

## DYNAMIC PROGRAMMING APPLIED TO AN OCEANOGRAPHIC CAMPAIGN PLANNING

*Programação Dinâmica Aplicada ao Planejamento  
de uma Comissão Oceanográfica*

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### ABSTRACT

Oceanographic survey planning, as hydrographic survey planning, involves optimizing the path of a survey platform. Such a platform must visit a set of target geographic positions, each of which must be visited a single time before the platform returns to the initial harbor. This problem is similar to the Traveling Salesman Problem (TSP), though distinct enough to warrant a dedicated solution. Here this problem is modelled as a sequential optimization problem and is optimally solved using a dynamic programming algorithm.

**Keywords:** Oceanographic campaign, Dynamic Programming, Traveling Salesman Problem.

### RESUMO

O planejamento de levantamentos oceanográficos, e também de levantamentos hidrográficos, envolve a otimização de uma rota para a plataforma que efetuará a coleta de dados. Esta plataforma (navio ou embarcação) necessita visitar, um conjunto determinado de posições geográficas, passando por cada uma delas uma única vez, antes de retornar ao porto inicial. Este problema é similar ao Problema do Caixeiro Viajante, apesar de suficientemente distinto para merecer uma solução dedicada. Neste trabalho, o problema é modelado como um problema de otimização sequencial e é resolvido por meio de um algoritmo de programação dinâmica.

**Palavras chaves:** Comissão Oceanográfica, Programação Dinâmica, Problema do Caixeiro Viajante.

## 1. INTRODUCTION

The Brazilian Navy (Marinha do Brasil), through the Development Plan of the Project Ocean (Plano de Desenvolvimento do Projeto Oceano), periodically collects scientific data along the Brazilian coast. This sampling provides data to the National Oceanographic Database (Banco Nacional de Dados Oceanográficos - BNDO) and assists researches at Brazilian public universities. For instance, academic researchers from partner institutions can take part in research cruises called “OCEANO” oceanographic campaigns.

The feasibility of an oceanographic research depends on proper planning and a key factor is to keep costs as low as possible. The importance of cost is greatly accentuated in a negative economic scenario, in which spending cuts impact the operation of data sampling (e.g. Anon. 2008). In this work, our objective is to optimize the planning of oceanographic or hydrographic data survey campaigns in terms of operational cost, which involves financial, human and material aspects. From practical viewpoint, this cost can be summarized by the number of days that the Brazilian Navy need to

commit an oceanographic vessel to this kind of activities.

In particular, we intend to determine the route with the shortest execution time to perform the oceanographic study. For this purpose, the problem is modelled as sequential decision problem and solved using a dynamic programming algorithm. This approach allows us to obtain the optimal solution for a typical midsize campaign, which is often found in real-world applications.

Here the data set used corresponds to the Oceano E campaign, which collects oceanographic and meteorological data from the geographic region corresponding to an area between the Brazilian states of Sergipe and Espírito Santo, bounded by the parallels  $10^{\circ}40.7'S$  and  $19^{\circ}40.0'S$  and the meridians  $033^{\circ}55.8'W$  and  $039^{\circ}30.0'W$ . To collect the data, samplings are obtained from 112 oceanographic stations arranged by a perpendicular line to the coast. A profile is a set of stations arranged by a perpendicular line to the coast. The maximum distance from the coast reaches 225 nautical miles (approximately 440 km). Figure 1 shows the layout of the oceanographic stations.

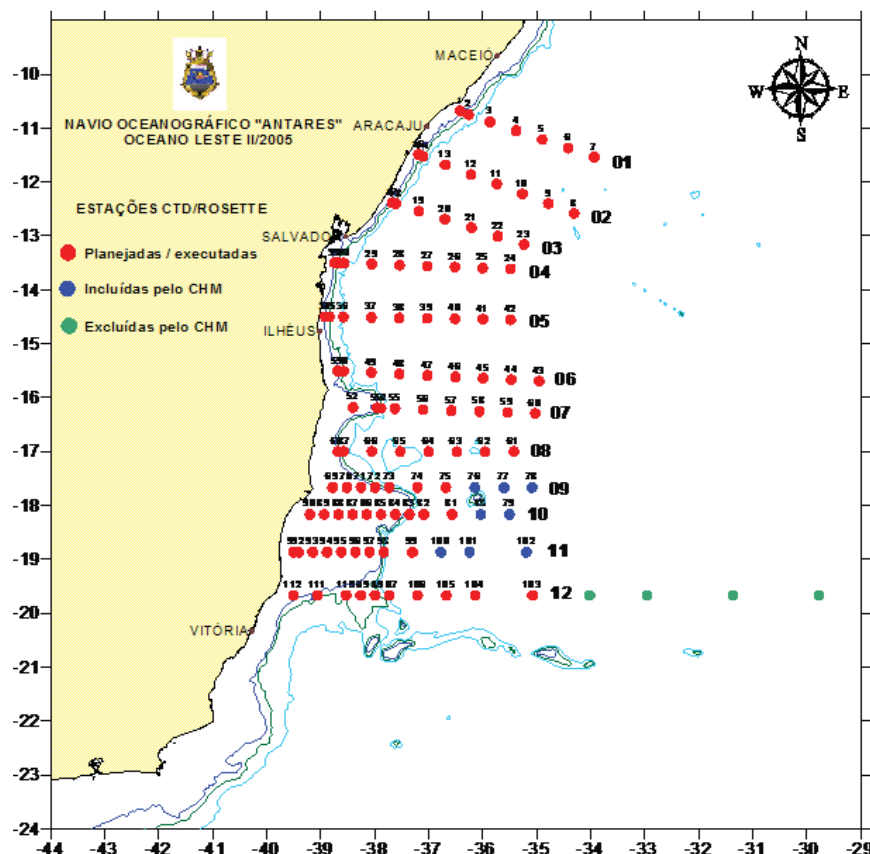


Fig. 1 - Scheme of oceanographic stations on a map of the Brazilian coast between Vitória e Maceió.

A complete sampling involves a single visit to each of the oceanographic stations and eventually returning to the initial base. Therefore, this problem can be explained as a variant of the classic Traveling Salesman Problem (TSP): given a set of targets to be visited and the costs associated with the displacement between all pairs of targets, the aim is to find the lowest cost to traverse all targets and return to the starting point. Applegate *et al.* (2006) presents a historical perspective of the TSP and solution techniques.

However, in the present situation the edges that connect vertices of the same profile must be used, besides the unique and mandatory visit to all vertices. The proposed problem here also involves operational constraints which refer to the autonomy to stay at sea without logistic support (e.g. fuel and food). The studied problem then has specific characteristics that distinguish it from a classic TSP and require us a specific formulation.

Notwithstanding the existence of polynomial algorithms for the solution of special TSP instances (Çela *et al.* 2012, Deineko *et al.* 1994), the classic version of the TSP belongs to the class of NP-complete problems (Papadimitriou, 1977). Due to its strong connection with the TSP, the oceanographic planning problem has then difficulties of at least the same magnitude.

Proposed in the 1950s, dynamic programming (Bellman & Lee 1984) is a suitable technique to treat multi-stage decision problems, such as the problem studied in this article. It provides experts an easy explanation of the model and makes the corresponding implementation accessible even for modest computational resources (Bellman, 2003). Lew & Mauch (2007) discussed important aspects of modelling and computational implementation for dynamic programming applications. There are several examples of dynamic programming in the maritime field, from stochastic modelling to determine the optimal habitat of species (Kirby *et al.* 2000) to the routing of ships.

Shao *et al.* (2011) use dynamic programming to optimize fuel consumption during a sea journey taking also into consideration the crew welfare based on weather forecasts and particular characteristics of a ship.

Millar & Russell (2012) analyses the dispatch of fishing inspection patrols. Although it models the routing of vessels, this study is different from our problem because it contemplates the selection of several targets for visitation constrained by cost or time. Furthermore, this approach does not consider a refuelling port different from that of the origin, which is a crucial condition in our application.

## **2. THE PROPOSED MODEL**

In this section, the model -the objective function and constraints- is discussed. The structures of input and output data are also presented.

### **2.1 Structure**

Our decision problem is supported by a fully connected graph  $G=(V,E)$  whose set of vertices  $V=\{A,...,Z\}$  has 26 geographic points. Only the start and the end point of each profile, the base port and the refuelling port are considered vertices.

The problem is then modelled by a sequence of stages. Each stage represents a step where a decision about the next point to be visited is required. This decision has to consider the current position, the set of vertices already visited and the actual autonomy of the vessel. In our model there are 14 stages. This fact implies more than  $2.5 \times 10^{13}$  possible solutions. Being a symmetrical problem -distances and travel times between each pair of points do not depend on the direction of travel- a path has the same cost of its inverse one. Consequently, the space of possibilities is reduced by half.

In each state, the constraints are defined by the set of vertices not yet visited, which may represent the start of a profile, a refuelling port or the decision to return to the initial port.

When a new profile is chosen, the vessel must execute the profile sampling without interruption. The final position that results from this choice will be the vertex that represents the end of the chosen profile. Note that the decision to proceed to the refuelling port can only be taken after the end of a profile. Recall also that it is only possible to return to the base port after visiting all other vertices.

The cost associated to each edge  $(x_1, x_2)$  in the graph  $G$  corresponds to the time spent

during a travel between the two points  $x_1$  and  $x_2$ . This is an important aspect of this problem because of the oceanographic stations. Delay time is then defined as the cost of executing the corresponding analyses in each oceanographic station of a profile. Then the cost between the start and end points of each profile is not only defined by the geographic distance between them but also by the sum of all delay times associated with the oceanographic stations between them.

Let  $S$  be the current position,  $i$  be the next vertex to be visited (which will be chosen as the starting point for executing a profile) and  $f$  be the vertex corresponding to the end of the chosen profile. The cost of stage  $C_i^n$  that represents the consequence of decision in the stage from is therefore

$$C_i^n = \frac{d_{si}}{SOA} + \frac{d_{if}}{SOA} + \sum t_{if} \quad (1)$$

where:

$d_{si}$  is the geographical distance between  $s$  and  $i$  (current vertex and chosen vertex);

$d_{if}$  is the geographical distance between  $i$  and  $f$  (profile starting point vertex and profile end vertex);

$SOA$  is the standard speed of a survey ship; and  $t_{fi}$  is the delay time for each station belonging to the profile.

Thus, the vertex of stage  $n+1$  corresponds to vertex  $f$  of the previous stage. If the decision is to proceed to refuel or to return to the base port, then  $i$  and  $f$  coincide, and there is no delay time.

The goal is to minimize the total time using a recursive function with forward formulation, as shown below:

$$\begin{aligned} F_0(0) &= \{0\} \\ F_1(i)_{i \in S_0} &= \{F_0(i) + C_i^1\} \\ &\vdots \\ F_k(i)_{i \in S_{k-1}} &= \{F_{k-1}(i) + C_i^k\} \end{aligned} \quad (2)$$

This formulation consecutively excludes the vertices already traversed from the set of possible choices. The return to the home harbor is enforced in the last stage by the expression

$$F_N(i)_{i \in S_{N-1}} = \{C_{base}^{N-1} + F_{N-1}(i)\} \quad (3)$$

We then have that the problem is formulated as

$$\begin{aligned} &\min_{i \in S_{N-1}} F_N(i) \\ &\min_{i \in S_{N-1}} \{C_{base}^{N-1} + F_{N-1}(i)\} \end{aligned} \quad (4)$$

Recall that there are other constraints that must be addressed. Autonomy constraint establishes that a decision is feasible if there is enough autonomy to execute it. It is necessary to test if this condition holds at each stage.

Let  $a_i$  be the autonomy available for the  $i$ -th state. Assuming that the ship is refuelled at a  $k^{\text{th}}$  stage, we then have that

$$\begin{aligned} a_0 &= \text{autonomy} & a_1(i) &= a_0 - C_i^1 \\ & & & \vdots \\ a_k(i) &= a_{k-1} - C_i^k \\ a_{k+1}(i) &= \text{autonomy} - C_i^{k+1} \\ & & & \vdots \\ a_{k+2}(i) &= a_{k+1} - C_i^{k+2} \\ & & & \vdots \\ a_N(i) &= a_{N-1} - C_{base}^N \end{aligned} \quad (5)$$

Note that these conditions must preserve the non-negativity of  $a_i$ .

Other restrictions such as a required sequence of profiles can be added if necessary. For instance, suppose that 2 specific profiles have to be executed consecutively. This constraint can then be replaced by a single profile whose cost should be given by the sum of all delay times and geographic distances corresponding to both profiles.

### 3. IMPLEMENTATION AND RESULTS

The first step to solve the problem is to calculate the delay times as described in Section 2. We assume that the maximum autonomy is 400 hours. This value corresponds to 2/3 of the maximum operational radius of a ship under standard speed as defined in Brasil (n.d.).

Figure 2 depicts the steps of our algorithm. The pre-processing includes the following steps:

- The calculation of the time delay matrix;

- The initialization of the minimum cost obtained, which is considered much higher than is expected for the solution;

- The definition of the autonomy; and  
 - The determination of the set of possible points to be visited.

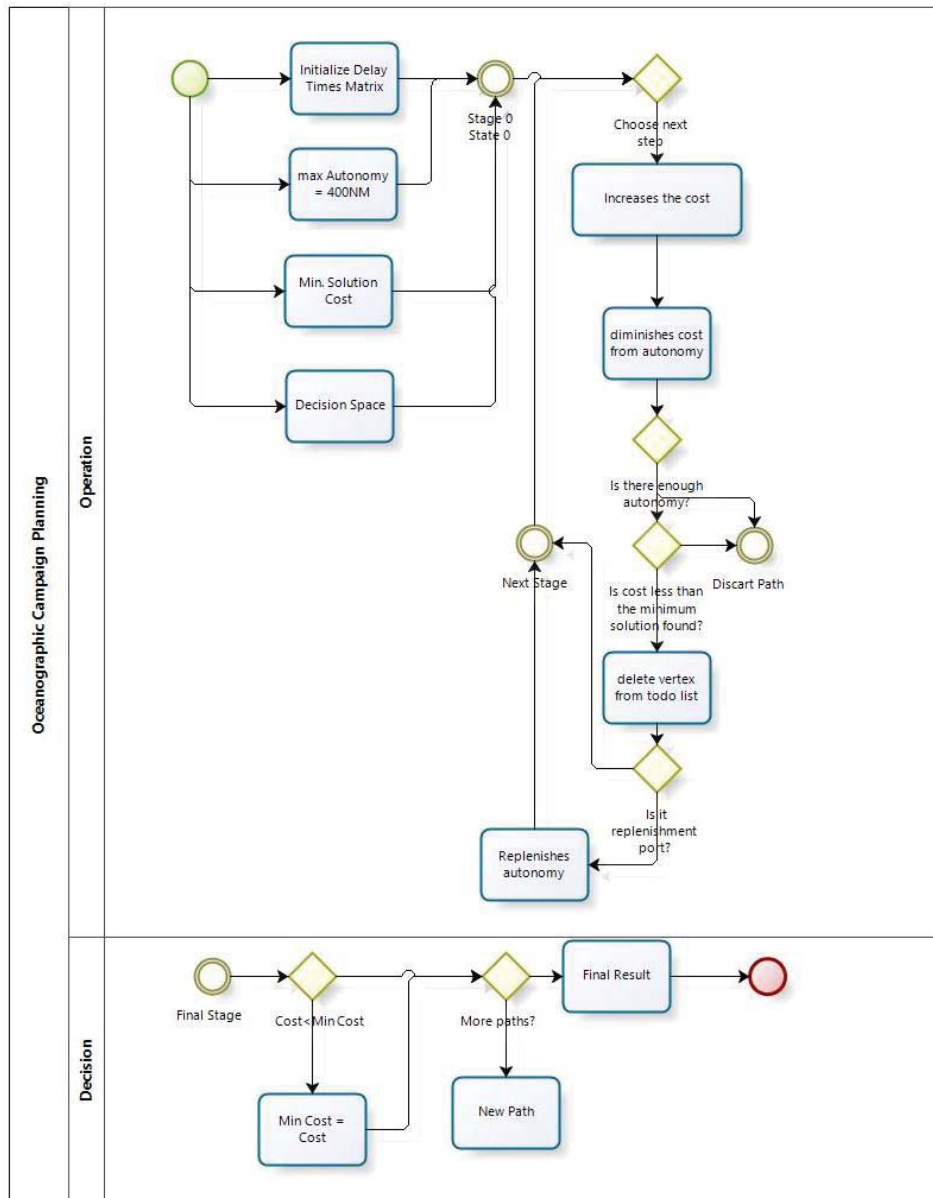


Fig. 2 - Dynamic Programming Algorithm for the Oceanographic Planning Problem.

Next, the program sets the current stage to 0 where the vessel is at the initial port (vertex A). At each subsequent stage, the algorithm needs to decide the best point to visit on the next stage. All points not yet visited are assessed and the one with the smallest cost is selected. For each decision, the chosen point is accepted if the autonomy allows the vessel to go from the current state to it. In this case, the travel time is subtracted from the autonomy time of the vessel. If the decision is to proceed to a refuelling port (vertex Z), its autonomy is reset to its full value.

This process is recursively followed until the end of a cycle.

It is possible to compare the total cost obtained with the minimum total cost recorded in the previous steps. Note that this procedure guarantees that each station is visited at most once and that the result is optimal, i.e., the minimum obtained is the global minimum.

Initially, a deep-search-first algorithm in which the vessel can refuel at any stage was used. However, it was observed that its implementation was extremely time-consuming. Since the



number of stages is fixed, if the refuelling is performed in step  $k$ , there must be only  $N-k$  subsequent possible steps that comprise the unused profiles and the return to base. Using this idea, the following construction proven to be more efficient:

- The refuelling point is fixed at  $k=7$ , and the results are obtained.
- The refuelling point is fixed at  $k=8$ , and the results for  $k=6$  are obtained by symmetry reasoning.
- The refuelling point is fixed at  $k=9$ , and the results for  $k=5$  are obtained by symmetry reasoning.

Figure 3 presents the solutions found by our algorithm for  $k=7,8,9$ . The search for the solution started at  $k=7$  and stopped at  $k=10$  since all candidate solutions do not respect the autonomy criterion. It is then not possible to determine viable solutions to refuelling using more than 9 steps and the space of solutions can be considered completely examined. The best solution has a

total length of 733 hours and is obtained for  $k=7$ . Figure 4 shows it on a map. Note that this solution is one (and not “the”) optimal solution because there may be other solutions with the same cost (for example, the reverse route).

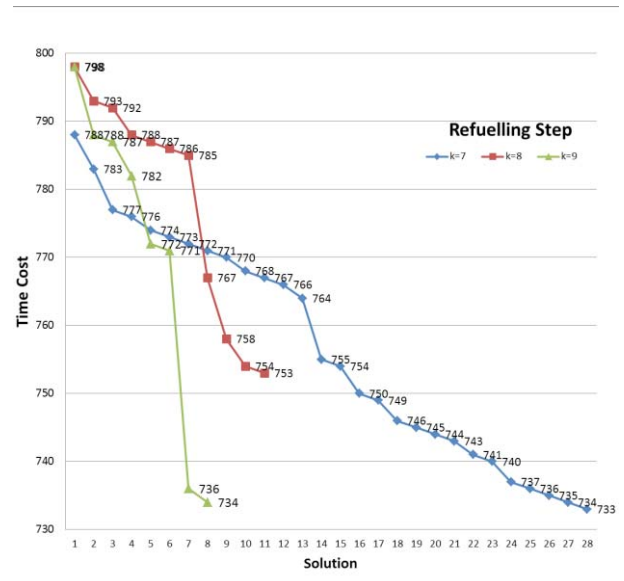


Fig. 3 - Solutions found at each refuelling step.

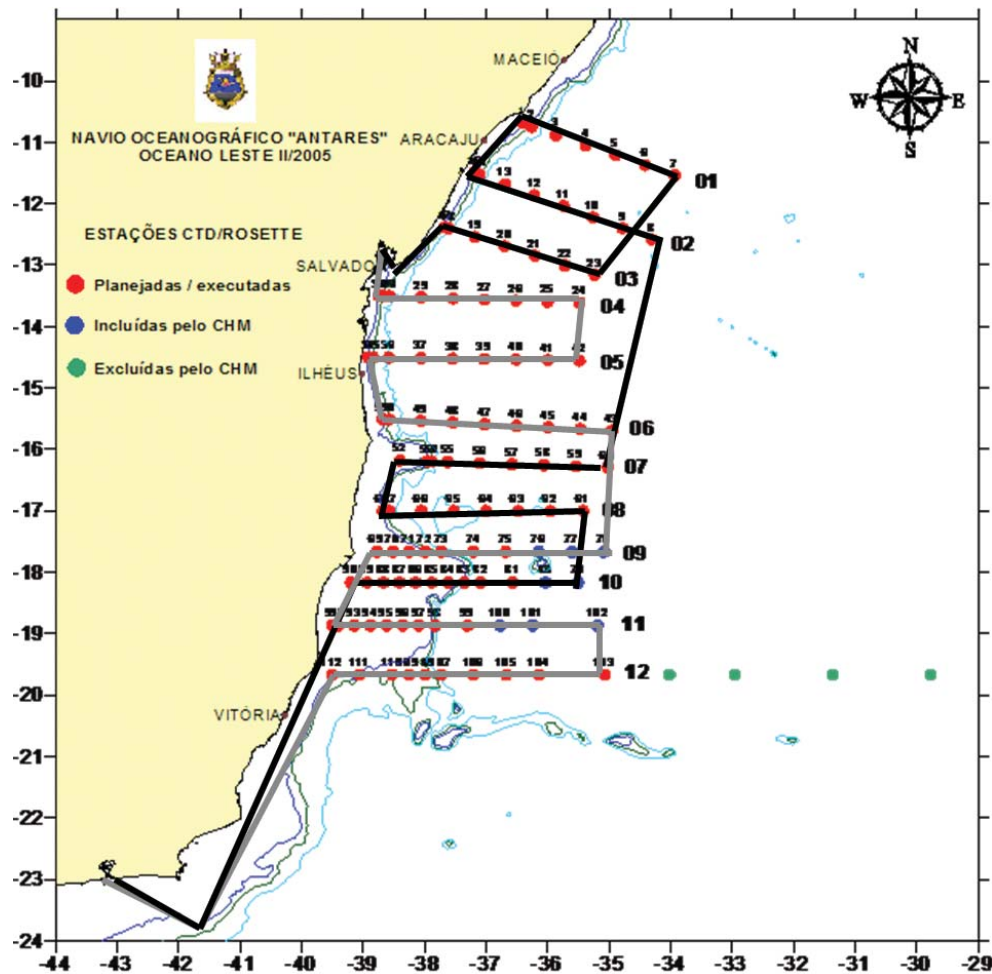


Fig. 4 - Best Solution.

The minimum cost is quite robust in terms of setting the refuelling points. In fact, the higher best cost (753 hours) was found for  $k=8$ . This solution increases the oceanographic travel only by 20 hours (less than 1 day). Note that an optimum travel lasts at least 31 days. However, higher values of  $k$  reduce significantly the computational time to find a best solution. This happens because the autonomy restriction excludes many solutions. Finally, each refuelling point provides a distinct family of tours, which decision makers often appreciate to evaluate.

#### 4. CONCLUSION

In this work, a mathematical model to optimize the cost of an oceanographic travel was developed. A dynamic programming algorithm found not only a best solution but also a set of different viable solutions whose scores are equally good. It allows decision makers to incorporate other political, economic and technical aspects that are not addressed in the mathematical formulation. This range of solutions can also be useful in case of unexpected events that may happen during the mission, such as a bad weather, an evacuation due to healthy complications with a crew member or an operational problem with the ship.

An important aspect of this study is to highlight the rich common background for interchange between Operational Research and Hydro-Oceanographic researches. The authors believe that these synergetic interactions can contribute to improve the solutions of important real-world problems and to encourage the developments of new technologies.

An interesting research topic is to increase the number of possible refuelling ports. The employment of a set of survey vessels dispatched at different times is also another promising extension.

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