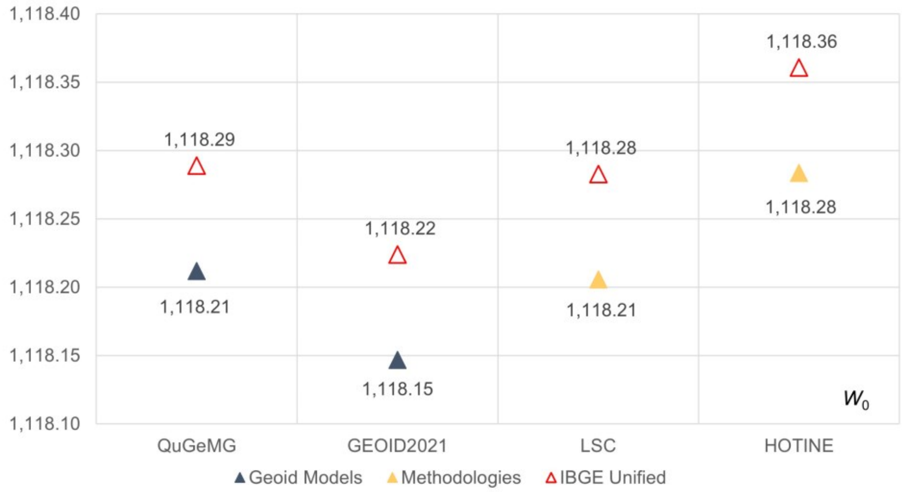


Source: The authors (2022).

As shown in Figure 9, the difference between the lowest value (1118.05 m) and the highest value (1118.28 m) is 0.23 m. The QuGeMG model was computed using LSC methodology, having the same value (1118.21 m) for the result estimated with the LSC. Although QuGeMG and GEOID2021 models have the same input data (gravity stations, GGM and DTM), the GEOID2021 was computed using Stokes Integral via Fast Fourier Transform (FFT). Thus, even if the input data are the same, different computation methodologies may result in different results (see the example in Guimarães et al. (2014)). In addition, a relevant point in the computation process is the computational tools. It is well known that each researcher or group has its computer programs. In this case, even if the methodology and input data are the same, the results found by two research groups, for example, may be different since each uses its tools.

Figure 9 also presents an "offset" between heights linked to W_0 and $W_{Imbituba}$. However, the third option for computing the potential values, shown in section 3.1, deals precisely with the unification and densification of the IHRF concerning the existing vertical systems. Sánchez and Sideris (2017) presented some values for W_{0DVL} in South America, and for the case of the Imbituba Local Datum, the computed value is 62,636,849.61 ± 0.18 m^{2}s^{-2} and the "datum parameter" (δW_{0DVL}) is 3.79 ± 0.18 m^{2}s^{-2} (0.387 ± 0.018 m). In this sense, from the W_P obtained in each of the models and methodologies presented in Figure 9 (except for the GGMs solution), the C_{PDVL} was computed according to Eq. (7). Subsequently, the C_P^{IHRF} value was obtained (connected to the IHRF). Finally, the height value was computed by dividing C_P^{IHRF} by the average value of the normal gravity (red triangles in Figure 10). In this experiment, it was considered the height value of station RN 2369V.

Figure 10 – BRAZ station normal heights (blue and yellow triangles) and RN 2369V (red triangle) are referred to W_0 (in meters).



Source: The authors (2022).

In Figure 10, the "datum parameter" is applied to the station RN 2369V, and the heights of new values are linked to the IHRF. A difference of 0.07 to 0.08 m is observed concerning the heights obtained by the four methodologies. This difference can be explained by the combination of systematic and random errors arising from the geometric leveling procedure, the GNSS ellipsoidal height in the gravity stations, the gravity values, the DTM, and the GGMs errors of omission and commission.

4 FUTURE PERSPECTIVES AND FINAL REMARKS

Establishing and maintaining a geodetic infrastructure aiming at future realizations of geodetic references are fundamental for all countries. In this sense, there must be governmental viability regarding continuous investments in the implementation and maintenance of geodetic stations over time, as well as in the constant training of resources. As mentioned in section 3, a geodetic reference system that is consistent, accurate, and stable over time returns benefits not only to Geodetic Sciences but also to society (social, environmental, and economic) in the global, regional, and local context.

Regarding the establishment of the IHRF, the next step is to establish the best strategy for countries/regions to compute potential values, aiming at the first realization consistent with each other. One possibility ventilated by the geodetic community would be the implementation of national/regional centers responsible for the computation of strategies (methods, computer programs, and input data) previously defined and standardized. SIRGAS GNSS data processing centers are an example of this structure. After the completion of the establishment of the IHRF from the first realization, another challenge inherent to the geodetic community will be the temporal modeling of height. In this sense, there is an opportunity to investigate the effects that interfere with the variation of heights over time, seeking to propose answers to the questions that will arise.

International connections based on GNSS on leveling stations in border regions, the link of the national vertical network at each IHRF station, and in the Local Datum of each country are recommended for institutions to make possible in the following years, aiming at the first realization of the IHRF. In addition, the gravity densification around each station is essential for the quality of the result of the potential values that will be computed in the first realization. In this way, specialists and institutions responsible for Geodesy in each country should discuss gravity resolution at each IHRF station, as Sánchez et al. (2021) recommended.

Regarding the national scenario, Brazil has been involved in activities related to the IHRF, either through institutions that produce geodetic information or through specialists presenting their investigations at events and participating in working groups on the subject. Efforts have been made to improve the geodetic infrastructure. Concerning the results of the experiments presented, heights computed using only global geopotential models are not supported now. There was no GGM that was better adjusted about the average of the GGMs (Figure 6), in addition to the result of the comparisons with the BRAZ station and the RN 2369V station (Figure 9) being the most discrepant. The results from the GBVP approach, either through existing geoid models or using the CMQ or Hotine's Integral, responded consistently.

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Authors' contribution

The first author contributed to the conceptualization, project administration, acquisition of funding, and writing (initial draft). The second and third authors contributed to the manipulation of the data, the computational process of the solutions, and the statistical analysis. The fourth author helped in the computation of one of the solutions presented. The last author contributed to the visualization (generation of figures), and the analysis of the infrastructure of the IHRF stations in the country. All co-authors participated in the critical review and comments.

Conflict of interest

The authors declare that there are no conflicts of interest.

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Biography

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