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Pedagogical approaches regarding the quadratic equation in 8th and 9th grade Teacher's Manuals

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ABSTRACT

Considering that curriculum materials offer different pedagogical approaches to Mathematics content, in addition to guiding the roles of teachers and students in the teaching and learning processes, this study sought to analyze task-solving strategies and the roles of teachers and students in the approach given to the quadratic polynomial equation in Teacher's Manuals for the 8th and 9th grades. Based on a document analysis of eight Teacher's Manuals from four collections of curriculum materials, the techniques for solving quadratic equations, the tasks related to them, and the corresponding teaching guidelines for teachers were mapped and analyzed. The findings show that, among the 235 tasks analyzed, less complex algebraic techniques predominated, as did tasks positioning students as respondents (54%) and describers (41%), whereas the role of argumenter appeared in only 5% of the tasks.

KEYWORDS: Curriculum Materials. Teacher's Manuals. Quadratic Equation. Role of Teachers. Role of Students.

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Abordagens pedagógicas referentes à equação do 2º grau em Manuais do Professor do 8º e 9º anos

RESUMO

Considerando que materiais curriculares oferecem diferentes abordagens pedagógicas dos conteúdos de Matemática, além de orientarem os papéis de professores e estudantes nos processos de ensino e de aprendizagem, buscou-se analisar estratégias de resolução e papéis dos professores e estudantes na abordagem dada à equação polinomial do 2º grau em Manuais do Professor do 8º e 9º anos. Pautada em uma análise documental em oito Manuais do Professor de quatro coleções de materiais curriculares, foram mapeadas e analisadas as técnicas de resolução de equação do 2º grau, as tarefas relacionadas a elas e as orientações de ensino correspondentes aos professores. Evidencia-se que, das 235 tarefas analisadas, predominam técnicas algébricas menos complexas e tarefas que posicionam os estudantes como respondentes (54%) e descritores (41%), enquanto o papel de argumentador aparece em apenas 5% das tarefas analisadas.

PALAVRAS-CHAVE: Materiais Curriculares. Manuais do Professor. Equação do 2º Grau. Papel dos Professores. Papel dos Estudantes.

Enfoques pedagógicos para las ecuaciones cuadráticas en los Manuales del Profesor de 8º y 9º grado

RESUMEN

Considerando que los materiales curriculares ofrecen diferentes enfoques pedagógicos para el contenido matemático, además de orientar los roles de docentes y estudiantes en los procesos de enseñanza y aprendizaje, el estudio analizó las estrategias de resolución de tareas y los roles de docentes y estudiantes en el abordaje de la ecuación cuadrática en los Manuales del Profesor para 8º y 9º grado. A partir del análisis documental de ocho Manuales del Profesor pertenecientes a cuatro colecciones de materiales curriculares, se identificaron y analizaron las técnicas para resolver ecuaciones cuadráticas, las tareas relacionadas y las correspondientes pautas didácticas para docentes. Los resultados muestran que, de las 235 tareas analizadas, predominan técnicas algebraicas menos complejas y tareas que posicionan a los estudiantes

como respondedores (54 %) y descriptores (41 %), mientras que el papel de argumentador aparece en solo el 5 % de las tareas analizadas.

PALABRAS CLAVE: Materiales Curriculares. Manuales del Profesor. Ecuación Cuadrática. Rol del Profesor. Rol del Estudiante.

* * *

Teaching quadratic equations

Any equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are real numerical coefficients and $a \neq 0$, is a Quadratic Equation. Solving such an equation consists of determining a numerical value for x such that the equality is satisfied. As prescribed by the *Base Nacional Comum Curricular* [National Common Core Curriculum — BNCC] (Brazil, 2017), students' first contact with this equation occurs in the 8th grade, through an introductory approach and in its incomplete form, using two solution techniques: factoring and square root extraction. The complete form and the remaining solution techniques are incorporated into the skills prescribed for the 9th grade.

In its early development, solving the equation consisted of determining two numbers given their sum and product. Throughout the history of Mathematics, different methods of solution emerged. Its teaching is usually conducted in a simplified manner and through tasks with varying levels of cognitive demand.

In the *Parâmetros Curriculares Nacionais de Matemática* [National Curriculum Parameters for Mathematics — PCN] (Brazil, 1998), a set of documents implemented in the 1990s and no longer in force, this content was addressed within the Numbers and Operations strand, with guidelines recommending that it be taught through problem situations, enabling students to represent them using equations and inequalities, assimilate parameters, unknowns, and variables, become familiar with formulas, and understand the rules for solving an equation.

According to the BNCC, "the techniques for solving equations and inequalities, including those in the Cartesian plane, should be developed as a way of representing and solving certain types of problems, rather than as objects of study in themselves" (Brazil, 2017, p. 271). This implies that it may be more productive to propose tasks that lead students to mobilize what they already know and develop strategies for determining solutions, rather than merely manipulating situations repetitively.

Solution techniques are taught in schools through algebraic expressions and manipulations. In a previous literature review (Silva, 2025), we identified different techniques, namely: the conventional method, half-sum and product, alternative method, square and difference method, Viète's method, substitution of variables, Euler's method, differential method or vertex-coordinate method, Fan-Fan or Horner's method, transformation method, among others.

We understand solution techniques as resources involving mathematical properties and relationships used to solve problems. In the case of the Quadratic Equation, such techniques are important to the teaching process, since this process reverberates in learning.

This same importance for the processes of teaching and learning Mathematics also applies to curriculum materials, which are widely used in planning and conducting lessons and can contribute to teachers' and students' learning of the content prescribed in official documents. Teacher's Manuals, in particular, contain teaching guidelines that support teachers in implementing the tasks incorporated into the materials (Rocha, 2025). Regarding this type of equation, the guidelines may present different solution techniques, explain their origins, make explicit the underlying properties and relationships, and indicate possibilities for classroom implementation.

Within curriculum materials, task statements and teaching guidelines, particularly those found in Teacher's Manuals, also include pedagogical approaches that envision the interactions teachers and students may develop in classroom contexts. This means that different roles are envisioned for

teachers and students to assume in the processes of teaching and learning Quadratic Equations. These interactions are important because they shape relationships with the material and define how participants are expected to engage, or even behave, in the processes of teaching and learning.

Although studies exist on the teaching of Quadratic Equations, solution techniques, and Mathematics curriculum materials, investigations that analyze, in an articulated manner, the techniques mobilized in tasks and the roles addressed to teachers and students in Teacher's Manuals remain scarce (Silva, 2025). This articulation is relevant because teaching guidelines and proposed tasks not only present mathematical procedures but also communicate expectations regarding how teachers and students should act in teaching and learning processes. Thus, understanding the pedagogical approaches present in these materials may contribute to broadening the debate on the teaching of Algebra in Middle School.

In this sense, the study presented in this article[3] was guided by the objective of analyzing solution strategies and the roles of teachers and students in the approach to the Quadratic Equation in 8th and 9th-grade Teacher's Manuals, based on the assumption that these Manuals present methodological guidelines and concepts that may influence the practices of planning and conducting lessons, particularly with regard to solution techniques.

Pedagogical Approach

When using curriculum materials as tools to support classroom instruction, teachers read and interpret the teaching guidelines and evaluate and select tasks for implementation. The selection of tasks aims to create conditions that enable students to construct their learning. In making these selections, teachers evaluate not only the underlying content but also the pedagogical approaches.

[3] This article is part of the master's dissertation defended in the Postgraduate Program in Education at the State University of Montes Claros, organized in a multipaper format, written by the first author and supervised by the second author.

According to Remillard and Kim (2020), pedagogical approaches are the expectations and assumptions underlying, particularly, the Teacher's Manual, with the purpose of adapting the learning opportunities available to students in Mathematics teaching practices. In the study reported in this article, we considered the following as pedagogical approaches to mathematical tasks: Quadratic Equation solution techniques; the role teachers are expected to assume in the teaching process; and the role students are expected to assume in the learning process.

Regarding techniques for solving a Quadratic Equation, the relevant literature — and curriculum materials in particular — presents a wide variety of approaches. This means that, in the teaching process, teachers may draw upon different procedures to create opportunities for student learning. Conversely, in the learning process, students may have access to a variety of procedures and choose the one they consider most suitable for solving a given problem.

Vale (2013) argues that the resolution of this type of equation has been limited to a single solution technique and to the relationship between roots and coefficients, while other procedures are not considered in teaching practices. In this sense, different techniques may be presented and taught to students so that they can choose the one best suited to a particular solution. Along the same lines, Flóes (2013) reiterates that students need to know and use the various existing techniques for solving tasks involving Quadratic Equations until they reach an understanding and generalization applicable to any equation of this nature.

Mathematics curriculum materials, particularly those designed for the 8th and 9th grades, also incorporate different techniques when proposing Quadratic Equation tasks. In Teacher's Manuals, accompanied by teaching guidelines, this provides teachers with opportunities to become familiar with different procedures and approaches; in the Student Book, the variety of techniques equips students with a repertoire from which they may select the one they consider most appropriate for a given solution.

These materials present mathematical ideas and concepts with the aim of promoting the learning process, contributing to the relationship teachers establish with them. The teacher-curriculum materials relationship is dynamic, as both are agents whose actions affect one another (Januario; Lima, 2019). In selecting these materials, Souza (2024) adds that teachers appropriate them according to resources arising from their own experiences and knowledge. Teachers' conceptions may influence how they read, interpret, evaluate, and select materials (Souza, 2024). The instructional proposals contained in these materials require adaptation by teachers in light of students' learning needs. This may occur when teachers evaluate the approach to mathematical concepts, teaching strategies, and the skills and guidelines associated with task implementation.

In addition to incorporating solution techniques, curriculum materials envision different forms of interaction between teachers and students, whether in task statements or teaching guidelines, especially in Teacher's Manuals. Regarding the expected or prescribed roles students assume in the learning process, Januario, Perovano, and Lima (2025) discuss three roles: respondent, describer, and arguer. In the respondent role, students tend to present or confirm answers similar to those provided by the teacher. In the describer role, students list some or all of the procedures used in their solutions. In the arguer role, students explain the procedures adopted and verbalize their reasoning.

Across the different techniques for solving a Quadratic Equation, the actions embedded in the tasks are addressed to students so that they engage in classroom discussions, participate by asking questions, interact by answering what is asked, express or confirm the expected answer, follow instructions, perform procedures, and reproduce information (Rocha, 2025; Januario; Perovano; Lima, 2025).

Regarding teachers, different roles may be implicit or explicit in curriculum materials and are envisioned for these professionals when implementing tasks. For Remillard and Kim (2020), the role of teachers is to support students' learning opportunities during classroom instruction.

According to Brousseau (1996), the role of teachers is to bring students' knowledge to life as a reasonable response to a particular situation and, furthermore, to transform this response into an extraordinary cognitive event. This role is important because teachers can propose meaningful tasks, monitor and guide the search for solutions, coordinate discussions among different solutions, value distinct procedures that lead to the same solution, validating them or showing situations in which certain techniques may not work (Romanatto, 2012).

By sharing what they know, conducting lessons, engaging students, and encouraging their active participation, teachers may assume different roles (Januario; Perovano; Lima, 2025). As transmitters, teachers communicate what they know as they develop the lesson. As reproducers, they reproduce the information, procedures, and answers contained in the curriculum material. As facilitators, teachers encourage students to participate in the lesson, facilitating their interaction with tasks and content. As coordinators, they stimulate students' active participation by explaining, analyzing, and discussing techniques and by establishing relationships among ideas.

In general, curriculum materials embody intended pedagogical practices and classroom structures. Different roles are addressed to both teachers and students.

Methodological Design

The study adopts a qualitative approach. According to Minayo (2007), this approach is grounded in the idea that, in the production of knowledge about human and social phenomena, our primary interest lies in understanding and interpreting their meanings rather than explaining them. We sought to understand and interpret the pedagogical approaches embedded in curriculum materials, specifically in 8th and 9th-grade Teacher's Manuals, by analyzing the approach given to the Quadratic Equation and the roles envisioned for teachers and students to assume.

This study is a documentary analysis, defined by Fiorentini and Lorenzato (2012) as research conducted primarily on written documentation. Information regarding the phenomena under investigation was collected from the Teacher's Manuals of curriculum materials. We analyzed the Manuals and surveyed the tasks related to the Quadratic Equation contained within them, as well as the corresponding teaching guidelines.

For the analysis, we selected four collections of Mathematics curriculum materials for the 8th and 9th grades of Middle School (Table 1), recommended by state and municipal schools in the city of Brasília de Minas, Minas Gerais, Brazil. The four collections were evaluated and approved in the 2024 edition of the *Programa Nacional do Livro e do Material Didático* [National Textbook and Educational Materials Program].

TABLE 1: Teacher's Manuals analyzed

Título	Editora	Ano de publicação	Autoria
A Conquista Matemática [The Mathematics Achievement]	FTD	2022	José Ruy Giovanni Júnior
Araribá Conecta [Araribá Connect]	Moderna	2022	Mara Regina Garcia Gay (Managing editor)
Matemática nos Dias de Hoje [Mathematics in Today's World]	SEI	2022	Daniel Romão da Silva, Jefferson dos Santos Cevada
Teláris Essencial [Essential Teláris]	Ática	2022	Fernando Cesar de Abreu Viana, Luiz Roberto Dante

Source: Own elaboration

For the analysis, we considered tasks related to the Quadratic Equation — exercises, problems, and exploration-investigation activities — as well as the corresponding teacher guidance texts incorporated alongside the tasks in the L-format. In September 2024, we mapped the tasks through a data-recording protocol containing the following information: page number, task, mention of the solution technique in the Student Book, and mention of the solution technique in the L-format

guidelines. While reading the task statements and the corresponding teaching guidance texts, we also sought to identify evidence of the roles addressed to teachers and students in classroom practice.

In the next section, we present the analysis of the 8th and 9th-grade Teacher's Manuals that address the content of the Quadratic Equation, seeking to identify the solution techniques associated with this content.

It should be noted that the results are not intended to represent the entirety of the materials approved by the PNLD but rather to understand trends present in the set of selected Manuals. This delimitation is a characteristic of the documentary research conducted and should be taken into account when interpreting the results.

Analysis

The mapped tasks were read and catalogued according to the page on which they were located, their numbering was identified, and any mention of the solution technique in the task statement was recorded.

In three collections, the approach to the Quadratic Equation begins in a specific chapter with a contextualization problem, followed by the definition, examples, and a sequence of tasks. The solution techniques are presented at the beginning of each chapter section, one at a time, followed by a sequence of tasks. In *Teláris Essencial*, the approach to the Quadratic Equation is incorporated into a chapter that includes other topics: *Algebraic Expressions, Equations, and Proportionality* (8th grade) and *Special Products, Factoring, and Quadratic Equations* (9th grade).

Solution techniques

Curriculum materials offer different approaches to Mathematics content while also guiding the roles of teachers and students in teaching and learning processes. The Teacher's Manual becomes a vehicle for communicating mathematical content and pedagogical practices (Remillard; Kim, 2020).

The techniques presented in the Teacher's Manuals are introduced at the beginning of the chapter, one at a time, followed by a sequence of tasks such as problems, exercises, and, more rarely, games. It is important to note that in all four collections analyzed, the equation is introduced through a contextualization problem, followed by its definition, detailing its elements and respective coefficients. For incomplete equations, the techniques used in all four collections are factoring and square root extraction.

With regard to complete equations, all of the analyzed 9th-grade Manuals present the techniques of completing the square and the quadratic formula. The collections *A Conquista Matemática* and *Araribá Conecta* present the sum-and-product technique. The technique of factoring a perfect square trinomial is described in *Araribá Conecta* and *Teláris Essencial*. Each of these techniques is described individually in a chapter section, followed by examples and sequences of problem-solving and exercise tasks. Both algebraic techniques and those requiring geometric interpretation are approached in the same way, namely through references to episodes from the history of Mathematics. However, algebraic techniques are more frequently used in task solutions.

For the technique of factoring a perfect square trinomial, two 9th-grade volumes (*Araribá Conecta* and *Teláris Essencial*) provide a sequence of examples to help students identify an equation that can be written in the factored form of a perfect square trinomial. There is also an association between this technique and Al-Khwarizmi's method of completing the square, as illustrated in Figure 1.

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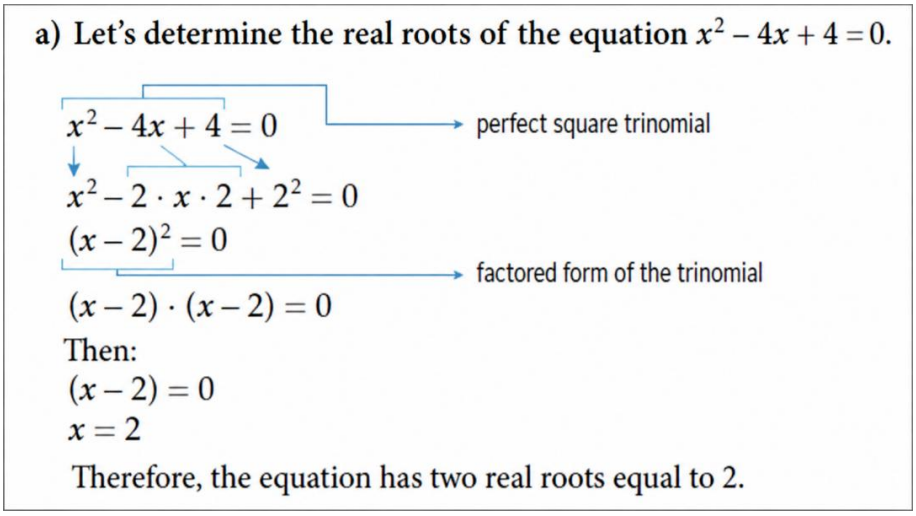
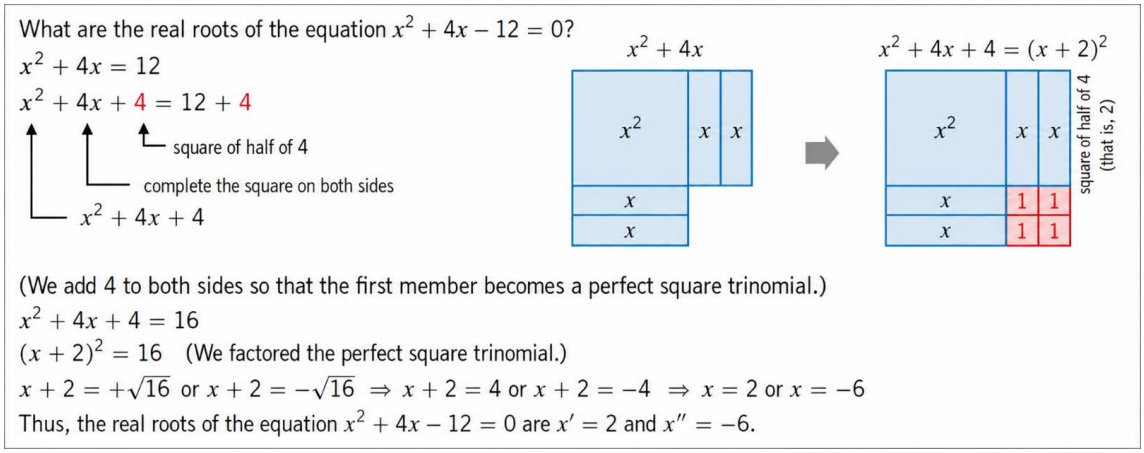
FIGURE 1: Example of the use of the *perfect square trinomial* technique



Source: Araribá Conecta, 9th grade (2022, p. 179)

The technique of completing the square is described in all 9th-grade volumes through the geometric idea of completing squares by means of area measurement; that is, the technique consists of constructing a figure whose area represents the first member of the equation. This is followed by tasks containing equations and problem-solving situations that employ the technique described, as illustrated in Figure 2.

FIGURA 2: Example of the use of the *completing-the-square* technique



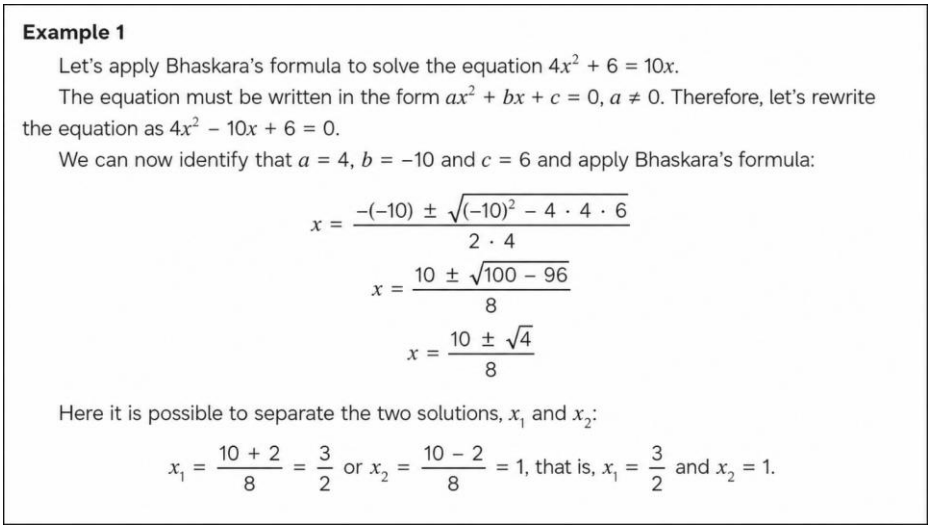
Source: Teláris Essencial, 9th grade (2022, p. 54)

We also observed a relationship between the historical account and the technique presented, enabling students not only to learn the technique but also to become familiar with its origins:

> Babylonian texts, written approximately 4,000 years ago, already referred to problems that we now solve using Quadratic Equations. One of the most common problems in Babylonian writings involved determining two numbers when their sum and product were known. The solution to these problems was strictly geometric: the product of two numbers was considered as an area, and their sum as the semiperimeter of a rectangle. The side lengths of the rectangle corresponded to the given numbers, which were always natural numbers. This geometric treatment was lengthy and laborious, leading the Greeks, and later the Arabs, to seek a simpler procedure for solving such problems. In the ninth century, al-Khwarizmi, an Arab mathematician, developed a process for solving these problems, giving rise to what became known as Geometric Algebra. In the twelfth century [...] the Hindu mathematician Bhaskara (1114–1185) presented a purely algebraic process that made it possible to solve any Quadratic Equation. Based on this process [...] mathematicians were able to derive a formula [...] known as the quadratic formula for Quadratic Equations (A Conquista Matemática, 9th grade, 2022, p. 90).

Regarding the quadratic formula, or Bhaskara's formula, teachers are encouraged to present the history of this technique so that students may read and reflect upon it. In his work *Lilavati*, Bhaskara presented many problems that are solved through Quadratic Equations, which may explain why, in Brazil, the formula became known as *Bhaskara's Formula* (Gay, 2022). Immediately afterward, an example is provided showing how to apply the quadratic formula to solve Quadratic Equations, followed by a sequence of tasks in all 9th-grade volumes, as illustrated in Figure 3.

FIGURE 3: Example of the use of the *quadratic formula*



Example 1

Let's apply Bhaskara's formula to solve the equation $4x^2 + 6 = 10x$.

The equation must be written in the form $ax^2 + bx + c = 0$, $a \neq 0$. Therefore, let's rewrite the equation as $4x^2 - 10x + 6 = 0$.

We can now identify that $a = 4$, $b = -10$ and $c = 6$ and apply Bhaskara's formula:

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \cdot 4 \cdot 6}}{2 \cdot 4}$$

$$x = \frac{10 \pm \sqrt{100 - 96}}{8}$$

$$x = \frac{10 \pm \sqrt{4}}{8}$$

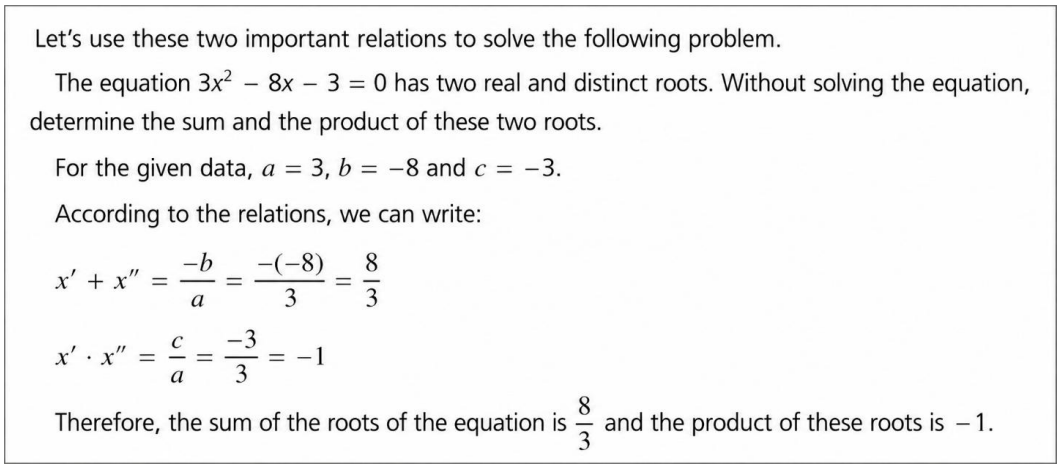
Here it is possible to separate the two solutions, x_1 and x_2:

$$x_1 = \frac{10 + 2}{8} = \frac{3}{2} \text{ or } x_2 = \frac{10 - 2}{8} = 1, \text{ that is, } x_1 = \frac{3}{2} \text{ and } x_2 = 1.$$

Source: Matemática nos Dias de Hoje, 9th grade (2022, p. 110)

Solving an equation of this type using the sum-and-product technique is a quick and efficient method that avoids the numerous calculations required by the quadratic formula. The relationships involving the sum and product of the roots of Quadratic Equations apply to any equation of this degree, since these relationships are derived from the general form of the equation. Several examples using this technique are provided, followed by a sequence of tasks in which students apply the relationships studied. This technique is described in the 9th-grade volume of *A Conquista Matemática* (Figure 4) and is addressed in the L-format guidelines in the corresponding 9th-grade volume of *Araribá Conecta*.

FIGURE 4: Example of the use of the *sum-and-product* technique



Let's use these two important relations to solve the following problem.

The equation $3x^2 - 8x - 3 = 0$ has two real and distinct roots. Without solving the equation, determine the sum and the product of these two roots.

For the given data, $a = 3$, $b = -8$ and $c = -3$.

According to the relations, we can write:

$$x' + x'' = \frac{-b}{a} = \frac{-(-8)}{3} = \frac{8}{3}$$

$$x' \cdot x'' = \frac{c}{a} = \frac{-3}{3} = -1$$

Therefore, the sum of the roots of the equation is $\frac{8}{3}$ and the product of these roots is -1.

Source: A Conquista Matemática, 9th grade (2022, p. 109)

In the task introductions, we observed that 200 tasks (85%) explicitly mention the technique to be used by students; in tasks without such indication, students may choose the technique they consider most appropriate. The L-format guidelines provide proposed solutions, explanations regarding the use or non-use of specific techniques, and indications of how students are expected to develop the techniques they have learned:

> During this activity, investigate whether any student is constructing the Quadratic Equation differently from the method presented and, at the end of the activity, if possible, ask them to share it with their classmates. If a student is shy or uncomfortable doing so, consider reproducing the method developed by that student on the board. [...] In Challenge 14, verify whether students understand that, in order to use the sum-and-product technique for the roots, the coefficient a must be equal to 1. Therefore, all terms of the equation must be divided by 3: $x^2 - 5x + 4 = 0$ (A Conquista Matemática, 9th grade, 2022, p. 110).

After the presentation of all techniques, a section is included to revisit work on Quadratic Equations. Across the eight volumes, this section is entitled *Revisiting What You Have Learned* in *A Conquista Matemática*; *Review Activities* in *Araribá Conecta*; *Ponto de Checagem* in *Teláris Essencial*; and *Check-Out* in *Matemática nos Dias de Hoje*. This section contains tasks that encourage students to share the techniques they have learned and exchange ideas with classmates regarding those they still find challenging. For teachers, guidelines are included to help them identify difficulties and intervene to support students in their solutions.

Throughout the analysis of the curriculum material collections, we identified a relationship between the techniques described and those discussed in the dissertations and theses we mapped (Silva, 2025). The

techniques are the same; however, in the Teacher's Manuals, pedagogical approaches are visible in the task texts, limiting student participation while requiring reasoning and the use of strategies and concepts already known to them. Learning involves interaction between the new and the familiar (Ongstad, 2006) as well as diversified strategies, as discussed by Oliveira and Lopes (2023).

The techniques presented in the Manuals are limited both in their definition and in their application within tasks, focusing on the less complex approaches — completing the square, Bhaskara's formula, and sum and product. In these tasks, we observed a predominance of algebraic forms of solution, while geometric approaches are seldom used. We have no doubt that presenting techniques for solving a Quadratic Equation in curriculum materials is important for student learning; however, learning still needs to be enriched through more meaningful approaches. Teaching guidelines provide teachers with direction and a pathway to follow, but they also require supplementation.

The predominance of these techniques may be related to the way school Algebra has traditionally been organized in curriculum materials. Procedures such as factoring, completing the square, sum and product, and the quadratic formula exhibit strong curricular stability and allow for the proposal of more standardized tasks with predetermined answers. However, this predominance tends to restrict students' exposure to other ways of thinking about and solving equations, limiting opportunities to compare strategies and discuss the mathematical foundations underlying the procedures employed.

We also observed that the way techniques are presented in the Manuals is associated with the roles assigned to students. In some tasks, procedures are already defined or strongly suggested, leading students to reproduce previously taught strategies. This characteristic tends to favor the roles of respondent and describer, to the detriment of situations that require argumentation, comparison of procedures, or the production of original strategies.

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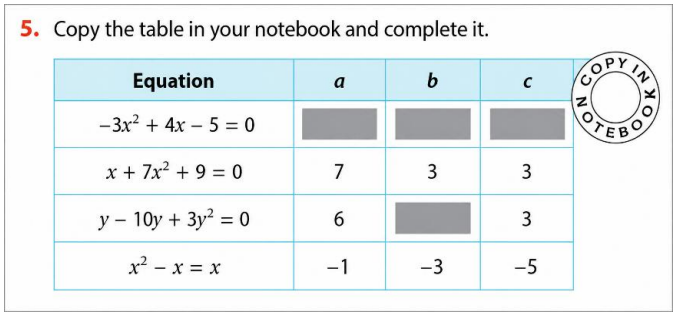
Students' roles

The analysis of the tasks included in the Teacher's Manuals made it possible to identify 235 tasks across the eight volumes of the four collections. Of these, 43 (18.3%) position the student as a respondent, 41 (17.4%) as a describer, and 6 (3%) as an arguer, totaling 90 (39%) tasks in the *A Conquista Matemática* collection. In the *Araribá Conecta* collection, we identified 22 (9.4%) tasks positioning the student as a respondent, 27 (11.5%) as a describer, and 1 (0.4%) as an arguer, totaling 50 (21%) tasks. In *Teláris Essencial*, we cataloged 45 (19%) tasks positioning the student as a respondent, 21 (9%) as a describer, and 5 (2%) as an arguer, for a total of 71 (30%) tasks. In the *Matemática nos Dias de Hoje* collection, we cataloged 16 (7%) tasks positioning the student as a respondent, 8 (3%) as a describer, and none as an arguer, totaling 24 (10%) tasks.

In the task presented in Figure 5, there are objective instructions that require students to provide answers without an explanation (Remillard; Kim, 2020). In solving this task, and others of a similar nature, students present a solution without the need to verbalize how they arrived at it, generally employing a familiar technique (Januario; Perovano; Lima, 2025; Rocha, 2025). Expressions and terms such as *solve*, *complete*, *identify*, *determine*, and *answer* are common in these tasks. These are characteristics of tasks that position students as respondents.

FIGURE 5: Example of a task positioning the student as a respondent



5. Copy the table in your notebook and complete it.

Equation	*a*	*b*	*c*
$-3x^2 + 4x - 5 = 0$			
$x + 7x^2 + 9 = 0$	7	3	3
$y - 10y + 3y^2 = 0$	6		3
$x^2 - x = x$	−1	−3	−5

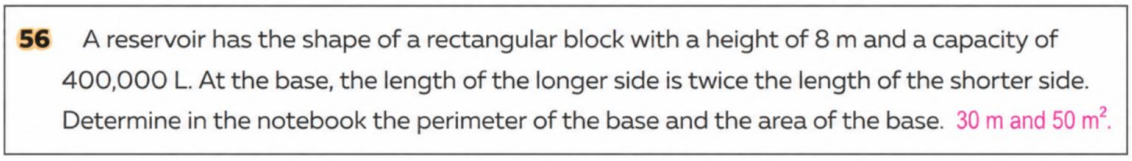
Source: Araribá Conecta, 9th grade (2022, p. 194)

The task privileges the direct identification of elements of the equation and the production of objective answers, without requiring explanations or justifications, a characteristic associated with the role of the student as a respondent.

A respondent student is a reproducer; they follow the task instructions without exercising authorship in their learning process. Respondent students are not given opportunities to exercise autonomy and creativity, since they follow the model taught by the teacher or presented in the material. Consequently, this type of task tends to restrict opportunities for students to mobilize their own strategies, formulate explanations, or communicate their reasoning.

Tasks in which students are expected to assume the role of describer are those in which they are asked to follow a set of steps (Remillard; Kim, 2020). In these tasks, students list some or all of the procedures used in their solutions. We observed terms and expressions commonly associated with such tasks: *describe*, *describe the process*, *observe*, and *demonstrate*. We identified that, in this type of task, students need to understand what they are doing and which procedure they will use, as illustrated by the task presented in Figure 6.

FIGURE 6: Example of a task positioning the student as a describer


56 A reservoir has the shape of a rectangular block with a height of 8 m and a capacity of 400,000 L. At the base, the length of the longer side is twice the length of the shorter side. Determine in the notebook the perimeter of the base and the area of the base. 30 m and 50 m².

Source: Teláris Essencial, 9th grade (2022, p. 54)

In this task, the focus shifts from obtaining the answer to making the adopted procedure explicit. This characteristic brings the student closer to the role of describer, as it requires organizing and communicating the steps used to solve the proposed situation. The following excerpt was taken from the teacher guidance and reinforces the role of students as describers.

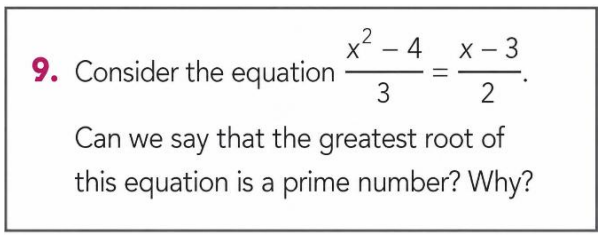
> During the exploration of the example presented in the Student's Book, verify whether students understand that, during the solution process, the complete Quadratic Equation presented becomes a perfect square trinomial. If deemed necessary, revisit this factorization case so that they can continue their study without doubts. (A Conquista Matemática, 9th grade, 2022, p. 97).

However, the role of describer tends to require students to possess knowledge of the procedures used, since it is necessary to describe the steps and organize ideas in order to solve the proposed task. Describer students do not reproduce procedures mechanically; rather, they understand which procedure will be used and how to use it. They may depend on the teacher's explanation for certain procedures, but they are able to demonstrate confidence when solving tasks.

In the role of arguer, students explain the procedures they adopted and verbalize their reasoning, communicating their strategies and understandings (Januario; Perovano; Lima, 2025). The analysis shows that this was the least emphasized role in the tasks, totaling only 12 (5%) tasks across the four collections. Expressions and phrases such as *justify your answer*, *why?*, *explain how you arrived at the answer*, and *discuss with your classmates* are common in these tasks.

Half of the tasks that encourage the role of arguer are found in the 9th 9th grade volume of *A Conquista Matemática*. We did not identify the presence of this type of task in the 8th grade volumes of the collections analyzed. Figure 7 presents an example in which the student must solve the equation using algebraic procedures to obtain it in reduced form; next, they may use whichever solution technique they consider most appropriate; and finally, after identifying the largest root of the equation, determine whether it is a prime number.

FIGURE 7: Example of a task positioning the student as an arguer



Source: A Conquista Matemática, 9th grade (2022, p. 104)

Unlike tasks addressed to respondents and describers, this proposal requires students to mobilize arguments to support the solution presented. The task creates conditions for them to explain their choices, justify procedures, and communicate their mathematical reasoning, characteristics associated with the role of arguer.

Students who act as arguers develop independence from both the teacher and the curriculum material. They are creative and autonomous, demonstrate the steps they followed, and are able to verbalize their reasoning. Tasks that emphasize this role, although infrequent in the manuals analyzed, produce a positive effect on student behavior, as they position students as authors of their own learning rather than leaving them dependent on and limited by the guidance contained in the materials.

In summary, of the 235 tasks analyzed, 126 (54%) were addressed to students as respondents. As describers, there were 97 (41%) tasks, with none identified in the 8th grade volume of *Matemática nos Dias de Hoje*. Of the total, 12 (5%) tasks were addressed to students as arguers, all located in the three 9th grade volumes and absent from the 8th grade volumes. This predominance of tasks focused on obtaining answers and applying already known procedures, to the detriment of argumentation, reveals more limited opportunities for students to explain, justify, or defend their mathematical reasoning and to assume more active roles in the construction of knowledge (Moura et al., 2021).

This result highlights the predominance of tasks centered on producing answers and describing procedures. The limited presence of tasks that

position students as arguers contrasts with perspectives that advocate greater student participation in the processes of constructing mathematical knowledge (Silva; Santos; Cruz, 2020).

The roles envisioned for students are related to the roles assigned to teachers. When tasks privilege predetermined answers and predefined procedures, teaching guidance tends to position the teacher as a transmitter or reproducer of knowledge. Conversely, tasks that encourage explanations, discussions, and justifications require greater teacher involvement as a facilitator or coordinator of classroom interactions.

Teachers' roles

The teacher's primary objective is to create the conditions for students to construct learning. For this objective to be achieved, it is important to understand what students already know and how they learn. According to Brousseau (1996), teachers need to recontextualize knowledge by seeking situations that give meaning to the concepts being taught. Considering the task statements and the corresponding teaching guidance, it is possible to identify different roles envisioned for teachers.

According to Giovanni Júnior (2022), the teacher's role is to provide students with access to different ways of doing Mathematics and to support them in developing skills and constructing knowledge so that they can (re)signify the Mathematics of their daily lives, as well as appreciate the beauty of Mathematics itself.

In the analysis, we identified expressions in the Teacher's Manuals that convey these roles, such as: *comment with the class*, *reproduce the examples on the board*, *if deemed necessary, propose to the students*, *emphasize the importance*, *read*, *expand the proposal and show it to the students*, *observe*, and *evaluate*.

As a transmitter, the teacher conveys what they know while conducting the lesson. In this role, according to Remillard and Kim (2020), the teacher explains, models, and guides students through the stages and procedures,

serving as the primary source of knowledge. We observed that, at the beginning of the approach to the Quadratic Equation, this role becomes evident, as the teacher poses questions to students, explains what they know, and expects students to understand it. In the 9th grade volume of *A Conquista Matemática*, there is guidance in which the envisioned role of transmitter is implicit: "Present the definition of a Quadratic Equation, detailing its elements and respective coefficients" (p. 91). Here, the teacher reiterates the information contained in the text, that is, the definition presented in the Manual, ensuring that students understand what is written.

The role of transmitter leads the teacher to explain and guide students through the procedures described in the Manual. There is little interaction between teacher and student, since the teaching process is determined by the material. The teacher has limited autonomy and little opportunity to develop creativity, becoming dependent on and restricted by the guidance contained in the Manual.

Across the four collections analyzed, we cataloged 105 (45%) tasks in which teachers are guided, even if implicitly, to perform this role. Approximately 195 (83%) of these tasks are found in the 9th grade volumes, as illustrated in the following excerpt:

> During the resolution of this activity, emphasize that only after developing the equations and simplifying like terms can we identify their degree. If deemed appropriate, present equations in which like terms have not been simplified and ask students what the degree of the equation is. Then simplify the like terms and ask again about the degree of the equation, so that they perceive the importance of carrying out this process. (Teláris Essencial, 9th grade, 2022, p. 48).

We identified terms and expressions commonly found in the guidance that envision teachers in the role of transmitter: *emphasize*, *if deemed appropriate*, *ask students*, *request that students*, *guide*, and *propose*.

As a reproducer, the teacher conducts the lesson by reiterating information, procedures, and answers contained in the curriculum material being used (Januario; Perovano; Lima, 2025). Terms and expressions such as *highlight*, *revisit*, *present*, *recall*, and *introduce* were identified in the teacher guidance as being associated with this role.

In the manuals analyzed, we cataloged 54 (23%) tasks in which the role of reproducer was envisioned for teachers when implementing instructional practices. In three 8th grade volumes, it was not possible to identify this role in the tasks: *A Conquista Matemática*, *Teláris Essencial*, and *Matemática nos Dias de Hoje*. We observed the role of reproducer in the following guidance:

> Students are expected to establish a relationship between the fraction of the area occupied by the bird drawing and the denominator of p, as well as conclude that negative values may be solutions to the equations. Mediate students' conclusions, weighing and correcting possible conceptual errors. (Matemática nos Dias de Hoje, 9th grade, 2022, p. 105).

As a facilitator, the teacher engages and encourages students' exploration in the construction of meaning, stimulating and supporting them in making connections or overcoming difficulties while working on tasks. Here, students are the primary source of knowledge (Remillard; Kim, 2020).

In the 9th grade volumes, we identified 36 (15%) tasks in which teachers are envisioned as performing this role. In the 9th grade volumes, we identified 14 (6%) such tasks in three collections: *Araribá Conecta*, *A Conquista Matemática*, and *Teláris Essencial*.

The following guidance exemplifies the role of facilitator. It can be observed that there is guidance aimed at supporting students' understanding of the task, thereby facilitating their interaction with it.

> Activity 1 promotes the development of skill EF09MA09. Some students may have difficulty associating or representing a number as an unknown and, consequently, may be unable to formulate a sentence that fits the context. If this occurs, explain that the unknown represents, in the equation, a number whose value is not known. Check whether they understand the meaning of words such as increased, decreased, double, triple, and so on. In such cases, one student may be asked to explain it in their own words, allowing them to participate in the lesson and assist their classmates. (A Conquista Matemática, 9th grade, 2022, p. 93).

This role favors dialogic interactions and broadens student participation in the construction of mathematical meanings. The teacher draws upon students' prior knowledge while encouraging them to express their ideas and attribute meaning to the completion of tasks. The teacher has autonomy in the teaching process and may use creativity to interact with students.

In the role of coordinator, the teacher creates a classroom environment in which students model, explain, or discuss strategies as a community (Remillard; Kim, 2020). The teacher performs an instructional role but participates less directly in the discussions.

> This activity addresses the solution of all types of Quadratic Equations, including incomplete ones. If possible, encourage students to solve the activities in pairs so that they can share their doubts and solution strategies with their classmates. (Teláris Essencial, 9th grade, 2022, p. 54).

In this guidance, we perceive the role of coordinator, since the task is to be carried out in pairs, involving discussions of the solution techniques

used and the exchange of strategies. There is no direct teacher intervention in these discussions; they are left to the students.

In this role, teachers assume the position of observers rather than being the primary source of knowledge. Students have autonomy in their learning process, while teachers make occasional interventions and facilitate dialogue. Teachers are not restricted to the Manual's guidance and are therefore more independent. This was the least emphasized role among the cataloged tasks, appearing in 26 (11%) tasks across the four collections. Of these, 24 are found in 9th grade volumes and only 2 in the 8th grade volume of *Teláris Essencial*.

Among the 235 tasks analyzed, we cataloged 105 (45%) that positioned the teacher as a transmitter, 50 (21%) as a facilitator, 26 (11%) as a coordinator, and 54 (23%) as a reproducer. These roles are more evident in the 9th grade volumes than in the 8th grade volumes, indicating a clearer emphasis on how teachers are positioned within the pedagogical proposals and on the type of interaction expected of them when implementing the tasks.

A further difference can be observed between the 8th grade and 9th grade volumes. In the materials intended for 9th grade, both the roles assigned to students and those envisioned for teachers appear in a more explicit and diversified manner. One possible explanation lies in the centrality that the Quadratic Equation assumes at this grade level, requiring more detailed pedagogical guidance and more elaborated task proposals.

Considerations

Based on the understanding that the Teacher's Manuals of curriculum materials present instructional guidelines and methodological concepts that may influence the practices of lesson planning and classroom implementation, considering both equation-solving techniques and the roles expected of teachers and students as they interact in teaching and learning processes, the study presented in this article sought to analyze solution

strategies and the roles of teachers and students in the approach to quadratic polynomial equations in 8th and 9th-grade Teacher's Manuals.

To this end, we analyzed techniques for solving quadratic polynomial equations, as well as the tasks associated with these techniques, in eight volumes of Teacher's Manuals from four Mathematics curriculum collections for lower secondary education, focusing on their implications for teaching and learning processes.

Across the four collections analyzed, the presentation of quadratic equations is diverse, yet superficial. In some cases, the approach is connected to historical contexts, with the presentation of problem situations, examples, an exposition of the solution technique, and a list of tasks. In other cases, the approach is more direct, introducing the solution technique followed by examples and tasks that encourage procedural reproduction by students, which may hinder conceptual development. The solution techniques were limited to the most well-known and less complex ones, while those requiring greater cognitive demand were not incorporated. In all descriptions of the techniques, worked examples are followed by lists of tasks that position students in the roles of respondents and describers, with only a small number of tasks addressed to students as arguers.

Based on the teaching practices observed in the Teacher's Manuals, meaningful interaction among teachers, students, and the proposed tasks is essential. Through such interaction, teachers are able to identify students' difficulties and individual potential. This enables teachers to adapt tasks, select those that mobilize students' reasoning, and foster their ability to develop strategies for determining solutions. It is important that students engage with tasks associated with all levels and roles.

When consulting the Teacher's Manuals, teachers encounter instructional guidance and teaching strategies that are important for lesson planning. The analysis revealed that these orientations predominantly position teachers as transmitters and reproducers of knowledge, whereas guidance directed toward facilitator and coordinator roles is considerably less

frequent. We recognize that consulting these Teacher's Manuals facilitates teaching practice; however, in some cases, it may also constrain it. It is important that teachers develop autonomy, creativity, and independence in the classroom; seek different teaching strategies; propose varied solution techniques; and implement diverse tasks that encourage students to question, create, and reason.

The findings also have implications for teacher education and the production of curriculum materials. For teacher education, they reinforce the importance of a critical reading of Teacher's Manuals, recognizing both the possibilities and limitations of the instructional guidance they provide. For authors and publishers of curriculum materials, the results indicate the need to broaden the diversity of solution techniques and to propose tasks that foster argumentation, autonomy, and more active student participation in learning processes.

The study highlights the importance of adopting different equation-solving techniques and proposing tasks that mobilize students' knowledge. Employing varied teaching strategies and approaches contributes to more meaningful learning. Curriculum materials constitute an important source of knowledge for teachers and students; however, teachers need to critically analyze the orientations and approaches contained within them and adapt this information to the different roles that they and their students assume in the processes of teaching and learning Mathematics, particularly with regard to quadratic equations.

The findings underscore the importance of analyzing not only the mathematical content present in curriculum materials but also the ways in which teachers and students are positioned within instructional proposals. Future research may examine how these orientations are effectively interpreted and enacted by teachers in practice, thereby expanding our understanding of the relationships among curriculum materials, Mathematics teaching, and learning.

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