Orbit Improvement and Generation of Ephemerides for the Global Positioning System Satellites: A Summary

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ABSTRACT

The ephemerides that describe the orbits of the Global Positioning System (GPS) satellites represent the solution of the equations of motion of the satellites. They require initial conditions (position and velocity) and a model which describes the forces that govern the motion of the satellites. A small offset in the initial conditions may cause hundreds, or even thousands, of metres of error in satellite position after a few days of integration. To avoid this problem, the initial conditions, plus some parameters of the force field, must be adjusted through a process known as orbit improvement. The improved initial conditions can then be used for the generation of the post-fitted ephemerides. This paper focuses on the orbit improvement and generation of ephemerides for GPS satellites. For our analysis, we processed a network of North American stations contributing to the global network of the International GPS Service for Geodynamics (IGS). To obtain a measure of accuracy of the geodetic network solution we have compared the resulting baselines with published International Earth Rotation Service Terrestrial Reference Frame ITRF92 values, and the improved orbits with the IGS orbits.

RESUMO

As efémerides que descrevem a órbita dos satélites GPS representam uma solução particular das equações de movimento. A solução destas equações requer condições iniciais (posição e velocidade, agrupados no chamado vetor de estado inicial) e um modêlo que descreva as forças que governam o movimento dos satélites. As condições iniciais têm que ser consistentes com a órbita a ser gerada. Uma pequena diferença nas condições iniciais podem acarretar em erros na posição dos satélites na ordem de quilômetros após alguns dias de integração. Para evitar-se este problema, as condições iniciais, bem como alguns parâmetros do modelo de força (por exemplo, os parâmetros da pressão da radiação solar) devem ser ajustadas dentro de um processo aqui chamado de determinação de órbitas. Neste contexto, as condições iniciais e os parâmetros do modelo de força constituem os parâmetros orbitais. No procedimento de determinação de órbitas, os parâmetros orbitais são estimados usando-se observações coletadas por estações cujas coordenadas sejam conhecidas, ou que sejam estimadas junto com os parâmetros orbitais. As condições iniciais ajustadas podem então ser utilizadas para a geração das órbitas dos satelites GPS. Este artigo se ocupa da determinação e geração de órbitas para os satelites GPS. As órbitas são determinadas usando-se dados de uma sub-rede do Serviço Internacional GPS para a Geodinâmica (IGS), composta por estações no Canadá e EUA. A partir desta solução, órbitas regionais são geradas, e comparadas com as órbitas do IGS. Uma avaliação da precisão externa do ajustamento desta rede é possivel através da comparação das bases ajustadas com os valores publicados no referencial ITRF92.

1 Introduction

The orbit of a satellite is the solution of a second order differential equation system, known as the equations of motion. The equations of motion can be numerically integrated provided initial conditions at an initial time t_0 are given. The initial conditions are a vector composed of the initial position and velocity of the satellite, or its equivalent osculating Keplerian elements, at the initial epoch. In this paper, this vector of initial conditions is referred to as the state vector. The solution of the equations of motion yields a set of satellite positions and velocities, at any other time, as a function of these initial conditions.

The equations of motion describe the motion of a satellite in two parts: (1) an elliptical orbit, in which the satellite is under the influence of the central part of the earth's gravitational field only, and (2) some perturbing accelerations which cause departures from this elliptical orbit. The perturbing accelerations are caused by: the non-central part of the earth's gravitational field; gravitational effects of the moon, the sun, and other celestial bodies; the direct and indirect effects of solar radiation pressure; the atmospheric drag effect; ocean and earth tides; relativistic effects; electromagnetic effects; thruster firings and out-gassing. If these perturbations were perfectly modelled, the integrated orbit would pinpoint the satellite position at any given time.

The three sources of error in orbit determination are: (1) the numerical integration technique, coming from the stability of the integrator itself or from the numerical integration step size; (2) the force model used; and (3) the initial conditions (a small offset in the initial conditions may cause hundreds or even thousands of metres of error after a few days of integration). These errors can be reduced by: (1) choosing a stable integrator that makes use of a step size large enough to save computing time and to avoid integration errors; (2) adopting a sophisticated force model that accounts for all significant perturbations; and (3) improving the initial conditions with respect to observations to the satellite, a process known as orbit improvement.

In the research described in this paper, we concentrate on GPS orbit improvement. By orbit improvement, we are talking about the procedure in which orbital parameters of a satellite (initial state vector and solar radiation pressure parameters) are estimated using observations to this satellite collected by stations whose coordinates are known, or estimated together with the satellite's orbital parameters. For the work reported in this paper, we used a regional GPS network. The technique of orbit improvement helps us to obtain better results in the network adjustment by allowing the orbital parameters to "learn" from the satellites' trajectories defined by the observations and, regarded as extra parameters in the adjustment, helps to absorb possible mis-modellings of the observations. The numerical integration technique and the force model used in this orbit improvement analysis are capable of overcoming the error sources 1 and 2 enumerated above.

2 Adopted Model

In order to carry out the objective of this paper, a model was adopted. This model uses: the geopotential contribution represented by the GEM-T3 model [*Lerch et al.*, 1992] up to 8th degree and order; the sun and moon regarded as point masses according to *Rizos & Stolz* [1985]; the direct and y-bias effects of the solar radiation pressure [*Beutler et al.*, 1986]; the solid earth tides [*Rizos & Stolz*, 1985]; and, the relativistic effect [*International Earth Rotation Service*, 1992]. The effects caused by the earth's reflectivity, ocean tides, atmospheric drag, satellite maneuvering and gravitational field of the planets were disregarded.

The solution of the equations of motion requires that the numerical integration be carried out in an inertial coordinate system (ICS). The adopted ICS for the numerical integration of the equations of motion is the true right ascension (TRA) system at a reference epoch t_0, which is the initial epoch of the equations of motion.

The integration techniques applied was the Störmer-Cowell methods, of 11th order [*Velez & Maury*, 1970]. The starting values required by these methods were computed following *Velez & Maury* [1970].

3 Results

For the present analysis, we used GPS data covering the full 24 hours of day 003 (GPS week 730) collected by 8 IGS stations. Figure 1 shows the geographical distribution of these stations. They are: Algonquin (ALGO), Penticton (DRAO), Fairbanks (FAIR), Goldstone (GOLD), Pie Town (PIE1), Richmond (RCM5), Saint John's (STJO) and Yellowknife (YELL). This network was used to form

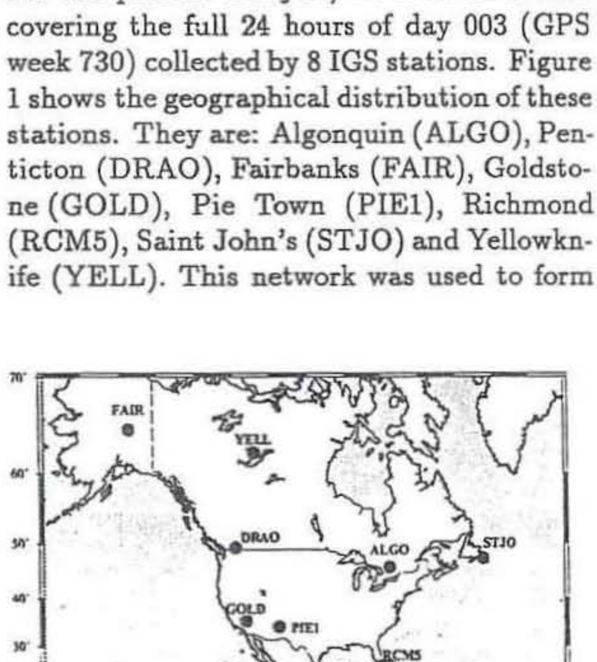

Figure 1: North-American network (based on IGS stations).

the baselines ALGO-STJO, ALGO-PIE1, GOLD-PIE1, PIE1-RCM5, GOLD-DRAO, FAIR-DRAO and YELL-DRAO. The criteria for selecting these baselines were: first, maximum number of double-differences; second, shortest baseline length. The station coordinates are defined in the ITRF92. We have followed the IGS choice of fiducial stations [*Kouba*, 1993]

The orbit improvement was carried out using the new version of the Differential Positioning Program package (DIPOP) [*Vaníček el al.*, 1985], which is capable of handling observations from different baselines simultaneously allowing for the full mathematical correlation between baselines to be taken into account [*Santos*, 1995].

The accuracy of the adjusted station coordinates was measured by comparing the components of the baselines with their published ITRF92 counterparts. A summary of the accuracy, by means of the relative error in baseline length, is shown in Figure 2. The average relative error is 2.27×10^{-8}; the lowest is equal to 2.57×10^{-10}; and the highest is equal to 4.76×10^{-8}.

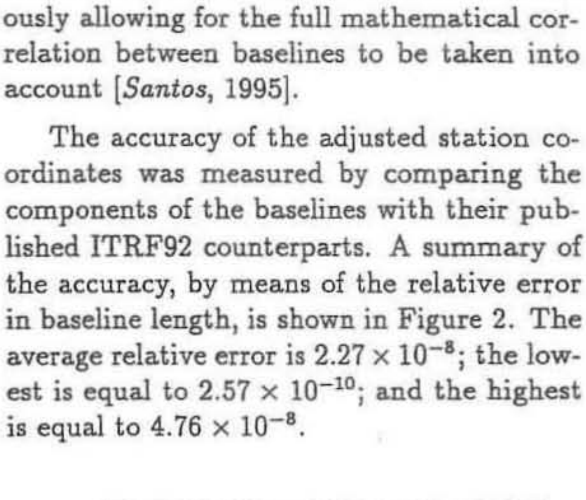

Figure 2: Relative error in baseline length.

The set of improved initial satellite orbits conditions was used to generate the postfitted (improved) ephemerides. These ephemerides were then compared with the published IGS orbits, regarded in this study as a benchmark. The differences, termed the "orbital residuals", were expressed in radial, along-track and cross-track components in a satellite-centered coordinate system. Figures 3 and 4 depict the orbital residuals for satellite PRN 28. Due to the regional extent of the North American network, the GPS satellites have not been observed continuously by all stations throughout the observation session. This lack of simultaneous observations for a particular satellite for a certain period of time results in a larger orbital residual for the period during which the satellite was not observed. The orbital residuals shown point out the difference between the strategies used to generate the orbits being compared: the IGS orbits are generated based on a global network whereas the orbits we have generated come from a regional network.

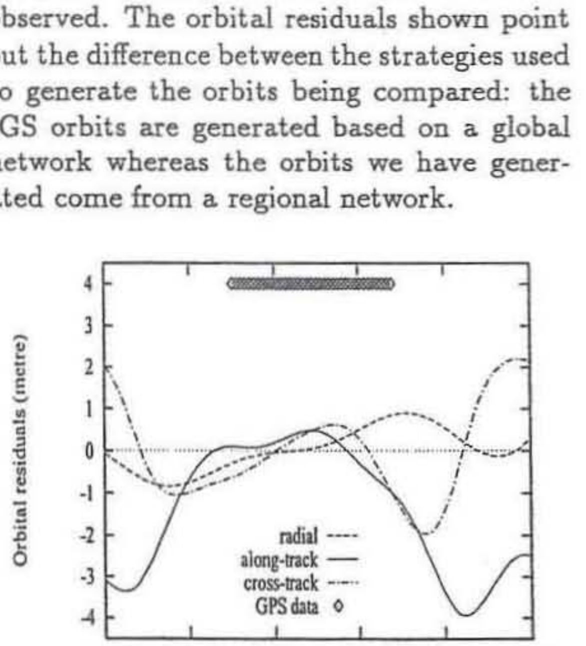

Figure 3: Orbital residuals for PRN 28 – the whole day.

4 Conclusions

The technique of orbit improvement with consequent generation of ephemerides for GPS satellites has been applied in this paper. A test orbit improvement was carried out based on a regional network composed of some North American IGS stations. The results of this combined adjustment of stations and orbits was assessed by using the published ITRF92

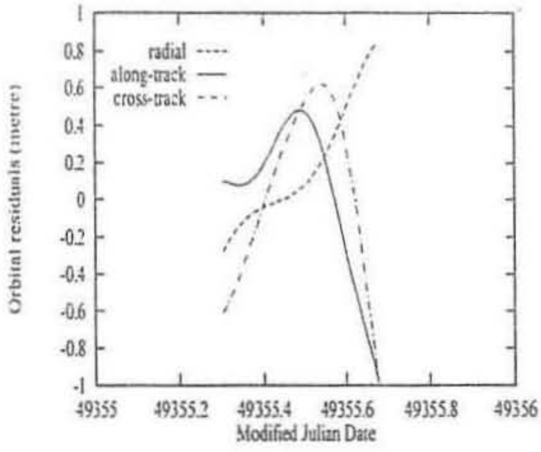

Figure 4: Orbital residuals for PRN 28 – data coverage only.

coordinates and the IGS orbits as benchmarks. Baselines with relative error of the order of 0.02 ppm were obtained. The generated regional ephemerides agree with the IGS at the single metre level. The comparison with the IGS orbits also shows that regional orbits can be of sufficiently good quality for the period of time when observations are collected.

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