

*Teacher's Specialised Knowledge and teacher's
mathematical discourse in the scope of
decomposition of natural numbers:
an example of a discursive task for
Teacher Education¹*

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ABSTRACT

Mathematics teacher's knowledge has specific characteristics which are specific to their professional practice, being specialised, from the perspective of Mathematics Teacher's Specialised Knowledge. This Specialised Knowledge materializes in the actions and mathematical discourse used by teachers, which supports their communication with students. To develop such specialised knowledge, Tasks for Teacher Education are conceptualized and implemented. Here, aiming a better understanding on the content of teacher's knowledge we focus on a Discursive Task for Teacher Education, involving mathematical topics in which student's (and teacher's) reveal inadequate or incomplete knowledge, and the topic is decomposition of natural numbers. We present and discuss the content of the Discursive Task for Teacher Educations that is associated with developing teachers' Specialised Knowledge within the scope of procedures, resources, teaching strategies and techniques, tasks and

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examples, which impact the elements of mathematical discourse within the scope of endorsed narratives and ritualistic routines on the decomposition of natural numbers.

KEYWORDS: *Mathematics Teacher's Specialised Knowledge*. Mathematical discourse. Task for Teacher Education. Natural number decomposition.

Conhecimento Especializado e o discurso matemático do professor no âmbito da decomposição de números naturais: um exemplo de uma Tarefa para a Formação Discursiva

RESUMO

O conhecimento do professor de matemática possui especificidades próprias de sua prática profissional, sendo especializado, na perspectiva do *Mathematics Teacher's Specialised Knowledge*. Esse Conhecimento Especializado materializa-se no discurso matemático utilizado pelo professor, que sustenta sua comunicação com os alunos no contexto escolar. Para desenvolvê-lo, conceitualizam-se Tarefas para a Formação, aqui, foca-se em uma Tarefa para a Formação Discursiva, envolvendo tópicos matemáticos em que alunos (e professores) revelam um conhecimento inadequado ou incompleto, como a decomposição de números naturais. Apresentamos e discutimos o conteúdo de uma Tarefa para a Formação Discursiva que se associa a desenvolver o Conhecimento Especializado do professor no âmbito dos procedimentos, recursos didáticos, estratégias e técnicas de ensino, tarefas e exemplos, que impactam nos elementos do discurso matemático nos escopos das narrativas endossadas e rotinas ritualísticas considerando a decomposição de números naturais.

PALAVRAS-CHAVE: *Mathematics Teacher's Specialised Knowledge*. Discurso matemático. Tarefas para a Formação. Decomposição de números naturais.

Conocimiento Especializado y el discurso matemático del profesor de matemática en el ámbito de la descomposición de números naturales: un ejemplo de Tarea para la Formación

RESUMEN

El conocimiento del profesor de matemática tiene especificidades de su práctica profesional, siendo especializado desde la perspectiva del *Mathematics Teacher's Specialized Knowledge*. Este Conocimiento Especializado se materializa en el discurso matemático utilizado por el profesor, que sustenta su comunicación con los alumnos. Para desarrollarlo, se conceptualizan Tareas para la Formación, y acá nos centramos en una Tarea para la Formación Discursiva, involucrando tópicos matemáticos en los que los alumnos (y profesores) revelan un conocimiento inapropiado o incompleto – la descomposición de números naturales. Presentamos y discutimos el contenido de una Tarea para la Formación Discursiva que se asocia con el desarrollo del Conocimiento Especializado del profesor en el ámbito de los procedimientos, recursos didácticos, estrategias y técnicas de enseñanza, tareas y ejemplos, que inciden en los elementos del discurso matemático en el ámbito de las relatos acreditados y rutinas repetitivas considerando la descomposición de números naturales.

PALABRAS CLAVE: *Mathematics Teacher's Specialised Knowledge*. Discurso matemático. Tareas para la Formación. Descomposición de números naturales.

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Introduction

Mathematics Teacher's Knowledge plays an essential role in students' mathematical learning (Nye; Konstantopoulos; Hedges, 2004). In the professional practice of the mathematics teacher, there is a set of routines that are unique, uncommon (inexistent) to other professions, such as formulating and solving mathematical problems, explaining and discussing mathematical topics, assigning meaning to the productions of others,

conceptualizing and implementing mathematical tasks. It is essential, therefore, that the mathematics teacher possesses a Specialised Knowledge, understood here from the perspective of the Mathematics Teacher's Specialised Knowledge – MTSK (Carrillo et al., 2018).

MTSK considers the knowledge of the mathematics teacher as specialised. It includes, for example, knowing the procedures involved in each mathematical topic, the definitions and properties that support them, and the ways of representing these topics. In addition to these, it includes knowing the teaching resources that (and how) support discussions with students, mathematical tasks, strategies, techniques, and examples to be used, optimizing the development of students' mathematical knowledge in each topic and at each moment. Such knowledge materialized through the mathematical discourse that the teacher uses (e.g., Bakhtin, 1992), and which supports communication about mathematical objects with their students (Ponte et al., 1998).

Discourse corresponds to the different forms of communication that allow individuals (from the same social group) to interact with each other (Sfard, 2008). Discourse, specifically mathematical discourse, includes words and symbols used to generate endorsed narratives – mathematical verbal sequences considered true by the scientific community – and ritualistic routines – repetitive actions essential to developing the subjects' ways of thinking mathematically (Sfard, 2008).

Here, it is assumed that learning mathematics consists of delving deeper into this discourse, progressing within a pre-existing mathematical discourse or adhering to the discourses of subjects considered more experienced (Sfard, 2008). In the school context, this subject is the mathematics teacher (Graff; Ripardo, 2023), making it essential to develop the teacher's Specialised Knowledge, since their discourse influences the students' discourse (e.g., Vygotsky, 1987) and their learning – this "influence" should enable mathematically adequate discussions of a high

level and cognitive demand – that is, discussions in which mathematical content is explored in a broad and deep manner.

Since the teacher's Specialised Knowledge does not develop in classroom practice (Ribeiro; Mellone; Jakobsen, 2013), there is a need for teacher education aimed at developing it (Ribeiro; Almeida; Mellone, 2021) and, associated, developing the mathematical discourse in which this knowledge is materialised (Sfard, 2008).

For this movement towards specialisation in teacher education, the Tasks for Teacher Education (Ribeiro, 2021; Ribeiro; Almeida; Mellone, 2021)– are conceptualized in the research and teacher education group Conhecimento Interpretativo e Especializado do professor de Matemática (CIEspMat)⁵ as a resource for teacher education and a data collection instruments for research (assuming research and teacher education in an intertwined manner). They support specialised discussions in teacher education contexts and are conceptualized by assuming practice as the origin and destination of associated specialized formation, prioritizing the development of teacher's Specialised Knowledge in a given topic, to improve the quality of mathematical discussions. When the Tasks for Teacher Education also focus on developing the teacher's mathematical discourse, they are called Discursive Tasks for Teacher Education, since the teacher's knowledge impacts their actions, what the teacher says and does, and how they do it, in their professional practice (Ribeiro; Carrillo, 2011).

As one of the generating elements for Task for Teacher Education, we consider the Tasks for Students which are conceptualized from the identification of incomplete or inadequate knowledge of students (usually associated with the difficulties they reveal) in the topic under discussion (Ribeiro, 2021). Among the topics on numbers and in the scope of developing Algebraic thinking, we identified the decomposition of natural numbers in

⁵ CIEspMat is a research and Teacher Education group that develops work focused on the development of Interpretive and Specialised teacher's knowledge – from kindergarden to upper secondary. [@ciespmat_formacao](http://www.ciespmat.com.br).

which students (e.g., Lerner; Sadovsky, 1996) and teachers (e.g., Curi, 2011) reveal incomplete or inadequate knowledge.

To this end, we present and discuss the structure and content of a Discursive Task for Teacher Education associated with the Teacher Education goal of developing the teacher's Specialised Knowledge and mathematical discourse in the context of the decomposition of natural numbers. Associated with this Discursive Task for Teacher Education the research question refers to: which Specialised Knowledge associated with procedures, teaching resources, strategies, techniques, tasks and examples, and on the elements of mathematical discourse, endorsed narratives and ritualistic routines.

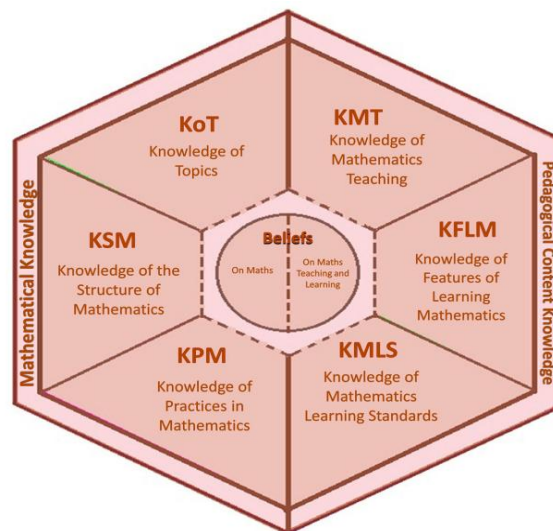
Some theoretical discussions

The decomposition of natural numbers expresses a number in different equivalent forms (Cebola, 2002). In the national curricular documents – Base Nacional Comum Curricular, BNCC (Brazil, 2018) – one can find recommendations to work with this topic since year one from primary. It is also part of the official documents of several other countries (e.g., Portugal (DGE, 2018); Chile (MINEDUC, 2015); USA (NCTM 2010)).

Although it is an essential topic for understanding the Decimal Number System, students reveal inadequate or incomplete knowledge (usually associated with difficulties in the topic), for example: (i) knowing the difference between the positional value and the absolute value of each digit of a number (Curi, 2011) – in 23, the tens digit is two, and there are two tens; (ii) knowing the positional value of each digit in the number (Lerner; Sadovsky, 1996) – in 23, they decompose as $2 + 3$ instead of $20 + 3$; (iii) knowing different ways of decomposing a number – the usual way of decomposing 23 is $23 = 20 + 3$, and unusual ways are, for example, $23 = 10 + 13$ and $23 = 0 + 23$.

In order for students to develop their mathematical knowledge adequately in the context of decomposing natural numbers, it is necessary for teacher's to be in possession of Specialised Knowledge, assumed here from the Mathematics Teacher's Specialised Knowledge – MTSK (Carrillo et al., 2018) and its two domains of knowledge: Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK), each of which consists of three subdomains.

FIGURE 1: Domains and subdomains in MTSK model



Source: Carrillo *et al.*, (2018, p. 241)

Knowledge of Topics (KoT) is discussed, highlighting knowledge of the category of procedures; and the Knowledge of Mathematics Teaching (KMT), focusing on knowledge of the categories of teaching resources (physical and digital) and strategies, techniques, tasks, and examples. Examples of knowledge content within the scope of the decomposition of natural numbers are presented for each of these categories.

Knowledge of Topics (KoT) includes the teacher's knowledge of mathematical topics, such as procedural knowledge, meanings and senses, and the properties that underpin the topic (Carrillo et al., 2018). It consists of four categories: (i) procedures; (ii) definitions, properties, and foundations; (iii) representation registers; and (iv) phenomenology and applications. All

are discussed in a synthetic way, but the focus of attention is on (i) procedures, as it will be discussed in the Discursive Task for Teacher Education presented later.

(i) Procedures include the teacher's knowledge related to how to do something – conventional or alternative algorithms –, when to do something – necessary and sufficient conditions for generating the algorithms –, and why something is done – the principles that design these algorithms (Carrillo et al., 2018). In the decomposition of natural numbers, this includes, for example:

- (a) Knowing that a procedure for decomposing a natural number involves adding the positional value of the digits, such as $23 = 20 + 3$.
- (b) Knowing that a procedure for decomposing a natural number on the abacus involves performing groupings of 10 associated with the use of addends. The number 23 has two addends (units and tens) and can be decomposed into $20 + 3$; $10 + 13$ or $0 + 23$ (Ribeiro, 2022).

In (ii), definitions, properties and foundations, the teacher's knowledge of the different definitions of a topic is included, as well as the set of properties that characterize mathematical objects, essential to support the definitions, and among these properties, those fundamental to the topic (Carrillo et al., 2018). For operational reasons, they are discussed in this order: (a) properties; (b) foundations; and (c) definitions.

- (a) Properties involve knowing a set of mathematical attributes that are invariant to a given topic. In the decomposition of natural numbers, this includes, for example, knowing that the absolute and positional values of the digits coincide only in the unit's place.
- (b) Foundations refer to knowing a set of essential properties for a topic to exist. In the decomposition of natural numbers, this includes, for example, knowing that without identifying the positional value of the digits, it is not possible to decompose the number.
- (c) Definitions involve knowing a minimal set of properties of a mathematical entity that allow it to be uniquely identified. In the

decomposition of natural numbers, this includes, for example, knowing that every number n with i digits that assume the value $a_i \in \{0, 1, 2, \dots, 9\}$ with $i = 1, 2, \dots, r$ is a polynomial represented by $n = a_0 \times 10^0 + a_1 \times 10^1 + a_2 \times 10^2 + \dots + a_r \times 10^r$ with $r \geq 0$ (e.g., Domingues, 1934).

(iii) Register of representation include knowledge of the different ways to represent a topic, involving numerical records, natural language, graphs, algebraic and pictorial representations (Duval, 1995). The decomposition of natural numbers includes, for example:

- (a) Knowing that the quantity 23 can be represented numerically (numerical record), for example, as $20 + 3$; $10 + 10 + 3$.
- (b) Knowing that the quantity 23 can be represented using natural language, for example, two tens and three units; 10 units plus 10 units plus three units.
- (c) Knowing that a quantity n can be represented algebraically, as $n = a_0 \times 10^0 + a_1 \times 10^1 + a_2 \times 10^2 + \dots + a_r \times 10^r$, where $r \geq 0$.

The (iv) phenomenology and applications, encompasses knowledge of the concepts associated with a topic and the different meanings involved, in a wide variety of contexts for teaching it. In the decomposition of natural numbers, this includes, for example:

- (a) Knowing that decomposing a natural number has a meaning of equivalence, since the same number can be represented using different decompositions. ($23 = 20 + 3 = 10 + 10 + 3 = 2 \times 10 + 3$).
- (b) Knowing that decomposing a natural number has the meaning of an operator, since decomposition can be understood as an operation to be performed – with the operation on one side of the equality and the result on the other side of the equality ($20 + 3 = 23$ or $2 \times 10 + 3 = 23$).

Knowledge of Mathematics Teaching (KMT) refers to the teacher's knowledge for teaching mathematics associated with the tasks, strategies, and techniques for teaching mathematical topics, as well as the resources that can be used to enhance students' mathematical understanding of each

specific topic (Carrillo et al., 2018). It consists of three categories: (i) theories of mathematics teaching; (ii) teaching resources (material or digital); and (iii) strategies, techniques, tasks, and examples. All are discussed in a synthetic way, but the focus of attention is on (ii) and (iii), as they will be discussed in the Discursive Task for Teacher Education presented later.

In (i), theories of mathematics teaching, knowledge of the theoretical elements that derive directly from specific studies in Mathematics Education (Carrillo et al., 2018) within the scope of teaching each topic is involved. In the decomposition of natural numbers, this includes, for example:

- (a) To understand Duval's (1995) Theory of Semiotic Representation Registers, in which there is an auxiliary representation register that can be used to understand the main representation register, with the abacus, for example, being a concrete resource (auxiliary representation) that helps in understanding numerical decomposition – abstract (main representation).

In (ii) teaching resources (physical and digital), the teacher's knowledge of the resources that can be used to enhance or teach two mathematical topics; and the limitations that may be involved in its use (Carrillo et al., 2018). The decomposition of natural numbers includes, for example:

- (a) Knowing that the use of the abacus resource has the potential to develop the understanding of positional value, and that to do so, it is necessary to predefine that the resource must be as well as to determine which haste will represent the units, as the other orders are determined (Ribeiro, 2022).
- (b) Note that when using an abacus, the hastes indicate the orders of the Decimal Numbering System and, therefore, the quantity of pieces that are placed in each haste corresponds to the quantity

of elements in that order and may be more than nine pieces⁶ (Ribeiro; Graff; Silva, 2024).

In (iii), strategies, techniques, tasks and examples, encompasses the teacher's knowledge about the type of tasks, and the associated discussion, which can be used to explore certain components of a topic, such as strategies, techniques and examples that can be employed to teach each of these topics. The decomposition of natural numbers includes, for example:

- (a) Knowing that a type of task with potential for students to understand the decomposition of natural numbers (and not only), must contain questions requesting the decomposition of a natural number using different forms, to think about the decomposition by adding the positional value of algorithms (Ribeiro, 2022).

The Specialised Knowledge of the mathematics teacher materializes through his/her mathematical discourse (e.g., Sfard, 2008). We, as humans, participate in the world from the production of social interactions (Vigotski, 2008), and thus, the discourse is perceived as a type of communication that helps individuals interact with their pairs – participants of the same social group.

This way, mathematical learning is optimized when the subject is inserted in mathematical discourse, developing the ability to talk about mathematics through interpersonal communication (Sfard, 2008), which in the school environment is the communication, for example, between the students and the teacher (the most experienced subject). Mathematical discourse has four elements (Sfard, 2008): (i) use of words; (ii) visual mediators; (iii) endorsed narratives; and (iv) routines.

In (i) use of words, it's included the words that name mathematical objects in the form of natural language and the way they are used (Sfard, 2008). In the decomposition of natural numbers, this involves, for example, using the expression "two tens and three units" to identify 23.

⁶ This knowledge leads to one of the so called Fake News in Mathematics Education (Graff; Silva; Ribeiro, 2024), which considers the game "Never 10" as an inappropriate strategy for students to understand positional value.

In (ii), visual mediators, we have symbols that designate a mathematical object with precision and clarity (Sfard, 2008). In the context of decomposing natural numbers, this involves, for example, using numbers and operation signs to form a numerical expression, such as $23 = 20 + 3$ or $23 = 2 \times 10 + 3$; and use algebraic expressions, such as $n = a_0 \times 10^0 + a_1 \times 10^1 + a_2 \times 10^2 + \dots + a_r \times 10^r$, where $r \geq 0$.

In (iii), endorsed narratives encompass verbal sequences that refer to mathematical objects and that are subject to approval or rejection by the community of intellectuals participating in that specific discourse (Sfard, 2008). Thus, endorsed narratives correspond to definitions, properties, theorems, proofs, and procedures. In the context of the decomposition of natural numbers, this involves, for example:

- (a) Using the endorsed narrative of the type procedures, in which the decomposition of a natural number is carried out from the addition of the positional value of its digits.
- (b) Using the endorsed narrative of the properties type, which states that the absolute and positional value of a digit coincide when the order is that of the units.
- (c) Using the narrative endorsed is of the fundamental type, stating that the positional value of the digits in a number supports its decomposition, since without identifying them, the number cannot be decomposed.
- (d) Using the endorsed narrative of the definition type, where every number n with i digits takes on the value $a_i \in \{0, 1, 2, \dots, 9\}$ it is a polynomial represented by $n = a_0 \times 10^0 + a_1 \times 10^1 + a_2 \times 10^2 + \dots + a_r \times 10^r$ that $r \geq 0$ (e.g., Domingues, 1934).

In (iv), routines correspond to the repetitive regularities' characteristic of a discourse, which uses words and symbols to create narratives according to its discursive needs. There are three types: (a) acts routines; (b) ritualistic routines; and (c) exploration routines. All are

discussed in a synthetic way, but the focus of attention is on (b), as it will be discussed in the Discursive Task for Teacher Education presented later.

In (a), acts routines, while producing narratives, do not focus on developing mathematical thinking – remaining solely in the realm of knowing how to do. In the decomposition of natural numbers, for example, this involves using the decomposition procedure associated with adding the positional value of the digits, without considering that this is not the only way to decompose a number and may not even be the most appropriate method for the specific situation in which the decomposition was performed (Cebola, 2002).

The (b) ritualistic routines, such as rituals, are a necessary occurrence for the mathematical learning process, as they develop mathematical thinking through successive interactions with subjects more experienced in mathematical discourse (Sfard, 2008). Furthermore, this includes, among other actions, increasing the cognitive demand of the examples used in relation to the topic, as students delve deeper into the discourse about a mathematical object; as well as using resources to deepen the understanding of a component of the topic to help practice the discourse (enhancing mathematical discussions).

In the decomposition of natural numbers, for example, this involves using the abacus in introductory lessons on the decomposition of natural numbers – as a way to allow understanding positional value – covering the predefined shapes on the abacus rods with stickers, so that, when starting the ritualistic routine, it is determined which rod will represent the units and the other place values are also determined (see, for example, Ribeiro, 2022); Upon completing the ritualistic routine (when the number is finished being decomposed), a new ritualistic routine begins (a new decomposition) by changing the stem unit (understanding that regardless of the place it occupies, upon determining it, the other orders assume their positions to its left).

In (c), exploration routines encompass the production of narratives (endorsed or not), which occurs when the subject becomes autonomous in

mathematical thinking after participating in successive ritualistic routines with more experienced subjects (Sfard, 2008). The decomposition of natural numbers involves, for example, using the algebraic visual mediator of a polynomial to define the decomposition of natural numbers.

There are three forms of mathematical discourse: (i) colloquial mathematical discourse; (ii) academic mathematical discourse; and (iii) literary mathematical discourse.

In (i) colloquial mathematical discourse, its included discourse about mathematical objects without the appropriateness of mathematically valid language. In the decomposition of natural numbers, for example, it involves using the word "houses" (inappropriate) when referring to the positions of the number, as in 23, the 2 is in the tens place.

In (ii) school mathematics discourse, we have the ideal discourse to be used in school (however, this is not yet always the case). In this type of discourse, more words and fewer symbols are used to refer to mathematical objects – always considering mathematical adequacy (Sfard, 2008). In the context of the decomposition of natural numbers, this involves, for example, using the word (appropriate) order to refer to the positions of the number, as in 23, the 2 is in the tens order.

The (iii) literate mathematical discourse encompasses linguistic rigor and a careful refinement of language, using more symbols than words to refer to mathematical objects (Sfard, 2008). In the context of the decomposition of natural numbers, this involves, for example, using the $\times 10$ to refer to the tens order of the number $n = a_0 \times 10^0 + a_1 \times 10^1 + a_2 \times 10^2 + \dots + a_r \times 10^r$.

In Graff and Ribeiro (2024), some relationships between the specialised knowledge of mathematics teachers and the mathematical discourse of this social group are discussed. Here, we revisit those that are the focus of discussion in the conceptualised Discursive Task for Teacher Education.

Procedures, definitions, properties, and fundamentals – endorsed narratives: procedures, definitions, properties, and foundations

(KoT) are considered endorsed narratives (mathematical discourse) because they are consensually accepted by the most experienced participants in the mathematical discourse – mathematicians, researchers, mathematics teachers, or members of the academic community.

Teaching resources (physical and digital) – ritualistic routines: teaching resources – physical and digital (KMT) – are considered ritualistic routines (mathematical discourse) since, in them, the subject of greatest authority – the mathematics teacher (Graff; Ripardo, 2023) – uses resources in mathematics classes and guides their students to practice some component of a given topic – the students usually end up using it in the way the teacher teaches (whether appropriate or not), because he is considered more experienced in its use.

Strategies, techniques, tasks, and examples – ritualistic routines: strategies, techniques, tasks, and examples (KMT) are ritualistic routines (mathematical discourse) because answering mathematical tasks is indispensable for learning mathematics (Graff; Ripardo, 2023), as they allow students to develop their knowledge and materialize it in their mathematical discourse, for example, by including questions in tasks that require the use of different strategies to solve a given problem (on a specific topic), or by increasing the cognitive demand in the sequence of examples included in the tasks.

Discursive Task for Teacher Education within the scope of the decomposition of natural numbers

Although the discussion of tasks is not recent within the scope of research in Mathematics Education (Tirosh; Wood, 2008), the focus of these tasks has been primarily on mathematical tasks for students (Delgado, 2013; Walls, 2005), discussing associated pedagogical approaches, such as problem-solving tasks and investigative tasks (e.g., Delgado, 2013; Ponte, 2005; Stein; Smith, 1998). Complementarily, a focus on Tasks for Teacher

Education has recently emerged, involving something more than Tasks for students and a discussion of how to implement them in the classroom (Nacarato, 2013), allowing a discussion that considers the specificities of the professional practice of the mathematics teacher, in order to do things differently from what has been done (Ribeiro; Almeida, 2022), so that students understand mathematics and begin to Think Mathematically.

Going against the grain of proposals that focus essentially on general pedagogical knowledge, disregarding the specificities of the areas of knowledge that teachers teach and, therefore, without concern and focus on the specificities of this practice so that students understand specific mathematical topics (Ribeiro, 2018), we consider that the practice of the mathematics teacher is specialised and, in this sense, requires Tasks for Teacher Education that are associated with specific goals to develop teacher's knowledge, competencies and discourse, to enable students to understand, transforming mathematics classes into places of thought.

Thus, considering this specialised knowledge of teacher's, and the fact that it does not develop over the years in classroom practice with students (Ribeiro; Mellone; Jakobsen, 2013), the so-called Tasks for Teacher Education are conceptualized (Ribeiro, 2021; Ribeiro; Almeida; Mellone, 2021). These Tasks for Teacher Education usually consider a Task for students as one of the starting points for discussion (only the triggering event and not the priority focus), allowing a transposition to the expected future professional practice.

Every Tasks for Teacher Education is understood simultaneously as a resource for teacher education and an instrument for data collection for research, and therefore always has an associated research question. The Tasks for Teacher Education conceptualized in CIEspMat pursue the priority training objective of promoting discussions that seek to develop the teacher's Specialised Knowledge, so that they can implement pedagogically exciting and mathematically innovative practices (Ribeiro; Silva, 2024). When the Task for Teacher Education also focus on developing the teacher's

mathematical discourse, they are called Discursive Tasks for Teacher Education⁷. Discursive Tasks for Teacher Education corresponds to the task that teachers participating in a teacher education context receive and, considering the pedagogical specialised approach that is implemented, it is usually composed of two parts: a set of questions focusing one topic as a starting point to focus attention on the knowledge materialized in the form of mathematical discourse (Preliminary Part); a set of questions that seek to access and develop the teacher's Specialised Knowledge and, consequently, their mathematical discourse based on a Task for the students – which is inside a rectangle – within the scope of the topic being addressed (Part I).

This Discursive Task for Teacher Education considered here, has been validated in different contexts (with in-service and prospective kindergarten and primary teachers and with mathematic teachers of lower and upper secondary). This Discursive Tasks for Teacher Education was conceptualised considering the teacher education goal of developing the participants' Specialised Knowledge and mathematical discourse regarding the procedures (endorsed narratives) of the topic decomposition of natural numbers using a physical resource, the abacus (a ritual-type routine).

Here, for the purposes of this discussion, we present part of the Discursive Task for Teacher Education (Figure 2), in particular, two questions from the Preliminary Part and two questions from Part I. Those questions are related to an introductory Task for students on the topic of natural number decomposition, for year 4 of primary, considering what is referred at the Brazilian National Curricular Guidelines (Base Nacional Comum Curricular – BNCC, Brazil, 2018).

⁷ For a discussion related to the model that describes the conceptualization and validation process of Task for Teacher Education see, for example, Ribeiro and Almeida (2022).

Figure 2: Part of a Discursive Task for Teacher Education within the context of natural number decomposition.

PRELIMINARY PART

1. Imagine you are going to replace a teacher who has devised an assignment where students decompose the same number in different ways. To do this, he formulated a question associated with a 2-digit number, 23, and asked students to provide examples of different decompositions of that number. What do you expect the students to do?
[...]

3. Are there any differences between the feature in Figure 1 and the feature in Figure 2? Justify your answer.

Figure 1

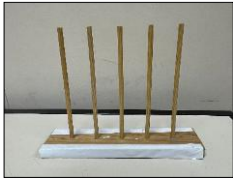



Figure 2



[...]

PART I

Task: Decomposing numbers on the abacus
(You should always explain your reasoning, describing the process you used to answer the question. You can do these using diagrams, words, calculations, ...)

1. Consider the quantity 105.

a) a) Using the abacus provided, decompose this quantity in, at least, three different ways and record what you did on your sheet (you can, for example, draw the abacus and its accompanying pieces), and make the corresponding numerical entries for these decompositions.

[...]

Consider the previous task:

- (i) Solve the task on your own without considering a teaching context.
- (ii) What do you consider to be the mathematical objective of this task? (What is the mathematical knowledge intended to be explored?) [...]

Source: author's own elaboration

In the Preliminary Part, the objective⁸ is to access and develop the participants' specialised knowledge regarding the procedures for decomposing a natural number (associated with the category of procedures –

⁸ All Discursive Task for Teacher Education have a research goal and a teacher education goal, and these goals are achieved by accessing and developing the participants' specialised knowledge and mathematical discourse. Each Discursive Task for Teacher Education question also pursues a specific goal which is always associated with accessing (to contribute to research on understanding what knowledge they reveal) and developing (to contribute to teacher education and research on understanding what and how this mathematical knowledge and discourse develop).

KoTp⁹). In conjunction, they also develop endorsed narratives of the procedural type (Nep)¹⁰ in mathematical discourse. Thus, it includes, for example, knowing the decomposition of natural numbers through the addition of the positional value of the digits of the number (Ribeiro, 2022). This Specialised Knowledge materializes in mathematical discourse by using the endorsed narrative, which involves the decomposition of a number by adding the positional value of its digits¹¹.

The second goal is to develop the participants' specialised knowledge regarding the use of the abacus in their classroom practices (physical and digital resources – KMTr¹²). In conjunction with this, ritualistic routines are developed, in which physical or digital resources are used (Rrr)¹³. Thus, for example, it includes knowing how to use the abacus without predefined orders on each rod to teach positional value. This way, one must decide and mark on the abacus which rod will represent the units order, and thus the other place values get defined (Ribeiro, 2022). This knowledge materializes in mathematical discourse through a ritualistic routine in which, when using the abacus without predefined orders on each rod, the teaching of positional value is enhanced. Thus, one must mark, at the beginning of the routine, which rod will represent the units order – after that, the other place values are already defined. Both questions in the Preliminary Part are associated with school and literary mathematical discourse.

In Part I, (i) aims for the teacher to solve the Task for students on their own, in order to establish a starting point of their own knowledge, identifying

⁹ KoTp this means that the question relates to developing Specialised Mathematical Knowledge within the KoT subdomain and, in particular, in the procedures category (p).

¹⁰ Nep means that the question is associated with endorsed narratives – Ne – of the procedural type – p.

¹¹ To specify the content of the Specialised Knowledge and mathematical discourse that we intend to develop, so as not to be repetitive here, the reader should return to the theoretical discussion that underpins, among other things, posing this question in the Discursive Task for Teacher Education.

¹² KMTr this means that the question relates to developing Specialised Pedagogical Knowledge within the KMT subdomain, specifically in the category of teaching resources (physical and digital) – r.

¹³ Rrr this means that the question is associated with ritualistic routines – Rr – of the type physical or digital resources – r.

the content and level of teacher's knowledge and, in association, the content and level of fluency in mathematical discourse, both school and literary¹⁴.

The first part of Task for students aims to contribute to developing knowledge associated with different strategies for decomposing a natural number using the abacus, thus discussing the procedures (Kotp). This knowledge materialized in school mathematical discourse¹⁵ from the use of endorsed narratives (NEP). Thus, for example, it includes knowing, in the abacus resource, that one of the procedures for decomposing natural numbers involves using groups of 10 associated with the use of addends – the number of addends must correspond to the number of orders of the number. This knowledge materializes in mathematical discourse when using the endorsed narrative, in which, in the abacus resource, a number is decomposed from the procedure that involves using groups of 10 associated with the use of addends – the number of addends must correspond to the number of orders the number has.

The question (ii) aims for the teacher to identify the mathematical learning objectives of the Task, in order to refine attention to the type, nature and objective of mathematical learning associated with the tasks (strategy, techniques, tasks and examples – KMTe¹⁶), because it is usual for teachers to list a set of objectives, the vast majority of which are broad and not specifically associated with the task, or not specific to the mathematical topic, or even to mathematical learning. This knowledge materializes in ritualistic routines when using mathematical tasks (Rrt). Thus, it is associated with identifying that Ta pursues the mathematical learning objective of knowing how to decompose a natural number in different ways (from different procedures) – making it possible to think beyond decomposition through the addition of the positional value of the digits of a

¹⁴ These questions are directed to the teacher, and therefore the use of school-based or literary mathematical discourse is expected.

¹⁵ Since this task is for students whose level of knowledge is compatible with that of the students, the use of school-based mathematical discourse is expected; however, since this task is used in teacher education, it is also expected that answers will emerge using literate mathematical discourse.

¹⁶ KMTe this means that the question relates to developing Specialised Pedagogical Knowledge within the KMT subdomain, specifically in the category of strategies, techniques, tasks, and examples.

number. This knowledge materializes in mathematical discourse when using ritualistic routines, of different ways of decomposing a natural number, making it possible to go beyond decomposition through the addition of the positional value of the digits of a number.

Final Considerations

Improving the teacher's mathematical practice in order to provide students with an increasingly better understanding of mathematics, becoming autonomous in generalizing ways of thinking and proceeding mathematically, requires developing the (mathematical and pedagogical) dimensions of their Specialised Knowledge and the elements of their mathematical discourse associated with this knowledge, so as to ensure that this discourse is always mathematically adequate. Discursive Task for Teacher Education, such as the one presented here, and other Tasks for Teacher Education that have already been conceptualized (e.g., Ribeiro, Gibim and Alves, 2021; Silva and Ribeiro, 2024) have this aim.

Specifically, the example presented here, of a Discursive Task for Teacher Education, focusing on two of the subdomains of MTSK and two elements of mathematical discourse, reflects the type of discussion considered when conceptualizing such tasks – contributing to developing and refining the teacher's Specialised Knowledge and mathematical discourse. Bringing such an example also seeks to enable the replicability of Discursive Task for Teacher Education conceptualization processes, abandoning the space for discussing generalist formative practices (which result in a lack of mathematical discussion that contributes to improving learning and results), and instead directing attention to the specificities of the mathematics teacher's mathematical practice, allowing what has not yet been done (Ribeiro; Almeida, 2022) to ensure that students understand and learn mathematics, reflecting in their results.

We presented a part of a Discursive Task for Teacher Education that forms part of the data collection instruments for a broader research project and a teacher education itinerary that has been implemented and validated in various territorial contexts. The expectation is that this can serve as a tool for other teacher educators and researchers, who seek to implement teacher education practices, and conduct research focused on the specificities of teacher's knowledge, enabling them to problematize their practice and implement it in the various formative contexts for which they are responsible, making adjustments, if necessary, according to the cultural, social, and historical context in which they operate, ensuring that the mathematical discussion is, in fact, specialised, and that the mathematical discourse is developed.

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