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The Logical-Historical Movement of Arithmetic Progression: a contribution to math teaching[1]

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ABSTRACT
This article describes aspects of the logical-historical movement of the concept of Arithmetic Progression (AP), identifying conceptual links (internal and external) with Algebra, which is taught in secondary school. The methodology is a bibliographical review of Brazilian sources in Portuguese and Spanish. The analysis of the results shows that the logical-historical movement of AP, didactically, enables students to develop mental actions linked to theoretical thinking through access to conceptual links. Thus, the teaching of AP is not just an operative technique, but an opportunity to develop a concept as a cognitive tool, developed from human needs. Some of the internal links in the concept of AP are common to other algebraic concepts, such as fluency, magnitude, variable, field of variation and interdependence, but we can add our own links, such as order (position), whose field of variation is the set of natural numbers, and the existence of a constant variation (difference) between the terms of the sequence.
KEYWORDS: Theoretical Thinking; Teaching Mathematics; High School; Arithmetic Progression.

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Aritmética: uma contribuição para o ensino de Matemática

RESUMO

Este artigo descreve aspectos do movimento lógico-histórico do conceito de Progressão Aritmética (PA), identificando nexos conceituais (internos e externos) de Álgebra, trabalhada no Ensino Médio. A metodologia é a revisão bibliográfica de fontes brasileiras em língua portuguesa e em espanhol. A análise dos resultados aponta que o movimento lógico-histórico de PA, didaticamente, possibilita aos alunos, pelo acesso aos nexos conceituais, desenvolverem ações mentais ligadas ao pensamento teórico. Assim, o ensino de PA não é apenas uma técnica operatória, mas uma oportunidade de desenvolver um conceito como instrumento cognitivo, desenvolvido a partir das necessidades humanas. Alguns nexos internos do conceito de PA são comuns aos de outros conceitos algébricos, como *fluência*, *grandeza*, *variável*, *campo de variação* e *interdependência*, podendo ser acrescidos nexos próprios, como a ordem (posição), cujo campo de variação é o conjunto dos números naturais, e a existência de uma variação constante (diferença) entre os termos da sequência.

PALAVRAS-CHAVE: Pensamento Teórico; Ensino de Matemática; Ensino Médio; Progressão Aritmética.

El movimiento lógico-histórico de la progresión aritmética: una contribución a la enseñanza de las matemáticas

RESUMEN

Este artículo describe aspectos del movimiento lógico-histórico del concepto de Progresión Aritmética (PA), identificando vínculos conceptuales (internos y externos) con el Álgebra, que se enseña en la enseñanza media. La metodología es una revisión bibliográfica de fuentes brasileñas en portugués y español. El análisis de los resultados muestra que el movimiento lógico-histórico del PA, didácticamente, permite a los alumnos desarrollar acciones mentales vinculadas al pensamiento teórico a través del acceso a los vínculos conceptuales. Así, la enseñanza del PA no es sólo una técnica operativa, sino una oportunidad para desarrollar un concepto como herramienta cognitiva, desarrollada a partir de necesidades humanas. Algunos de los enlaces internos del concepto de PA son comunes a los de otros conceptos algebraicos, como fluencia, magnitud, variable, campo de Aviación e interdependência, pero podemos añadir enlaces propios, como orden (posición), cuyo campo de variación es el conjunto de los números naturales, y la existencia de una variación constante (diferencia) entre los términos de la sucesión.

PALABRAS CLAVE: Pensamiento teórico; Enseñanza de Matemáticas; Escuela secundaria; Progresión aritmética.

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Introduction

According to Vygotsky (1988, p. 115), "a correct organization of the child's learning leads to mental development". Based on this premise, this article sought to describe important aspects of the logical-historical movement of the concept of Arithmetic Progression (AP), content expected to be taught in the Numbers and Algebra unit in High School, according to the National Common Curricular Base - BNCC (Brazil, 2018). It is an apparently simple concept; a numerical sequence in which the difference between a term and its predecessor is constant, but which has internal connections, linking it to many other concepts in the field of algebra, constructed in its logical-historical development.

It is understood that an organization of this mathematical content's teaching-learning process, based on the history of its development and the history of its knowledge, can contribute positively to students appropriating their conceptual, internal and external connections in order to develop their theoretical thinking and lead them to training that encompasses cognitive, emotional and psychological dimensions.

The teaching-learning process of a mathematical concept, in this case, that of AP, organized in accordance with this perspective, surpasses the teaching model proposed by traditional didactics (Davydov, 1982). This didactic encourages the teaching of Mathematics based on copies, exercise lists and the memorization of procedures for solving exercises, treating the concepts as if they were separate boxes. In this model, the teacher organizes and plans the teaching-learning process of a scientific concept in four stages:

1) definition of the concept; 2) presentation of how the concept works; 3) training of the concept through exercise lists; and 4) assessment to verify whether the student has memorized the content. Thus, there is a great possibility that the student will understand that the concept has no history, that it arose instantly, ready and finished, isolated and fragmented, and that it makes no sense for their life beyond school walls.

External connections are prioritized [4], leaving the internal connections of concepts, which are the essential aspects to be explored in the process of internalizing knowledge, in the background. Furthermore, according to Sousa (2018), in the classroom, external connections are presented "completely disconnected from the various areas of knowledge from the symbolic aspect. It is as if the symbols had a life of their own; as if they spoke for themselves" (Sousa, 2018, p. 42) and were not generated from a historical, social and cultural context.

In this sense, Lima *et al*. (2001), when analyzing 12 (twelve) collections of Mathematics textbooks most used in High School, point out that they, when presenting concepts, including AP, prioritize the manipulation of formulas and memorization. The concept itself is little explored, in the sense of enabling the student to think with it in situations in which they need this concept to work through them. As Bacaro and Sforni (2021) state, it is necessary to make the student think with the concept, so that they have the opportunity to internalize it. The student has the possibility of developing theoretical thinking, when thinking about/with the concept with its different ways of being expressed, with its relations with the different areas of knowledge and as knowledge of a reality in movement.

For the development of theoretical thinking, one possibility is to organize the teaching-learning process of scientific concepts, taking into account the logical-historical movement of the elaboration of these concepts

[4] The conceptual nexus, according to Sousa, Panossian and Cedro (2014, p. 96), is the "link between the ways of thinking about the concept, which do not necessarily coincide with the different languages of the concept". External nexuses are related to the perceptible, formal elements, to the representations of the object. Meanwhile, internal nexuses concern what is in the essence of the concept. In this way, it contains the logical-historical movement.

throughout human experience. According to Kopnin (1978), it is possible to understand the *history* of a scientific concept as its process of change, that is, its stages of emergence and development, and the *logic* as the way in which thought performs this task in the process of reflecting on history, so that logic reflects the main periods of the object's history (Sousa, 2018). The mental actions that are triggered so that the concept is synthesized in the main periods of history. The concept's logical-historical movement extends from its origin to the present day, since it is adapted to the contemporary needs of humanity.

A scientific concept does not emerge from nothing. Throughout human experience, it is improved in accordance with objective reality, which, according to Caraça (1951), presents two essential characteristics: interdependence and fluency. By *interdependence*, it is possible to understand that reality is a living reality, composing the one, with its various parts, which are related to each other and to the whole. *Fluency*, on the other hand, represents the movement that all the parts and the whole perform, in a permanent state of change and evolution. According to Heraclitus of Ephesus, everything transforms, everything flows, everything evolves. Davydov (1982) assures us that, when one captures, in this reality, the connections of a scientific concept, one is faced with a theoretical thought.

In the context of what has been exposed, a question arises: what is the logical-historical movement of the scientific concept of Arithmetic Progression that can be used as a teaching instrument in High School? What are its conceptual connections?

This is the question that led to the research that gave rise to this article, developed through bibliographic research of a qualitative nature. This type of research aims to improve and update knowledge about a given object through a scientific investigation of previously published works. For Minayo (2009), the qualitative approach, in the area of Education, is used in research whose main objective is to elucidate the logic that permeates the social practice effectively occuring in reality, "[...] because human beings are distinguished not only by acting, but also by thinking about what they do and by interpreting their actions within and based on the reality experienced and shared with their peers" (Minayo, 2009, p. 21). Thus, it allows the researcher to understand the various aspects of reality, allowing for the appropriation of the internal dynamics of processes and activities.

Bibliographic research can be carried out using published works on a theory as a source of research, guiding the scientific work. For Severino (2007, p. 122), this type of research is carried out by:

> [...] available records, resulting from previous research, in printed documents, such as books, articles, theses, etc. Data from theoretical categories already worked on by other researchers and duly registered are used. The texts become sources of the themes to be researched. The researcher works based on contributions from the authors; the analytical studies contained in the texts.

Thus, in this article, the theoretical assumptions of what constitutes the logical-historical movement, as a dimension of thought, were initially presented. Afterwards, the logical-historical movement of the Arithmetic Progression. And finally, the final considerations were presented.

The logical-historical movement as a dimension of thought

In order for the subject to understand the essence of a given object or phenomenon studied, as stated by Radford (2011), it is important to organize the teaching-learning process, making a link between "modern and historical conceptual development" (p. 74). In this sense, it is understood that, by making the connection between the modern form of the concept and its historical evolution, the subject is allowed to understand the needs that motivated its emergence, its elaboration and systematization, and also its implications and relations with other knowledge produced by humanity. Understanding these aspects allows the subject to see, as a premise of scientific knowledge, evolution, change, movement, fluency and the relationship between the various areas of scientific knowledge. Learning knowledge, based on these assumptions, allows the student to understand science as something in movement, in a constant process of updating, in the search to meet the contemporary needs of humanity, be they practical, intellectual or emotional.

Human development involves learning new knowledge, whether it be assimilated through the study of scientific concepts developed and systematized by humanity or through everyday concepts acquired through social experiences. At school, students are expected to appropriate scientific concepts and, when internalized, these concepts become the source driving their intellectual, emotional and psychological development.

Among the various forms and methods that didactics can offer, as a perspective for better learning of scientific concepts, it is understood that the logical-historical perspective is a viable and consistent possibility due to the number of productions that are based on it. This perspective values learning from the essential aspects of concepts, going beyond formality and appearance and placing emphasis on their internal connections. These aspects were/are developed at unique moments in the concept's history, characterized by changes in the way of thinking about it and expressing it. Understanding them means putting thought into motion. The historical is associated with human experience, with the external; while the logical is associated with the internal, with the particular, with forms of thought, languages and the concept's formation.

Kopnin (1978) and Davydov (1982) point out that a scientific concept's formation is permeated by cultural, historical and social aspects that influenced its systematization. Thus, it is understood that, for the student to appropriate knowledge, the teacher must organize the mediating elements, which are the object and the learner, planning the teaching-learning process of the concept while considering these aspects. This means that planning must be based on the logical-historical movement of the concept, bridging the gap between the current conceptual development and that of other moments, as recommended by Radford (2011).

When considering that the logical refers to the reproduction of history in thought, it is possible to agree with Saito and Dias (2013, p. 93), when they state that "such reproduction does not mean that thought should copy the steps of history, since reproduction in thought is formation, reconstruction and elaboration. The logic of history refers to dialectical logic, which is broader than formal logic". Therefore, it is important for students to understand the paths taken by scientific concepts; paths leading from their inception until reaching their most up-to-date forms. It is essential for students to understand that scientific concepts have always been part of a complex and dynamic reality, that is, knowledge is related to other areas and is not ready or finished, since it continues to undergo a process of change in an attempt to meet the needs of each historical period of humanity, including those of the 21st century.

Organizing a scientific concept's teaching-learning process, based on its logical-historical movement, means understanding its relationships and its changes throughout the history of its construction and evolution.

It is necessary to enable students to put their thoughts into motion, so that they can understand the needs that led to the development of a concept and what changes occurred both in the way of thinking about the concept and in the way of expressing it. By carrying out this process, students begin to understand the concept as the result of an activity of human experience and to understand its internal connections. This allows the student to develop an image of the concept in their mind, which will in turn be transformed into new knowledge and which can lead to cognitive, emotional and psychological development.

Davydov (1982, p. 296) defines thought as "a very complex spiritual activity", as it is a particular process of the subject, seeking to abstract essential aspects of the object and generalize them, in order to conceptualize it. Although it is not possible to "see" a student's thought, it is possible to identify signs of development of their theoretical thought. Furthermore, according to the author, when the student grasps the internal connections of a scientific concept, they are faced with theoretical thought. Internal connections, according to Davydov (1982), are the interdependent relationships that the scientific concept presents in an integrated and constantly moving reality. The content of theoretical thought is the reflection of the results of the knowledge acquired when studying scientific concepts. The main function of the content of theoretical thought is to clarify the essential aspects of the scientific concept and generalize them.

In this sense, it is important to study the concept of AP by taking singular moments in its history as references, moments that contributed to its evolution and improvement. It is also important to pay attention to the interferences that, for one reason or another, influenced the formation and logic of its evolution.

The concept of AP is inserted in a larger area of Mathematics, Algebra, being a particular case of the concept of sequences. Its logical-historical movement is, therefore, within this context.

Some aspects of the logical-historical movement of Arithmetic Progression

In the production of scientific knowledge about natural phenomena, man seeks, through observations, to find some type of pattern or regularity that allows him to formulate laws and make predictions, which can provide a way to control them. Regularity is associated with the act of repetition, the concept of movement, the idea of fluency and, in many cases, involves relationships between two quantities (position and element). This is a mathematical concept that we call "sequence", and which, according to the Aurélio dictionary, means "action of following, of following up, continuation; series" or "continuation that occurs after something has already been started; continuity". In teaching, this concept is included in the scope of algebra.

According to Eves (1997), the word algebra has its origins in Arabic, and its genesis is in the term *al-jabr* (meaning meeting). According to this author, Algebra is a mathematical tool that allows abstractions and generalizations to be made, fundamental aspects for the development of theoretical thought. Conceiving it in a broader sense within this branch of Mathematics, and not only as a tool or language, one can deal with important concepts such as *variables*, *unknowns*, *field of variation*, *interdependence* and *fluency*. These concepts, including the concept of AP, are the bases that enable the understanding of the various algebraic objects studied in high school.

Being aware of the history and logic of the creation and systematization of algebraic concepts brings revelations about their genesis, about the paths they have taken until reaching the present day. These are important aspects that can help students understand Mathematics as a living science, in motion and with truths that can be changed and transformed according to the needs of humanity. The development of language and algebraic thought throughout history is the result of the needs experienced at various times by individuals in a given historical, social and cultural context. Thus, it can be stated that the development of AP, inserted in the field of Algebra, is a product of human activity, and from the perspective of Leontiev (1983), as something conscious and intentional.

In the relationship between man and nature, the search for ways to master the reality around him in order to satisfy his needs motivates him to build physical and abstract tools that allow him to transform reality according to his objectives. This constitutes a dialectical process, since he also transforms himself. Knowledge and development are a dialectical pair, since when acquiring knowledge, the individual develops and, when developing, they seek new knowledge.

Sequences did not emerge as a specific field of Mathematics. Initially, they were developed to meet other needs of human activities, whether practical or intellectual. The concept of sequences was forged as mathematical content was systematized and classified into specific fields. It is possible to see that, in this movement, the internal connections of algebra are present, even though there was no language that could express the perceived generalities.

Initially, we can think of the expression “Arithmetic Progression”, a special type of sequence, as an action of advancing or moving forward, in a process of successive additions or subtractions, in order to build a numerical sequence. This could, perfectly well, be an idea developed by anyone who knows the meaning of the words “Progress” and “Arithmetic”, but it can also give us the idea of counting, based on sets that have the same number of elements. It is very likely that this was how ancient people began to develop the thought around something that we know today as Arithmetic Progression.

Thousands of years ago, primitive man felt the need to count and to systematize the concepts of size, shape and number. Traces of counting methods that primitive people used at a time when writing was still rudimentary have been found on stones, clay tablets and bones. These first forms of counting were constructed through a *biunivocal correspondence*[5] between the set of elements to be counted and the set of marks made on a piece of wood, or on a clay tablet, or even using the fingers and toes. At that time, there were no words or symbols (numbers) to represent the quantities of objects in a collection or set of elements (Eves, 1997), since there were only material, concrete objects. Biunivocal correspondence, an important mathematical idea, was already present, but a language was missing that would allow for going further, though that was not necessary at that historical and cultural moment.

In Figure 1, two bones are represented in which some carved grooves correspond to certain quantities, containing the idea of sequence and the important idea of correspondence.

[5]Also called one-to-one correspondence.

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FIGURE 1: Two views of the 8,000-year-old Ishango bone found at Ishango on the shores of Lake Edward in Zaire, showing numbers preserved by carvings on a bone.

Source: Eves (1997, p. 26)

The way numbers are represented, shown in Figure 1, shows that thought and language do indeed have an intrinsic relationship, since simpler and more complete forms of representation contribute to the gain in abstraction and generalization of thought.

From the Hindu-Arabic symbols, the decimal numbering system, and the creation of a number to represent the empty space (zero), natural numbers gained a greater level of generalization. This allowed the representation of numbers of any magnitude with a small number of symbols, just 10. The set of natural numbers, fundamental to the counting process, bring, in their internal connections, the concept of biunivocal correspondence, which is a relationship between sets that are equipotent, that is, that have the same number of elements, regardless of the nature of the elements that compose them. Furthermore, in the set of natural numbers, each symbol is associated with an order in the sequence formed by them. It is worth considering that, based on the needs that arose throughout the history of humanity, natural numbers, an arithmetic progression, were the basis for the expansion of numerical sets, until reaching what we call the set of real numbers. In this process of numerical set development, from natural numbers to real numbers, the concepts of movement and infinity are intertwined, which are important concepts in the context of Mathematics, specifically in the context of Arithmetic Progression, the subject of this article.

In this process, throughout human experience, studying the regularity of phenomena contributed to the development of sciences, among them Mathematics, in a movement of abstraction, generalization and creation of laws. This area of scientific knowledge is one of the main factors responsible for the technological, economic, cultural and political advancement enjoyed by society today. One of the benefits of having mathematical knowledge is that it allows man to develop models in various areas (Economics, Automation, Health, Environment, among others), which establish laws, which are syntheses of the movement of ascension from the abstract to the concrete.

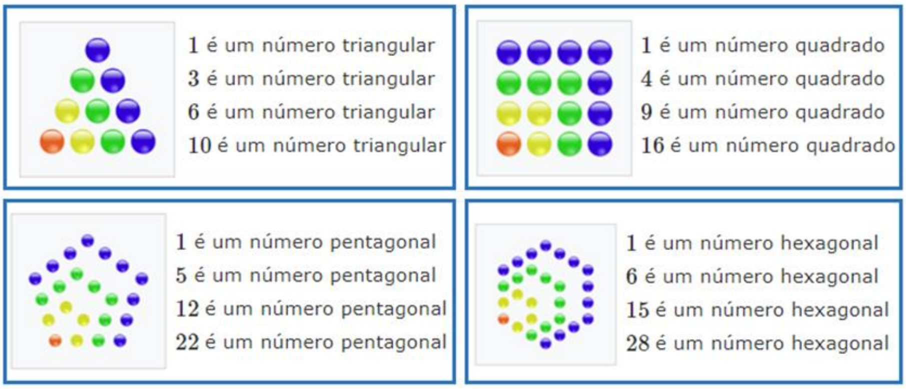
According to historians in the field of Mathematics, such as Eves (1997) and Boyer (1996), man's search for phenomena that present some kind of regularity dates back to the Babylonians and Egyptians, as well as the Asians, Europeans and other cultures. In this sense, language, algebraic thinking and the creation of symbols, as they developed, enabled a better understanding and representation of sequential models.

In ancient Egypt (around 3000 BC to 300 BC), the search for understanding the periods of flooding caused by the Nile River and the drought, when its waters receded, was very important for the survival of its people as knowing the pattern in which these seasons occurred was essential for the production of grains, the basis of their diet. Thus, understanding this regular movement of floods and droughts in the fertile lands along the banks of the Nile was intended to ensure an appropriate time for planting and to have a forecast for the harvest and how much could be expected for the final yield. Based on these studies, the Egyptians created the solar calendar, with a year consisting of 12 months and each month having 30 days. In addition to these 360 days, the year also had 5 more days of celebration, making a year of 365 days (Boyer, 1996). It can be seen that, through the observation of regular phenomena, the Egyptians were able, even at that time, to develop an annual calendar, very similar to what we have today. The months of the year formed a *sequence*, in which each month corresponded to a position in the calendar.

The Rhind papyrus, found in Egypt, also contains exercises that today could be solved using the concepts of linear equations and arithmetic progression. For example, this papyrus contains the following problem: "Divide 100 loaves of bread between 5 men so that the parts received are in arithmetic progression and one-seventh of the sum of the three largest parts is equal to the sum of the two smallest." (Cajori, 2007, p. 40).

Around the 6th century BC, in Greece, the Pythagoreans were already familiar with and developed studies on sequences, using mainly geometry and arithmetic (Eves, 1997). The best-known topic of these studies is figurative numbers, in which a sequence of flat geometric figures is constructed, each of which is formed by a certain number of points. Examples of figurative number sequences are triangular, quadrangular and pentagonal. Figure 2 shows some such numbers.

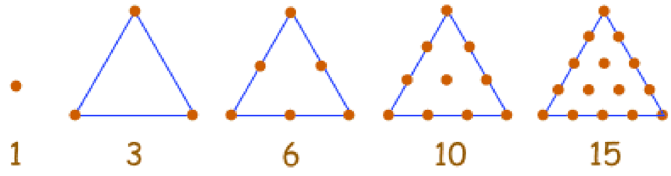
FIGURE 2: Examples of figurative numbers



Source: Images taken from Wikipedia.

With these arrangements, it is possible to work on the relationship between quantities, make recurrences, develop abstractions and generalizations. For example, taking triangular numbers, it is possible to observe important characteristics about these numbers from the figures. It starts with a dot. In the second figure, two dots are added, in the third, three dots, in the fourth, four dots, and so on, forming a sequence (Figure 3).

Figure 3: Sequence of triangular numbers



Source: Images taken from Wikipedia.

T1 = 1

T2 = 1 + 2 = 3

T3 = 3 + 3 = 6

T4 = 6 + 4 = 10

T5 = 10+ 5 = 15

.

.

.

Tn = T(n - 1)+ n

Thus, it can be observed that the number of points making up a given figure is related to the position that the figure occupies in the sequence formed by them. Here we have a relationship between two quantities, the position of each figure, represented by a natural number, and the other, represented by the quantity of points in each figure. Thus, it can be represented by means of a table:

TABLE 1: Relationship between position and number of points of each figure

Position (n)	*1*	*2*	*3*	*4*	*5*	....	*N*
Number of points of each figure	*1*	*1+2 = 3*	*3 +3=6*	*6 +4=10*	*10 + 5 =15*	....	$T_{(n-1)} + n$

Source: prepared by the author (2024).

When analyzing how the number of points in each figure was obtained, it is clear that all that is needed is to add the number corresponding to the position of the next figure to the number of points in the figure immediately before it. This is a recursive process, since it is necessary to refer to the previous term to obtain the next term in the sequence. Upon realizing this relationship, the subject begins the process of abstraction and generalization, which must go beyond external, empirical aspects. However, the symbolic language that can be used today to establish a general law, such as the one in the last cell of Table 1, did not exist at that time. The idea of sequence is present in these representations, even though they are not arithmetic progressions.

The Pythagoreans also had great admiration for the universe. They believed that it was possible to understand the Universe through the relationships between numbers, that is, through mathematical laws (Caraca, 1951). For the Pythagoreans, everything in the Universe could be expressed through a rational number, that is, it could be written as a fraction of whole terms. However, they were still unfamiliar with irrational numbers. This fact became a problem for them because when they tried to represent the diagonal of a square by the ratio between two whole numbers, they came across an object that was impossible to write as a fraction. Although this negation was the cause of the Pythagoreans' failure, it became a new negation, which ended up motivating the search for the expansion of numbers, which was important for the continuity of studies in the field of Mathematics. These studies later contributed to the development of the concepts of function and infinity (Caraça, 1951). It was a negation whose negation would create something new. This is the dialectical movement of knowledge. Both the concept of function and that of infinity have an internal relationship with the field of sequences.

It is also possible to find in the work *The Elements*, by Euclid of Alexandria (3rd century BC), problems related to the concept of Arithmetic Progression, such as, for example, “If a^2, b^2, c^2 are in Arithmetic Progression, then $b + c$, $c + a$, $a + b$ are in harmonic progression”.

Continuing the history of sequences, other civilizations, such as the Hindus and the Chinese, also made contributions. The Chinese present in the book *Nine Chapters on the Art of Mathematics*, among its 246 problems dealing with equations, surveying, engineering and other areas, some of which can be solved using the concept of sequences (Eves, 1997). The Hindus, on the other hand, were responsible for the symbols that today are called

Hindu-Arabic and that make up our decimal numbering system. This numerical set naturally forms a sequence and, from it, it is possible to construct infinite other sequences.

In the 3rd century AD, there lived another important Greek scholar in the history of Mathematics and Algebra, Diophantus of Alexandria. He was fundamental for the construction of symbolic algebra and its dissemination throughout Europe. His work consists of three works, the most important being *Arithmetic*, composed of thirteen volumes. Diophantus is considered by many to be a genius of Mathematics. In his book, *Arithmetic*, there are problems that are attractive and fascinating due to the way they were designed. An important detail is that, for him, a number means a positive rational number.

Another important mathematician who contributed to the development of Arithmetic and, consequently, Arithmetic Progression, was Leonardo of Pisa, better known as Fibonacci and mentioned in the previous topic. His book *Liber Abacci*, whose second edition was released in 1228 in Europe, contained a large number of problems related to Arithmetic and Algebra and had a strong influence on the old continent. It was through this book that Hindu numerals were introduced to the European continent.

Among the famous sequences, there is one that arouses the curiosity of many, known as the Fibonacci sequence (1, 1, 2, 3, 5, 8, ...), in which each term of the sequence from the third onwards is the sum of the two immediately preceding it. This sequence is applied in many problem situations in objective reality.

The Hindus were skilled mathematicians and were important for the consolidation of Arithmetic and Algebra. They made significant contributions to Algebra, quickly adding Arithmetic and Geometric Progressions. Bhaskara, who lived from 1114 to around 1185, is considered the last important medieval mathematician in India. His best-known work is the *Lilavati*, which was the name of his daughter. In this work, certain problems that deal with the concept of Arithmetic Progression can be found.

In Germany, at the end of the 15th century, Michael Stifel (1486 - 1567) was born, considered the most important German algebraist of the 16th century. His most famous work is *Arithmetica Integra (Renewed Arithmetic)*, which was published in 1544. According to Eves (1997), this work by Stifel is divided:

> [...] in 3 parts dedicated, respectively, to rational numbers, irrational numbers and algebra. In the first part of the book, Stifel highlights the advantages of associating an arithmetic progression with a geometric one, thus foreshadowing, almost a century later, the invention of logarithms (Eves, 1997, p. 301).

Stifel observed that the terms of a geometric progression (of ratio r) (r_0, r_1, r_2, r_3, ...) corresponded to the terms of an arithmetic progression (of ratio 1) (0, 1, 2, 3 ...), formed by the exponents, so that the multiplication of two terms of the geometric progression resulted in a term whose exponent represented the sum of the two corresponding numbers in the arithmetic progression, for example, $r_1 \times r_2 = r_{(1+2)} = r_3$. Based on his research on Arithmetic and Algebra, he formulated logarithms, independently from Napier, using approximations totally different from those used by the latter.

Some years later, Napier conducted studies on logarithms, in which he demonstrated the correspondence between Arithmetic Progression and Geometric Progression. This work was important, as it made it possible to develop and systematize logarithms. The great advance that occurred with the invention of logarithms was that they were a tool that facilitated, at a time when there were no computers or calculators, the operations of multiplication and division, which are reduced to addition and subtraction.

In the 17th, 18th and 19th centuries, the Frenchman Abraham De Moivre (1667-1754) and the German Johann Friederich Carl Gauss (1777-1855) made significant contributions to the field of Algebra. According to Eves (1997, p. 467), “De Moivre earned his living in England as a private tutor and became a close friend of Isaac Newton”. There is a legend about De Moivre that involves Arithmetic Progression:

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> [...] According to her, De Moivre had revealed, on one occasion, that from then on he would sleep, each day, 15 minutes more than the previous day. And when this arithmetic progression reached 24 hours he had in fact died (Eves, 1997, p. 368).

Gauss, from a very young age, demonstrated that he was a brilliant mathematician. According to several studies, Gauss, at the age of three, detected an arithmetic error in his father's draft. According to historians, during a math class, the students were talking, so the teacher decided to give the class, to keep them busy, the task of adding the numbers from 1 to 100. A few minutes after the teacher gave the exercise, Gauss placed the answer, which is 5050, on the teacher's desk, but without accompanying it with any calculation. According to Eves (1997, p. 519),

> [...] he had mentally calculated the sum of the arithmetic progression: 1 + 2 + 3 + ... + 98 + 99 + 100, noting that: 100 + 1 = 101, 99 + 2 = 101, 98 + 3 = 101, and so on with the 50 possible pairs in this way, the sum therefore being: 50 x 101 = 5050.

Thus, it was possible to present some significant moments in the movement of sequence/algebraic knowledge, notably, Arithmetic Progression, with the purpose of understanding the essence of this knowledge, which is logical and historical.

When studying the logical-historical movement of AP, it is necessary to consider important aspects of the logical-historical movement of the Affine Function, since the development of these two concepts is closely linked throughout human experience. The creation of one contributed to the development of the other, in a dialectical relationship. The separation of Mathematics into specific topics is recent, so that it was divided into only three large areas: Arithmetic, Geometry and Algebra.

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Of the unique moments in the history of the development of the affine function, the contributions of Galileo Galilei (1564 -1642) on the study of graphs, in search of representing one quantity in terms of another, stand out as relevant for the construction of this concept. Isaac Newton[6] (1643 - 1727), who, when studying curves that represented movements and mechanical phenomena, used the expression "fluent" to explain the concepts of function, making an important contribution to the evolution of this concept. Others who contributed significantly to the development of the concept of function were Leibniz[7] (1646 - 1716), Jean Bernoulli[8] (1667 - 1748) and Leonard Euler[9] (1707 - 1783). The term "function" was adopted for the first time in 1673, by Leibniz (1646-1716), to indicate quantities that varied along the curve, such as the tangent. In the 18th century, Bernoulli and Euler refined the concept of function, representing it by an algebraic expression.

It is interesting to study the concept of AP after students are familiar with the concept of affine function, since the first is understood as a particular case of the second concept. In this sense, the National Curricular Parameters for Natural Sciences, Mathematics and their Technologies for Secondary Education (Brazil, 2008) point out that "with regard to sequences, it is necessary to ensure an approach connected to the idea of function, in which the relationships with different functions can be analyzed" (Brazil, 2008, p. 122). It is important that students can relate the internal and external connections of these two concepts.

AP is an affine function whose domain is natural numbers. Thus, this is a restriction of the affine function. Lima (2001) states that

> sequences are functions whose domain is the set of natural numbers (infinite sequence) or the set of the first natural numbers n (finite sequence, with n elements) (Lima, 2001, p. 46).

[6] Scientist, chemist, physicist, mechanic and mathematician who discovered several laws of physics, including the law of gravity.
[7] Gottfried Wilhelm Von Leibniz, German mathematician and philosopher, "father of function".
[8] Swiss mathematician who contributed to the development of differential and integral calculus.
[9] He was the mathematician who developed the most work in history.

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In this sense, associating AP with its graph, a set of aligned points, but not a straight line, allows the student to understand the behavior of this sequence without simply needing to memorize information, and also to assimilate its algebraic representations. It is essential for the student to understand the concept of AP based on the concepts of variable, field of variation, relationship between two quantities, ordering and movement, which are also internal links of this concept, although they are links of other algebraic concepts.

In the current context of humanity, in which there is great technological advancement, one of the aspects that most contributed to this advancement was the creation of computers in the 1940s, the development of high-level programming languages from the 1960s onwards and the emergence of the internet in the 1990s. The development of these technologies is directly linked to Mathematics. Computational thinking requires the notion of space, time and the understanding of codes and symbols. The creation of machines involves, for example, the use of programming languages, which, according to Sebesta (2018), is a set of symbols and commands for the creation of programs, enabling communication between humans and computers.

The concept and techniques of PA can be used to develop various technologies necessary to meet the demands of today's society, such as the development of programs that can create machines that perform a certain task in equal amounts of time. In this sense, when teaching the concept of PA, it is necessary to provide students with activities that lead them to think about the possibilities of its use, based on the concept's appropriation.

Final considerations

The logical-historical movement of AP, discussed here, has aspects that can contribute to the teaching of Mathematics, which values the appropriation of scientific concepts, materialized in school content, and seeks their internal connections, having evidenced and substantiated the relationships among teaching, learning and human development. When dealing with logical-historical aspects of the AP concept, it was possible to perceive that this concept is inserted in a network and connected to other concepts with common internal links, but with some singularity, which characterizes it.

The bibliographic research carried out indicates that the logical-historical movement of AP, as a didactic perspective, can contribute to the teacher's organization of teaching, focusing on conceptual learning. This organization allows students to have access to the conceptual connections (internal and external) of AP and to develop mental actions that contribute to the development of theoretical thinking, involving abstraction, generalization and the formation of concepts. *Magnitude*, *interdependence* (relationship between two magnitudes), *fluency* (movement), *variable*, *field of variation*, are internal links of AP, which are also present in other algebraic concepts. However, the logical-historical development discussed shows that there are internal links that characterize the concept of AP, present since the creation of natural numbers, a special type of sequence. We can highlight the *position* (order), which assumes values in a discrete field of variation, and the existence of a *constant difference* between one term and the other of the sequence.

Another point to emphasize is that studying the concept of AP together with the concept of affine function is important for the student to understand the relationship between these two concepts, the internal and external links that are common and those that characterize each of the two concepts, highlighting the relationship between the singular and the general. It is an excellent opportunity to deal with a discrete function and a continuous function at this level of education.

These conclusions highlight the important role teachers play when it comes to organizing teaching, understanding the logical-historical movement of concepts and identifying internal and external conceptual connections, all of which are essential for making proposals that allow for the appropriation of knowledge produced by humanity and that contribute to the student's integral development.

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