An Assessment of the Effect of Mathematical Correlation on GPS Network Computation: A Summary

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ABSTRACT

This paper presents a summary of a study of the effect of mathematical correlations on GPS network data reduction using carrier phase double difference observations. For our analysis, a network with baselines of hundreds of kilometres in length was processed applying three distinct modes of observation correlation: (a) the correlations were ignored, (b) only the correlations within baselines were taken into account, and (c) all correlations, including those between baselines, were considered. This analysis used both broadcast and post-fitted orbits. It is shown in this study that the proper modelling of mathematical correlation typically yields a more realistic formal error estimates and better reliability of baseline component estimates.

RESUMO

Este artigo descreve um estudo sobre o efeito da correlação matemática no processamento de redes GPS, usando-se observações de dupla diferença de fase. Para tal, uma rede, composta por bases com comprimento de centenas de quilômetros, foi processada utilizando-se 3 modalidades distintas de correlação: (a) ignorando-as, (b) considerando somente aquelas entre as observações, e (c) considerando as correlações entre as bases, sendo esta modalidade a mais rigorosa. Este processamento utilizou-se de órbitas transmitidas e precisa, o que permite uma comparação entre a qualidade destes dois tipos de órbitas. Verificou-se que a correta modelagem das correlções matemáticas propicia, de modo geral, uma estimativa da precisão (desvio padrão) mais realista, bem como melhores resultados. Ignorando-se as correlações resulta em precisões irrealmente altas. Uma correta estimativa da precisão é importante em trabalhos de densificação de redes, e integração entre redes terrestres e GPS, bem como no processamento tri-dimensional de redes GPS.

1 Introduction

Two types of correlations affect the Global Positioning System (GPS) double difference carrier phase observations: the mathematical, and the physical correlations. The mathematical correlation is created when, for the sake of removing common errors and reducing partially correlated errors, the double difference observable is formed. The physical correlation is a consequence of the environment common to the observations, making them spatially and/or temporally correlated. The physical correlation is usually not taken into account.

The correlations affecting the double difference observations are accounted for via the covariance matrix of the observations. For a baseline, the mathematical correlation yields a block diagonal structure for the observation's covariance matrix; including the physical correlations, results in a fully populated covariance matrix. Taking into account the correlations yields better estimates and more realistic formal errors. It also allows an easier ambiguity resolution. Realistic formal errors are important for tasks of GPS network densification and integration between GPS and terrestrial networks, as well as for the 3-dimensional processing of GPS networks.

Earlier evaluations of the effect caused by the mathematical correlations in GPS network processing reported in the literature (e. g., *Vaníček et al.* [1985], *Beutler et al.* [1987], *Hackman et al.* [1989], *Hollmann et al.* [1990]) have been based on relatively small networks (with baselines of up to 60 km). They typically found little difference in the estimated coordinate values whether correlations were ignored or taken into account.

El-Rabbany [1994] investigated the effect of physical correlation on baseline determination and accuray estimation in GPS differential positioning. Among the several important conclusions drawn in his study, we quote: that the use of a scale factor to scale the overly optimistic covariance matrix was found to be inappropriate and that the physical correlation is typically inversely proportional to both observation sampling rate and baseline length. In this study, we used baselines of hundreds of kilometres in length and an observation sampling interval of 120 seconds. The physical correlation was disregarded and its effect assumed to be neglegible.

There are 3 basic ways of coping with the mathematical correlation: (1) to use the undifferenced carrier phases, which are mathematically uncorrelated, (2) to decorrelate the double difference measurements of one epoch by means of Cholesky decomposition [*Goad & Müller*, 1988], or (3) to keep the double difference untouched and compute the corresponding covariance matrix pertaining to one epoch directly from the differencing operator matrix used to form the double differences from the undifferenced observations [*Beutler et al.*, 1987]. From the mathematical point of view, these approaches are similar. For the analysis reported in this paper, we use the third approach.

2 Mathematical Correlation

The double difference carier phase observable has found great use in GPS computations due to the fact that it is capable of removing or eliminating errors and biases affecting the original (undifferenced) carrier phase observations such as satellite and receiver clock errors, atmospheric effects, and orbital biases [*Langley*, 1993]. A consequence of doubly differencing the carrier phases is that the observations become mathematically correlated.

When processing data from a network occupied by GPS receivers, 3 approaches can be applied for handling the mathematical correlation: (a) ignore it, (b) consider its effect within each individual baseline (the between-satellite correlation), or (c) consider its effect both within and between the baselines. Approach (c) is the most rigorous one, but applies only for simulteaneously observed baselines; approaches (a) and (b) do not have such a requirement and can be applied in the procesing of single baselines or networks.

Let the double difference observations for one epoch be represented as:

$$\nabla\Delta\underline{\Phi} = \underline{R}\ \underline{\Phi}, \tag{1}$$

where $\underline{R}$ is a matrix with entries 0's, +1's and -1's and $\underline{\Phi}$ is the vector of undifferenced carrier phase observations. Applying the law of propagation of variances [*Vaníček & Krakiwski*, 1986] we arrive at the covariance matrix of the double difference observations:

$$\underline{C}_{\nabla\Delta\Phi} = \underline{R}\ \underline{C}_{\Phi}\ \underline{R}^T, \tag{2}$$

where $\underline{C}_{\Phi}$ is the covariance matrix of the vector $\underline{\Phi}$. The undifferenced phases are assumed to be uncorrelated. If the mathematical correlation is totally disregarded, $\underline{C}_{\nabla\Delta\Phi}$ equals an identity matrix. If, in a network mode, the mathematical correlations of the double difference observations within individual baselines are considered, the diagonal sub-matrices, one for each baseline, will have a block diagonal structure in $\underline{C}_{\nabla\Delta\Phi}$, and all off-diagonal sub-matrices will be equal to zero. If all mathematical correlations are taken into account, there will be some non-zero elements in the off-diagonal submatrices, each representing correlations between receivers observing the same satellite. It goes without saying that matrix $\underline{C}_{\nabla\Delta\Phi}$ is scaled by the a priori variance factor of the double difference observations.

.The rigorous implementation of the covariance matrix, including the correlations between baselines, adds a great deal of computation to the processing. Efficient methods are needed, such as the one described by *Beutler et al.* [1987].

3 Effect of Mathematical Correlation on Network

The effect of mathematical correlation was assessed using a network composed of 4 stations, namely Goldstone (GOLD), Algonquin (ALGO), Penticton (DRAO) and Pie Town (PIE1). The geographical distribution of this network is shown in Figure 1. Three independent baselines were formed, with station GOLD common to all of them. The baselines, and their respective lengths, are: Goldstone-Algonquin (3402 km), Goldstone-Pie Town (810 km) and Goldstone-Penticton (1556 km). The data set used for the processing shown in this paper covers a period of one full day (day 003 of GPS week 730). We used the network oriented DIPOP software [*Santos*, 1995] for the data processing.

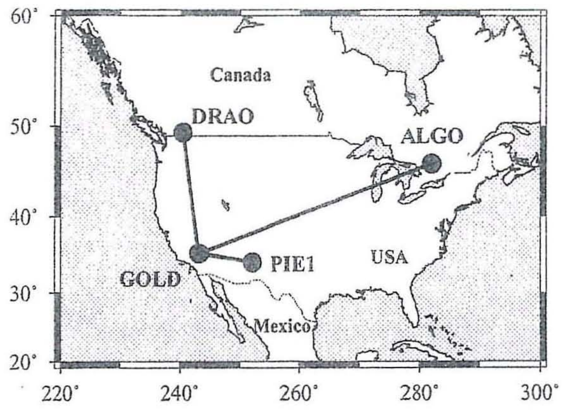

Figure 1: Geographical distribution of the stations.

As far as the mathematical correlation is concerned, the network was processed using the three different modes: (a) totally disregarding the mathematical correlation, i.e., assuming $\underline{C}_{\nabla\Delta\Phi}$ equal to an identity matrix, (b) modelling the mathematical correlation within each baseline, and (c) modelling the mathematical correlations between baselines. The results were compared in order to assess

Table 1: Formal errors (in millimetres) using broadcast ephemerides.

Station: Algonquin				
Corr. mode	Day 3			
	σ_ϕ	σ_λ	σ_h	σ_ℓ
(a)	22	55	111	44
(b)	22	61	123	49
(c)	27	85	150	64
Station: Penticton				
Corr. mode	Day 3			
	σ_ϕ	σ_λ	σ_h	σ_ℓ
(a)	27	37	87	24
(b)	28	41	94	26
(c)	41	49	128	34
Station: Pie Town				
Corr. mode	Day 3			
	σ_ϕ	σ_λ	σ_h	σ_ℓ
(a)	14	38	93	40
(b)	14	42	99	44
(c)	16	49	121	50

Table 2: Formal errors (in millimetres) using post-fitted ephemerides.

Station: Algonquin				
Corr. mode	Day 3			
	σ_ϕ	σ_λ	σ_h	σ_ℓ
(a)	1.9	4.9	9.7	4.0
(b)	1.8	5.0	10.0	4.2
(c)	2.2	6.9	12.0	5.3
Station: Penticton				
Corr. mode	Day 3			
	σ_ϕ	σ_λ	σ_h	σ_ℓ
(a)	2.2	2.8	7.1	2.1
(b)	2.2	2.8	7.0	2.0
(c)	3.2	3.3	9.6	2.6
Station: Pie Town				
Corr. mode	Day 3			
	σ_ϕ	σ_λ	σ_h	σ_ℓ
(a)	1.0	2.8	7.9	2.9
(b)	1.1	3.0	8.0	3.1
(c)	1.3	3.5	9.9	3.5

how the formal errors (precision) of the different solutions behave. Also, the adjusted baselines were compared to the corresponding published ITRF92 values, in an attempt to gauge the accuracy of the three distinct modes.

The effect of mathematical correlation in the precision estimation of the network solution, for the three correlation modes, is summarized in Table 1 for a solution using the broadcast ephemerides, and in Table 2 for a solution using the post-fitted ephemerides, for all 3 baselines. The post-fitted ephemerides were produced as a batch orbital solution covering the day used [*Santos*, 1995]. It can be seen that without the mathematical correlation the formal errors are, most of the time, smaller than those obtained using the mathematical correlations. Also, the formal error estimates using the broadcast ephemerides are larger than those obtained using post-fitted ephemerides.

The effect of mathematical correlation on the final results is seen in Figure 2 for the solution using the broadcast ephemerides, and in Figure 3 for the solution using the post-fitted ephemerides. These figures show the relative error in baseline length by comparing the baselines resulting from the adjustment with the published ITRF92 values [*Altamini & Boucher*, 1994]. Again, the three correlation modes (a), (b) and (c) were used. It can be clearly seen that the results improve with the proper modelling of the mathematical correlation by as much as 50%.

Another interesting feature to be noticed is the difference in the quality of the solution obtained using the broadcast and the post-fitted orbits. According to the rule of thumb [*Vaníček et al.*, 1985], the broadcast orbits used have an orbital bias in the 3 m range; whereas, the post-fitted orbit used is certainly below the single metre level. The use of post-fitted orbits results in a final solution one order of magnitude more accurate than that using broadcast orbits.

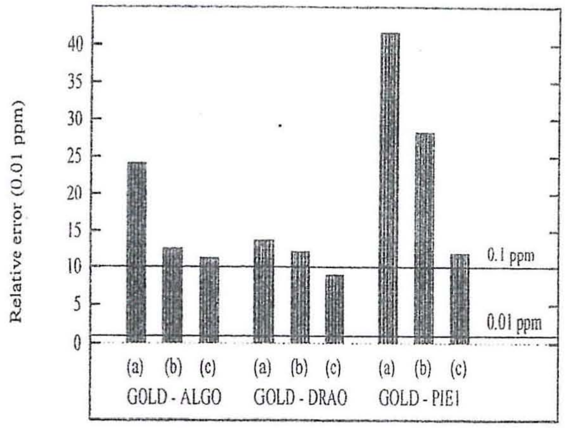

Figure 2: Relative error in baseline length, for correlation modes (a), (b) and (c) (using broadcast ephemerides).

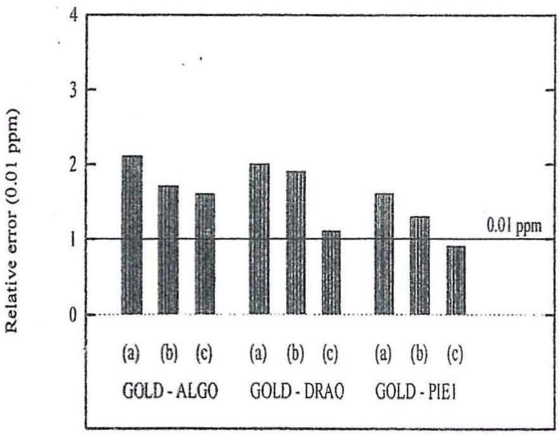

Figure 3: Relative error in baseline length, for correlation modes (a), (b) and (c) (using post-fitted orbits).

4 Conclusions

This paper gives a summary of an investigation into the effect of mathematical correlation in the processing of a data from a GPS network. The modes of mathematical correlation considered were: (a) correlations were ignored, (b) correlations within baselines were taken into account, and (c) correlations between baselines were accounted for. The conclusions are: (1) the proper modelling of mathematical correlations yields a more realistic accuracy estimation, typically, $\sigma_{(c)} > \sigma_{(b)} > \sigma_{(a)}$; (2) better results are obtained using modes (c), (b), (a), in this order; (3) the effect of the quality of the orbit used seems to be greater than that due to the proper modelling of the mathematical correlation itself. We intend to follow up this study with further testing of the effect of mathematical correlation on networks with a mixture of baseline lengths.

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