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Mathematical learning: exploring the concept of function through games[1]

Caroline dos Santos[2]
Anemari Roesler Luersen Vieira Lopes[3]
Ricardo Fajardo[4]

ABSTRACT

This article, which analyzes a teaching situation, developed within the scope of a master's degree research, aims to investigate indications of the formation of algebraic thinking by students when studying the concept of function, using the game Pega Varetas. The theoretical and methodological approach adopted is based on the historical-cultural perspective. The teaching situation aimed to promote the appropriation of the concept of function based on the conceptual nexuses of variable, field of variation and dependence. It was developed with students in the first year of the Technical Course in Agriculture Integrated into High School, at a municipal school in Ijuí-RS. The actions were organized from the perspective of a didactic experiment, considering records produced in portfolios as an instrument for capturing the empirical material. As results, the formation of algebraic thinking is evident in line with the development of algebraic language and the meanings attributed by students.

KEYWORDS: Logical-historical movement; Conceptual nexuses; Algebraic thinking.

[1] English version by Silvia Iacovacci. E-mail: siacovacci@gmail.com.

[2] Master's student in Mathematics Education. Universidade Federal de Santa Maria, Santa Maria, Rio Grande do Sul, Brazil. Orcid: https://orcid.org/0000-0003-2151-8071. E-mail: carolzinisantos@gmail.com.

[3]Post-Doctorate in Education. Universidade Federal de Santa Maria, Santa Maria, Rio Grande do Sul, Brazil. Orcid: https://orcid.org/0000-0002-4636-9618. E-mail: anemari.lopes@gmail.com.

[4]PhD in Mathematics. Universidade Federal de Santa Maria, Santa Maria, Rio Grande do Sul, Brazil. Orcid: https://orcid.org/0000-0002-9416-713X. E-mail: rfaj@ufsm.br.

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Aprendizagem matemática: explorando o conceito de função por meio do jogo Pega Varetas

RESUMO

O presente artigo, que analisa uma situação de ensino desenvolvida no âmbito de uma pesquisa de mestrado, tem como objetivo investigar indicativos da formação do pensamento algébrico por estudantes no estudo do conceito de função, com o uso do jogo Pega Varetas. A abordagem teórica e metodológica adotada baseia-se na perspectiva histórico-cultural. A situação de ensino proposta visava promover a apropriação do conceito de função a partir dos nexos conceituais de variável, campo de variação e dependência. Foi desenvolvida com estudantes do primeiro ano do Curso Técnico em Agropecuária Integrado ao Ensino Médio, de uma escola municipal de Ijuí-RS. As ações foram organizadas na perspectiva de um experimento didático, contemplando como instrumento para apreensão do material empírico registros produzidos em portfólios. Como resultados, evidencia-se a formação do pensamento algébrico em consonância com o desenvolvimento da linguagem algébrica e os sentidos atribuídos pelos estudantes.

PALAVRAS-CHAVE: Movimento lógico-histórico; Nexos conceituais; Pensamento algébrico.

Aprendizaje matemático: explorando el concepto de función a través del juego Pega Varetas

RESUMEN

Este artículo, que analiza una situación de enseñanza, desarrollada en el ámbito de una investigación de maestría, tiene como objetivo investigar indicios de la formación del pensamiento algebraico por parte de los estudiantes al estudiar el concepto de función, utilizando el juego Pega Varetas. El enfoque teórico y metodológico adoptdado se fundamenta en la perspectiva histórico-cultural. La situación docente tuvo como objetivo promover la apropiación del concepto de función a partir de los vínculos conceptuales de variable, campo de variación y dependencia. Fue desarrollado con estudiantes del primer año del Curso Técnico en Agricultura Integrada en la Enseñanza Media, de una escuela municipal de Ijuí-RS. Las acciones fueron organizadas desde la perspectiva de un experimento didáctico, considerando los registros producidos en portafolios como instrumento de captura del material empírico. Como resultados, se evidencia la formación del pensamiento algebraico en consonancia con el desarrollo del lenguaje algebraico y los significados atribuidos por los estudiantes.

PALABRAS CLAVE: Movimiento lógico-histórico; Nexos conceptuales; Pensamiento algebraico.

* * *

Introduction

Students' learning of algebraic concepts can be a major challenge in basic education, especially when the organization of teaching is based solely on the formality of concepts and does not consider connections with elements that enable the production of meanings, which can favor the reproduction of a mechanized and repetitive solving technique. We therefore argue that conceptual links should be the starting point for teaching algebraic concepts, with a view to considering their logical-historical movement and learning in their essence.

This article analyzes a teaching situation as part of a research project within the Postgraduate Program in Mathematics Education and Physics Teaching (PPGEMEF) at the Universidade Federal de Santa Maria. This is a Master's thesis in which three teaching situations were developed and analyzed in a first year high school class.

Specifically, in this article, we consider the situation involving the game Pega Varetas, with the aim of promoting the understanding of variables, the field of variation, and the relationship of dependence between quantities. Therefore, we aim to study evidence of the formation of algebraic thinking by students in the study of the concept of function using the game Pega Varetas.

Next, we briefly consider the theoretical contributions that underpin our work, followed by the organization of the didactic experiment. We then analyze the evolution of the proposed instructional situation by means of extracts from the students' recordings of the triggering of seven actions (calculating the score; representing the number of sticks; identifying fixed and variable values; relating the value of the score to the number of sticks; representing all the points; approaching the algebraic expression; approaching the definition). Finally, we present some reflections on the study.

Some theoretical notes

Our study is based on the assumptions of the Teaching Guiding Activity (TGA), first proposed by Moura (1997), based on Vygotsky's Historical-Cultural Theory and Leontiev's Activity Theory. For the author, TGA is a way of organizing teaching that involves the professor and the student in an activity where the professor has to teach content and the student has to acquire it. From this perspective, "its main content is theoretical knowledge, and its object is the constitution of the individual's theoretical thinking in the movement of appropriating knowledge" (Moura et al., 2010, p. 100).

Both subjects, professor and student, in their respective teaching and learning activities, seek to promote human development through the appropriation of theoretical knowledge (Moura; Sforni; Lopes, 2017). To this end, teaching organized on the basis of AOE assumptions aims to generate the need to appropriate concepts, promoting the collective search for a solution to a problem that can mobilize students in the learning activity.

Therefore, it is important that the intention of the professor is to allow the creation of reasons for them to solve the problem, because "it is the need to solve the problem that allows the AOE to become a learning activity for the student" (Moura; Sforni; Lopes, 2017, p. 90). These solutions must be carried out collectively, so there must be a sharing of actions to solve them (Moura et al., 2010).

According to Vygotsky (1998), higher psychic functions move from the interpsychic to the intrapsychic, so they exist first at the social level, and through the process of internalization, that is, "internal reconstruction of an external operation" (Vygotsky, 1998, p. 74), they move to the individual level. Therefore, for the author, social interaction is fundamental to a child's development because those actions that are initially carried out with the help of adults, once internalized, are carried out individually. In this way, the child appropriates the knowledge historically produced by humanity, which is essential for human development since man is a social and historical being.

Therefore, the fundamental role of the school is to enable students to appropriate the knowledge produced, validated and systematized throughout history. And this can be achieved through the Learning Triggering Situation (LTS), which allows students to participate actively and collectively in the search for solutions to problems, which must take into account the nature of concepts and their emergence (Moura et al., 2010).

In order to organize the teaching of concepts, it is important to be aware of their logical-historical movement. For Kopnin (1978, p. 183), history is "the process of change of the object, the stages of its emergence and development. History acts as the object of thought, the reflection of history, as content". Thus, it refers to the history of the concept itself, its emergence, and its development in certain social contexts. And the logical is the means by which thought aims to reproduce the real historical process in all its objectivity, complexity, and contrariness. "It is the reproduction of the essence of the object and the history of its development in the system of abstractions" (Kopnin, 1978, p. 183-184).

In the case of algebraic knowledge, bearing in mind that the focus of the teaching situation addressed in this article is on learning the concept of

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function, the logical-historical movement of concepts is fundamental in the organization of teaching, because,

> [...] to understand algebraic language and also the creation and use of the algebraic symbolic system and the development of language in its complexities and contradictions, it is necessary to understand its historical movement and its essence revealed by the movement of (logical) thought (Sousa; Panossian; Cedro, 2014, p. 117).

The conceptual links, or internal links, constitute the logical-historical basis of the concept and are defined as "a link between ways of thinking about the concept that do not necessarily coincide with the different languages of the concept" (Sousa; Panossian; Cedro, 2014, p. 96). However, they differ from external links, which are limited to the perceptible elements of the concept and are formal.

According to Sousa (2004, p. 61), "the conceptual links that underlie concepts contain the logic, history, abstractions, and formalizations of human thought in the process of becoming human through knowledge". The algebraic conceptual links (fluency, field of variation, and variable) take into account the logical-historical movement of algebra and can enable the appropriation of algebraic thinking and the concept of function, since "learning variation within limits, sets, boundaries, and defined conditions means relativizing variation, creating dependencies, creating from the variable, and expanding the concept of variable to the concept of function" (Sousa; Panossian; Cedro, 2014, p. 123).

The concept of function, according to Caraça (1951), can be understood as an instrument created to interpret reality. Its teaching should not be based solely on formalism or applications to objective reality. Therefore, its external links, such as the domain, the image, the law of formation, the graphic representation, and the points of intersection with the abscissa and ordinate axes, are not enough to make the concept appropriate in its essence. Teaching must be organized with the conceptual links of fluency, variable, variation field, and dependence as a starting point.

Based on the above, three instructional situations have been developed that take these connections into account when organizing the teaching of the concept of function, with the expectation that they would be potentially learning-inducing situations. One of these is discussed in this article.

The organization of the didactic experiment

The actions were organized from the perspective of a didactic experiment, which, as Vigotski (2003 apud Cedro, 2008, p. 105) points out, aims to study the development of higher psychological functions during the schooling process. Therefore, we designed and developed teaching situations in order to understand the phenomenon under study, i.e., students' learning. Here we present the Pega Varetas game and the first approaches to the concept of function.

Thirty-nine students participated in the study (we will call them E1, E2, ..., E39) from a first-year class of the Integrated Agricultural Technical Course in a municipal school in Ijuí-RS. The whole project took place in May, June, and July 2022, in 14 lessons with 29 periods of 50 minutes. The teaching situation presented in this article was developed in May in three lessons for six periods.

The students are residents of Ijuí or neighboring communities. About half of them live in rural areas, and most of them came to this school specifically for the technical course, so they came from different institutions. The first contact with the class was through the math professor.

At the beginning of the research, the researcher followed the math classes for about two months and interacted with the students. She wanted

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to get to know them and understand the context in which they lived in order to organize the teaching experiment.

The proposed teaching situations were based on the AOE, which, as an activity, is structured in such a way that it "allows subjects to interact, mediated by content, sharing meanings, with the aim of solving a problem situation together" (Moura, 2018, p. 159). Thus, throughout its development, the students were organized in groups that changed in each lesson, allowing for greater interaction among the class.

The empirical material consisted of audio and video recordings of all the lessons, individual portfolios provided by the researcher, a learning assessment worksheet at the end of the experiment, and a feedback form. In this article, we will use the recordings made in the portfolios, as shown below.

The stick game in functional teaching

On the first day, the researcher instructed the students to organize themselves into groups of three or four and told them the rules of the game. The game consists of 21 sticks, 5 yellow, 5 red, 5 green, 5 blue, and 1 black. The participant must pick up the sticks one by one, without moving the others, and get the highest score, which is given by color as follows: yellow (5 points), red (10 points), green (15 points), blue (20 points) and black (50 points). Actions were developed from the game, as described below: Calculating the score; Representing the number of rods; Identifying fixed and variable values; Relating the score value to the number of rods; Representing all the points; Approaching the algebraic expression; and, Approaching the definition.

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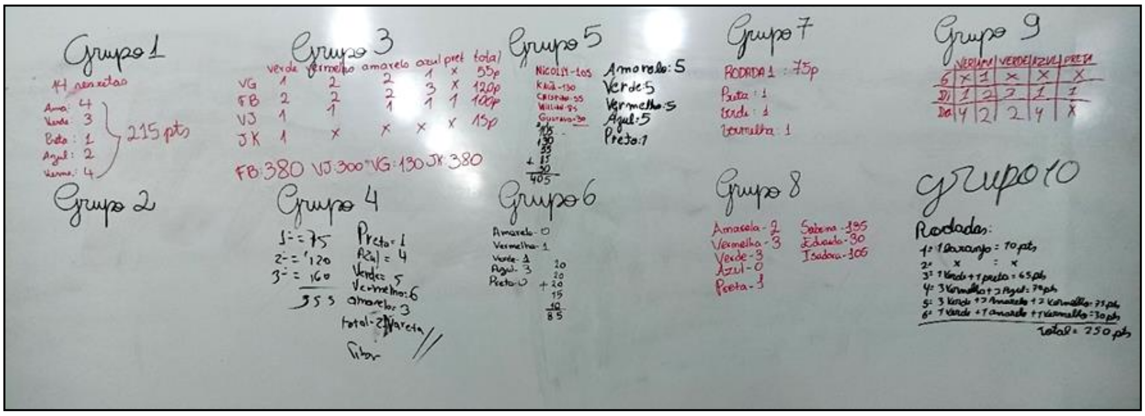
Calculating the score

Initially, the groups played freely to familiarize themselves with the game and its rules. The researcher then asked the students to record their scores at the end of each round, in whatever way they preferred. They had to count the number of sticks of each color and calculate the score for each player. After a few rounds, the groups socialized the way they recorded their scores by writing on the white board, as shown in Figure 1.

FIGURE 1: Records made by the groups



Source: Research data

After looking at the records, we discussed together the quickest and easiest way to record the scores. After a tumultuous debate, some students chose the tabular form for groups 3 and 9, but many disagreed, preferring to write down the colors and suggesting that the names be abbreviated, using only the first three letters, so that the record would be quicker.

It should be noted that the researcher did not interfere with the records . The responses were varied, but some groups didn't understand the suggestion. This was the case with group 5, which added up the scores of all members, and group 9, which recorded the number of sticks but didn't calculate the score of each player. This should be fixed in the next actions.

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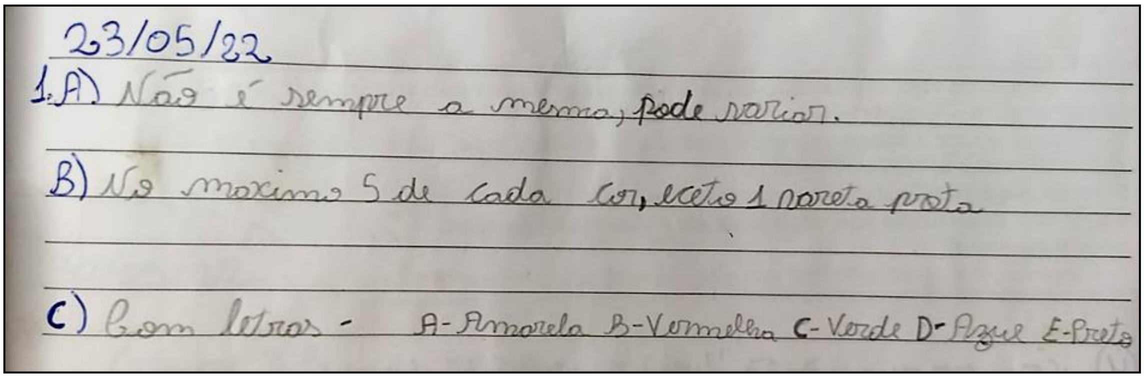
Representing the number of sticks

Then, with the intention of helping students understand variables and areas of variation, the following questions were suggested: Q. 1-a - *Is the number of sticks of each color received by each player always the same, or can it vary? Question 1-b - What are the outcomes of each color? Q. 1-c - How can we represent the number of sticks of each color if they don't have a fixed value?* These questions could be answered by looking at the results obtained in each round, and Figure 2 illustrates the answer given by one of the students.

FIGURE 2: Records made by E20 in response to question Q.1



Source: Research data

All the students identified, as did E20, that the number of sticks of each color obtained by each player at the end of the round and the final score are not fixed, i.e., they vary. Variables are therefore needed to represent these quantities. According to Caraça (1951), the correspondence between sets of numbers is the essence of the concept of function, and, to make it easier to handle, a symbolic representation is needed for the sets, introducing the concept of variable. Therefore,

> Let (E) be any set of numbers, a finite or infinite set, and we will conventionally represent any of its elements by a symbol, for example: x. This symbol representing any of the elements of the set (E) will be called a variable (Caraça, 1951, p. 127).

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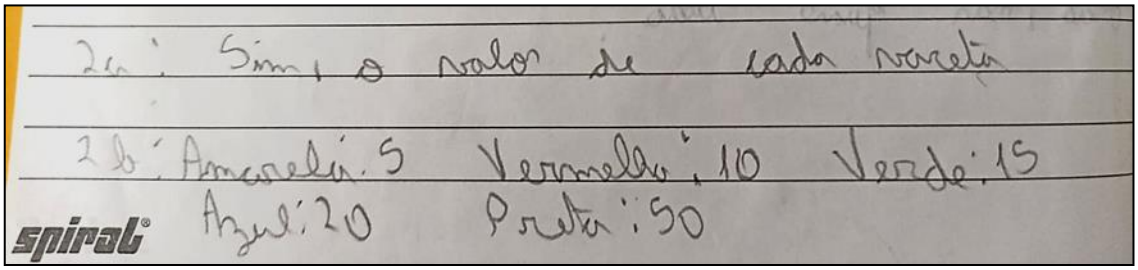
When asked how they could represent the number of rods of each color, most of the students mentioned the use of letters, stressing the need for them to be different for each color. As we hadn't yet defined the concept of a variable, the students referred to it as a letter, even though they understood that the values vary.

The existence of a variable is related to a field of variation, which determines the possible values it can take on in a given context. "The field of variation depends directly on the movement of the reality being dealt with. There is no ready and absolute answer, although a large part of the movements of reality seem to occur in the field of real numbers" (Sousa, 2004, p. 158).

Identifying fixed and variable values

Along with the above, the following questions were proposed*: Q. 2-a - Are there fixed values that do not change? Q. 2-b - What are these values?*, with the intention of students realizing that some values vary and others are fixed. Most of the students answered that the scores for each stick are fixed, as shown in Figure 3.

FIGURE 3: Records made by E34 in response to question Q.2



Source: Research data

Others cited values related to the scores of each stick, such as the total score that can be obtained with sticks of the same color, i.e., with five yellow sticks, a total of 25 points can be reached.

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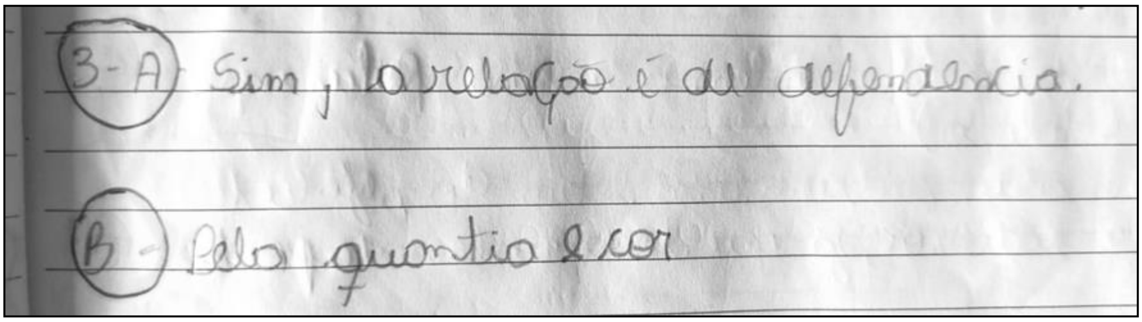
Relating the score value to the number of sticks

In order to broaden the discussions and make the students see that there is a relationship of dependence between the quantities, given that the final score depends on the number of sticks obtained, we proposed the following questions: *Q. 3-a - Is the value of the final score related to the number of sticks of each color? Q. 3-b - How are they related*? Although we hadn't mentioned the term dependency before, some students had already used it when answering the questions, as shown in Figure 4.

FIGURE 4: Records made by E29 in response to question Q.3

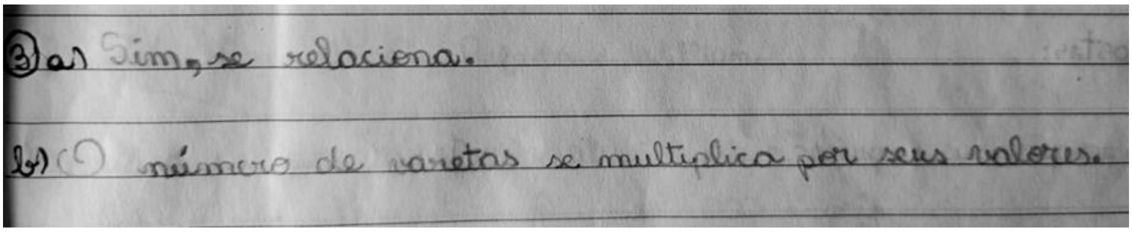


Source: Research data

In the case of E29, shown in Figure 4, we can see that she realized that the relationship of dependence exists and that the final score depends not only on the number of rods, but also on their colors, as the values are different. According to Caraça (1951), interdependence is one of the two essential characteristics of reality. According to him, "All things are related to each other; the World, all this Reality in which we are immersed, is a living organism, one whose compartments all communicate and participate in each other's lives" (Caraça, 1951, p. 109). And for Lanner de Moura and Sousa (2008), interdependence is translated into algebra by the dependence between variables.

Other students expressed the relationship of dependence based on the mathematical operations needed to obtain the final score without using the term, as shown in Figure 5.

FIGURE 5: Records made by E25 in response to question Q.3



Source: Research data

E25's answer shows that she recognized a relationship, even without mentioning the dependency, and explained the operation to be performed to calculate the final score, i.e., you have to multiply the number of sticks of a certain color by their respective value. Therefore, there is evidence that she understood that the score can only be obtained if the number of sticks of each color obtained in the game is known. Understanding the dependence between variables is fundamental to understanding the concept of function because

> The function is the motion itself, and every motion to be studied in depth has as its starting point the conditional if it has a premise. To study any movement in reality with a function, we must choose a field of variation, a numerical field. By creating it, we have the dependence, the interaction, and the relativization of the movement, even if this is the substance, the fundamental, because movement is relative. The field and the image have to do with the relationship of dependence that characterizes the function (Sousa, 2004, p. 175, emphasis added).

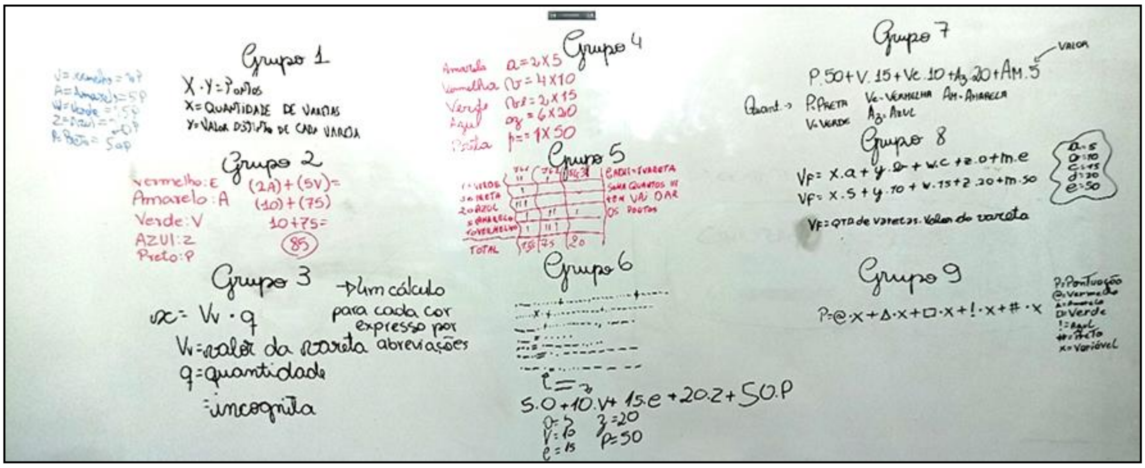
The questions raised above were answered by the students, organized in groups, in their portfolios. They were then debated collectively in the class, with a summary of the discussions held in each group being shared.

Representing all points

We then posed the following problem: *How can we represent the calculation of all the possible points in the game and the number of sticks in each round?* With this problem, we wanted to encourage the students to come up with an algebraic expression to calculate the final score obtained by a player based on the number of sticks of each color multiplied by their respective value.

In this situation, the students were expected to represent the final score and the number of sticks of each color as variables. After discussing the problem in groups, we shared the answers on the white chart, as shown in Figure 6.

FIGURE 6: Records made by the groups in response to the problem



Source: Research data

We see a wide variety of representations, which may indicate that the students were at different stages of conceptual development. This may

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be due to their different interactions with algebraic concepts in the years prior to high school.

Figure 6 shows us that Groups 2, 4, and 5 answered the problem with records from a specific round and were unable to write an algebraic expression that could generalize the game situation. This difficulty encountered by the students may have originated in "the use of algebraic language due to the characteristics of this symbolic system itself or due to the break between algebraic knowledge and arithmetic knowledge" (Sousa; Panossian; Cedro, 2014, p. 142-143).

With Group 2, the letters used in the register represent the fixed value of each color, i.e., they are unknown. Therefore, these fixed values should appear in the algebraic expression. For Group 4, the letters used represent the score obtained with each color, i.e., the result of multiplying the number of sticks of a given color by its value. However, the number of sticks of each color should be represented by a variable. Finally, group 5 represents the number of sticks of each color using dashes in a table and calculates each player's score in a given round without using variables.

The records of groups 1 and 3 in Figure 6 show us that the algebraic expression written does not take into account the different colors and their respective values, but rather the score obtained by multiplying the number of rods by their value. Group 9, on the other hand, chose to use different symbols to represent the colors of the sticks, including a legend to identify them. When they met with the class, they clarified that the symbols were used to identify the colors and to check the score. Since they didn't associate the symbol directly with the value of each color, this may have made it difficult to use the fixed values in the expression. In this sense, there may be indications of algebraic thinking in the writing of the expression, even if it is not in the most appropriate mathematical language, because

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> Based on the historical evolution of algebra, we can consider the existence of algebraic thinking prior to the use of the language that underpins symbolic algebra, and this thinking is enhanced as the student uses the most appropriate language (Sousa; Panossian; Cedro, 2014, p. 142).

In addition, students in Group 9 used the letter x, identified in the legend as a variable, to multiply each of the symbols, explaining that it represented the number of sticks of each color. After questioning, we realized that these students understood the idea of variable in the sense that the values of x could vary in the same algebraic expression, i.e., the same letter x could take on different values. For this reason, the group members didn't feel the need to use different letters for each color, since the number of sticks could be different.

Groups 6, 7, and 8 used the fixed values of each color when writing the algebraic expression and represented the number of rods of each color by variables. However, Groups 6 and 7 did not represent the final score in the expression, i.e., they did not write the dependent variable. Thus, they did not identify what would result from the calculation. We can assume that the students understood that the expression would result in the player's final score, but had not yet grasped the concept of the function that had just been introduced.

Groups 6 and 8 used different letters as variables to represent the number of sticks of each color. Group 8 used letters of the alphabet that had nothing to do with the name of the color, while Group 6 used letters that were part of the word but not its initials. Unlike Group 7, which used the initials of each color, they chose to include the second letter to distinguish between colors that start with the same letter, i.e., they used two letters to represent the same variable, for example, "az" for blue and "am" for yellow, as shown in Figure 7.

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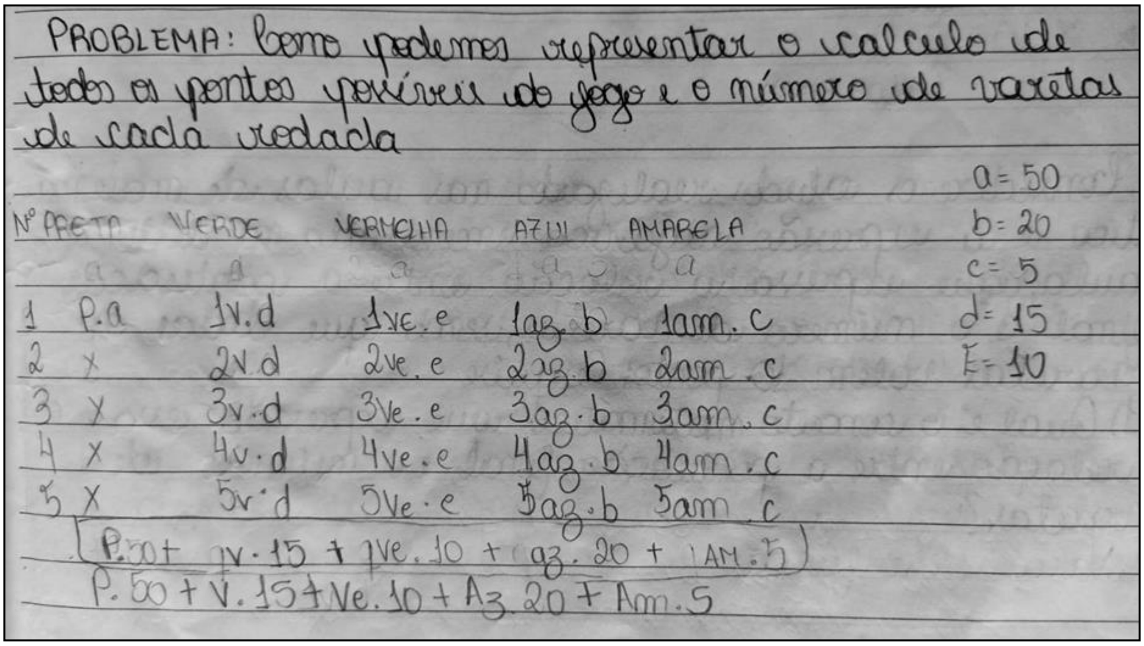
FIGURE 7: Records made by E9 in response to the problem



Source: Research data

In Figure 7, we see the path taken by Group 7, to which E9 belongs, to work out the algebraic expression. They started with the names of the colors and wrote down possible results from a few rounds to find a generalization, using only the initial letters of each color. In this way, there is a symbolic representation of each color, but it is still tied to the senses because the letters had to refer to the color, which can make it difficult to write in a formal symbolic language.

The logical-historical movement of the concept of variable is essential for its understanding, since it went through different representations until it reached the use of only one letter of the alphabet. According to Caraça (1951), the variable does not coincide individually with any of the elements of the set, but can represent all of them. Therefore, the variable is "[...] the symbol of the collective life of the whole, a life that is nourished by, but not reduced to, the individual life of each of its members" (Caraça, 1951, p. 127).

The students' difficulty in using only one letter as a variable may be related to the recent emergence of symbolic algebra, since

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> [...] Algebraic symbolism, as we know it today, emerged about 400 years ago. In its most synthetic form, the Frenchman Viète (1540-1603) is considered the "father" of symbolic algebra, as he used literal symbols for unknown quantities and also for given quantities, generating parameters. Viète used only letters to study motion (Sousa; Panossian; Cedro, 2014, p. 113).

When teaching algebraic concepts, it is essential that the professor takes into account their logical-historical movement when designing teaching situations, in order to enable students to experience this process of constructing the variable up to its more synthetic representation.

According to Sousa, Panossian and Cedro (2014), by considering the use of words, letters, signs, and symbols, it is possible to trace a path for algebraic language that passes through the non-symbolic - rhetorical, geometric, syncopated - until it reaches the symbolic. In rhetorical algebra, the word variable is considered; in geometric algebra, the figure variable; in syncopated algebra, the numeral variable; and in symbolic algebra, the letter variable. It is therefore understandable that students use other representations of a variable before using just one letter of the alphabet. "From the middle of the 17th century, symbolic algebra began to impose itself as scientific knowledge. The use of symbolism was intended to do more than simply synthesize writing, it was intended to facilitate the use of thought" (Sousa; Panossian; Cedro, 2014, p. 114).

Based on the above, we can say that Group 8 was the closest to the expected algebraic expression because it used only one letter of the alphabet to represent the variables referring to the number of sticks of each color, except for the final score, which did not refer to the name of the color, and also considered the fixed values.

While some groups came close to this expression, others were far from a generalization that could represent the relationship between the quantities involved. This leads us to understand that "it is necessary for the professor to be aware of the difficulties of the students' thinking in appropriating a formal symbolic language, which has been elaborated for centuries by various civilizations" (Sousa; Panossian; Cedro, 2014, p. 136). This idea guided our actions.

Approximating the algebraic expression

We then told the students that all sticks, regardless of color, would have the same score of 10 points. We gave them time to play again using the modified rules and asked the groups the following questions: Q. *4- Can we use the same algebraic expression as in the previous activity to represent the total score and the number of sticks of each color? Please explain. Q. 5- If not, what algebraic expression can we use? Q. 6-a - Is the final score related to the number of sticks? Q. 6-b - How? Q. 7 - Is it possible to get the final score without knowing the number of sticks? Explain.*

The groups answered the questions in their portfolios and then shared their ideas with the class so that we could reflect on the answers together. This activity was designed to help students identify the simplest expression that expresses the relationship between the final score and the number of bars, and to verify that it is a function. Figure 8 shows one student's answers.

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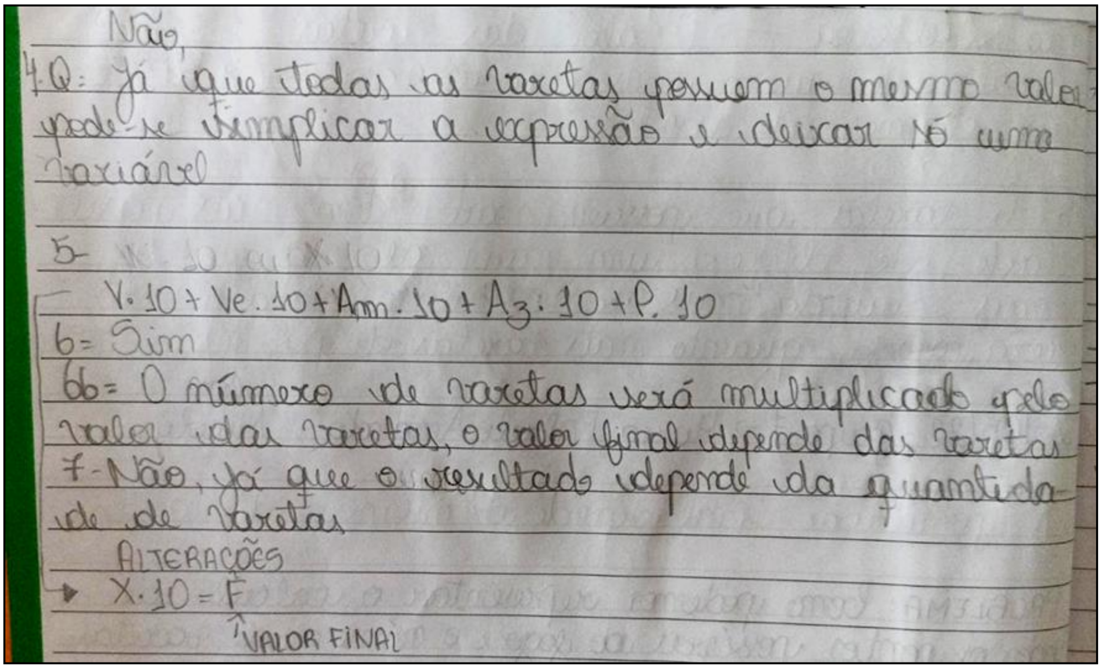
FIGURE 8: Records made by E9 in response to questions Q.4, Q.5, Q.6, and Q.7



Source: Research data

In response to question Q.4, the student showed that she understood that the algebraic expression could be simplified, i.e., since all colors would have the same value, it would not be necessary to know how many sticks of each color were obtained, only the total. When she explained that there would only be one variable, we assumed that she was referring to an independent variable, since it doesn't represent the dependent variable in the expression. She changed the expression and wrote, in a simpler way, a single variable to represent the total number of rods. During the socialization with the class, we noticed that most of the students managed to write an algebraic expression appropriate to the situation so that the colors didn't interfere with the result, differentiating one from the other only by the letters of the alphabet chosen as variables.

Writing the algebraic expression may have been facilitated by the perception of regularity of the results obtained, i.e., the number of rods obtained is always multiplied by 10. Regular movements can be understood using algebraic thinking because

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> From the moment we study and understand these movements, we create quantitative and qualitative movements based on laws. In certain studies, qualitative laws can be represented on the basis of quantitative ones, to such an extent that we create functions (Sousa, 2004, p. 94).

Also in Figure 8, the student considered the relationship of dependence between the quantities involved in her answer to question Q.6-b, clarifying that the final value would depend on the rods, which contributed to her answer to question Q.7, since the dependent variable would only be obtained from the value of the independent variable. We would point out that the students found it easy to answer the questions, and this may be because of the discussions held earlier.

Approaching the definition

At the end of the lesson, after socializing, the researcher asked the students to carry out a homework assignment to find out what mathematical concept was involved in the situation studied, considering the dependency relationship between the quantities involved and the algebraic expression found. The proposed questions were: *1) What is the mathematical concept that expresses this relationship between the final score and the number of rods? 2) What is the definition of this concept?*

In the next lesson, they recalled the algebraic expression they had found, and the quantities represented by the variables. Afterward, they shared the answers they had found as a homework topic. For the most part, they were able to identify that it was the concept of a function and present a definition, as illustrated in Figure 9.

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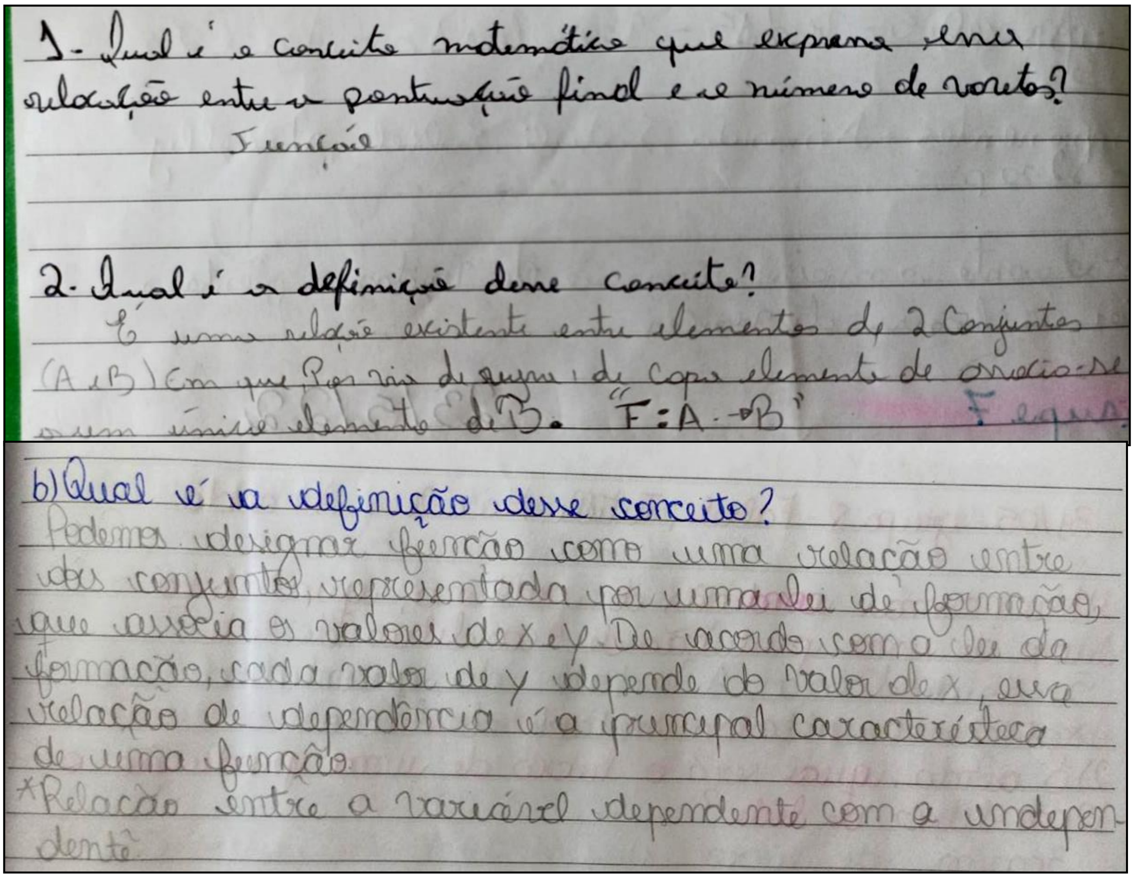
FIGURE 9: Records made by E2 and E9 in response to questions 1 and 2



Source: Research data

However, some of them mentioned the concept of an equation, which might indicate that they didn't attach any importance to the unknown. So the researcher asked them to work out a first degree equation, choosing '2 + x = 3', which was solved collectively. After questioning, they realized that there is only one number that can satisfy the expression, i.e., it is not a variable. Everyone then concluded that in the equation, the letter x took on the role of unknown, as it is important to distinguish it from the variable.

> Analyzing the historical movement of algebra, we can see that the notion of unknowns has been present since the singular moments of rhetorical algebra, which used natural language and generated particular methods for finding unknown values. However, variation and the notion of variable were not

> introduced at that time. It was only after the studies of Viète and later Descartes that the notion of variable was associated with its own symbology and developed in response to human needs to control the movement of quantities and advances in scientific knowledge (Panossian; Moretti; Souza, 2017, p. 142).

We then compared the equation with the algebraic expression found earlier (F = 10x). By identifying their differences, especially the relationship of dependence between the variables, the students realized that it was a function. So we systematized the definition of the function and identified the total score as the dependent variable and the number of rods as the independent variable.

Final considerations

The aim of this article is to study the formation of algebraic thinking by students when studying the concept of function, using the game Pega Varetas. To this end, we discuss a teaching situation aimed at introducing the concept of function, based on the conceptual links of variable, field of variation and the relationship of dependence between quantities.

The design and development of the teaching situation, in its different stages, allowed essential discussions on the constitution of the conceptual links and their relevance for the approach to the concept of function. The aim of this activity was to introduce this concept, which was studied in the following lessons.

From the analysis of the data, it was possible to identify indications of the formation of algebraic thinking by the students, because at the beginning, through the game Pega Varetas, the first records made and the questions proposed, the students realized that some values vary within a limit and that there is a relationship of dependence between the quantities involved. Next, they used these perceptions in the proposed problem, which required writing an algebraic expression, since it required the use of variables, taking into account the dependence between the final score and the number of sticks obtained of each color. Finally, the change in the score of the sticks, whose value became equal, made it possible to write a law of formation of a function with the dependent variable and the independent variable.

By analyzing the evolution of the teaching situation, we can see that there has been an improvement in the constitution of the algebraic language and in the production of meaning by the students of the actions triggered. Thus, both the use of variables, which went through non-symbolic algebraic representations before arriving at formal writing with the use of a letter of the alphabet, and the meanings attributed by the students in the recordings made, can show the formation of algebraic thinking.

Finally, two aspects should be emphasized. The first refers to the importance of the intentional organization of the teaching in this didactic experiment, in that the teaching situations were designed and developed on the basis of the Teaching Guiding Activity, with the aim of triggering the students' learning and enabling them to be active. The second refers to the importance of conceptual links in the organization of teaching, since concepts such as variables, which are not a specific teaching topic in the school curriculum but are essential for the development of algebraic thinking, cannot be discussed in classes that prioritize only technical and repetitive manipulations.

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