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Proposal for teaching binomial distribution for high school students incorporating technology[1]

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ABSTRACT

This paper describes the application process of a design for the teaching of binomial distribution which incorporates the use of *Fhatom* software. Thirty-five Chilean high school students (15 to 18 years old) participated in the experience. The teaching design includes manipulative, computational and algebraic representations, as well as some of the recommendations of the GAISE project. A task with a context close to the students is proposed, and its solution implies the generation of sampling distributions. Based on this information, several questions are proposed that allow conjecturing about the concept and characteristics of the binomial distribution, which are later formalized. Trough the development of the proposal, the students were able to understand some concepts of the binomial distribution.

KEYWORDS: Secondary Education; Binomial distribution; Statistics; Technology.

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Propuesta de enseñanza de la distribución binomial para estudiantes de Bachillerato incorporando la tecnología

RESUMEN

En este trabajo se describe el proceso de aplicación de un diseño para la enseñanza de la distribución binomial el cual incorpora el uso del software *Fhatom*. En la experiencia participaron 35 estudiantes chilenos de 3º curso de Educación Media (15 a 18 años). El diseño de enseñanza considera representaciones como la manipulativa, computacional y algebraica, además se recogen algunas de las recomendaciones del proyecto GAISE. Se plantea una tarea con un contexto cercano para los estudiantes y que su solución implica la generación de distribuciones muestrales. A partir de esta información se proponen diversas cuestiones que permiten conjeturar sobre el concepto y características de la distribución binomial que posteriormente son formalizadas. Los estudiantes a través del desarrollo de la propuesta de enseñanza pudieron comprender algunos conceptos de la distribución binomial.

PALABRAS CLAVE: Educación Media; Distribución binomial; Estadística; Tecnología.

Proposta de ensino da distribuição binomial para alunos do ensino médio incorporando tecnologia

RESUMO

Este trabalho descreve o processo de aplicação de um projeto para ensino da distribuição binomial que incorpora o uso do software *Fhatom*. Participaram da experiência 35 estudantes chilenos do 3º ano do ensino médio (15 a 18 anos). O projeto de ensino considera representações como manipulativas, computacionais e algébricas, e também algumas das recomendações do projeto GAISE. Propõe-se uma tarefa com um contexto próximo aos estudantes e a sua resolução implica a geração de distribuições amostrais. Com base nesta informação, são propostas várias questões que permitem conjecturas sobre o conceito e características da distribuição binomial que posteriormente são formalizadas. Através do desenvolvimento da proposta de ensino, os estudantes conseguiram comprender alguns conceitos da distribuição binomial.

PALAVRAS-CHAVE: Educação media; Distribuição binomial; Estatisticas; Tecnologia

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It has been argued that technology offers a range of affordances that could revolutionise statistics education but there are reasons why that transformation has not yet happened and indeed might never happen, except in certain pockets of good practice.
Pratt, Davies & Connor (2011, p.103)

Introduction

The future adults who form the educational centers must be able to make judgments for subsequent decision making, in this sense Gal (2005) states that the importance of probability for adults lies in its personal and social function, when facing situations that require interpretation of probability, generation of probability judgments, or decision making.

The teaching of statistics and probability has become a necessity as citizens require knowledge of quantitative data analysis to understand the information present in various fields and thus perform successfully in today's society (Engel, 2017; Ridgway, 2022).

In this sense, probability allows quantifying chance and forms the basis of statistical predictions, hence its special relevance in statistics, and also in mathematics (Bargagliotti et al., 2020). Batanero and Borovcnik (2016) highlight the need for the development of probabilistic reasoning, since they combine both mathematical and philosophical aspects, which could have an impact on difficulties for students, since they eventually associate theoretical and subjective ideas in the resolution of tasks.

In this line, various biases or misconceptions about probability have been identified in the resolution of tasks posed in contexts of uncertainty,

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such as the equiprobability bias, the isolated outcome bias, the perception of randomness, among others (Batanero; Sánchez, 2005).

This paper focuses on the binomial distribution, since it is one of the most widely used probabilistic models for dichotomous variables in everyday and professional situations (Sánchez; Landín, 2011). It considers performing a random experiment and observing whether a certain event occurs or not, where p corresponds to the probability that it occurs and q=1-p that it does not occur, therefore, the variable would take two values, 1 if it occurs and 0 if it does not occur.

On the other hand, the binomial distribution corresponds to performing a random experiment n times, where each experiment is independent of the previous one. In this case the random variable X follows a binomial distribution of parameters n and p denoted by X~ B(n, p) where n corresponds to the number of repetitions, while p is the probability of success.

Recognizing the importance of this probabilistic model, high school curriculum guidelines from different countries (Landín; Sánchez, 2010) incorporate the study of the binomial distribution.

In Chile, the binomial distribution is proposed in the 4th year of Secondary Education (17-18 years old) in which students are expected to be able to "base decisions in situations of uncertainty, based on the critical analysis of statistical data and based on the binomial and normal models" (Mineduc, 2019, p. 111). Likewise, the curricular guidelines for this course recommend the inclusion of technological tools in the study of the different topics.

In the same line, the GAISE II project points out that "modern statistical practice is inseparable from technology and many software tools and applications are freely available to enhance students' understanding, so it is recommended to embrace technology to the greatest extent possible" (Bargagliotti et al, 2020, p.73).

In addition, GAISE II makes recommendations for teaching statistics and probability with the idea of making sense of the data:

- Pose research questions that promote the collection and analysis of data to answer these questions.
- Consider different types of variables, since data collection, analysis and interpretation will be planned according to their type.
- Incorporate multivariate thinking at different educational levels.
- The importance of probability for probability quantification.
- The inclusion of technology in statistics education.
- The importance of assessment focused, especially, on the understanding of the concepts involved in the development of statistical reasoning.

Thus, the objective of this work is to present a proposal for teaching binomial distribution together with its implementation in a group of Chilean students who were finishing the 3rd year of Secondary Education (15 to 17 years old), although this subject is proposed in Chile for the 4th year of Secondary Education (Mineduc, 2019), the students already had the knowledge and skills to participate in the experience.

Fundamentals

This section describes the elements considered for the design and implementation of the binomial distribution teaching experience.

Background

There are studies on this subject that try to seek improvements in student learning, based on strategies that are far from memorizing concepts or performing algorithmic calculations, since this way of

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teaching does not guarantee an optimal understanding of the concepts (Batanero; Borovcnik, 2016; Batanero et al., 2000). In this context, García-García et al. (2018) develop a teaching experiment of the binomial distribution with Mexican high school students (16-17 years old). To do so, they applied the same questionnaire before and after the implementation of a formative activity. The students were presented with an activity in a coin-tossing context that incorporated computational simulation through the use of Fathom software. The results suggest that the use of the software allowed the students to improve the results obtained in the pre-test, since they considered variability as a relationship between what the theoretical distribution of the random variable predicts and what occurs in a series of experiences of that variable. The authors emphasized that the use of Fathom promoted a better understanding of the concept by facilitating the students' simulation of a large number of trials of the experiment, where they were able to observe data frequencies that they would not have been able to visualize if the experiment were done manually.

Regarding the difficulties in understanding the binomial distribution, Begué et al. (2019) analyzed the arguments used by high school students (17-18 years old) when performing a sample generation activity. Students were posed that when throwing one hundred thumbtacks 68 fell with the tip up and 32 with the tip to one side, and they were asked to conjecture about the results when repeating the same situation in four supposed cases. The responses showed that the students confused the concepts of randomness and probability, without noting that probability is the measure of the degree of uncertainty, coinciding with previous studies (Konold, 1989). They also observed that a low percentage of students argue based on the frequentist approach to probability, some alluding to the asymmetry of the object used, and almost half of the participants showed a limited understanding of randomness.

Another study on the subject is found in Begué et al. (2021), where the researchers posed a situation based on throwing a ball into a basket, but this time the situation is based on a small sample size (n = 10), where the random variable is defined by the number of hits on the basket. In this case the students were asked to estimate what was possible from the data in the statement. The results showed that more than 50% of the students estimated values arguing with the frequentist approach to probability, contrary to Begué et al. (2019). As in this occasion the sample size was limited only to n = 10, the sample variability presented greater range compared to larger sample sizes, so it was expected that students would argue alluding to this property, however, most of them did not justify in this sense. As in Begué et al. (2019), students did not distinguish between randomness and probability. The authors attributed it that for students randomness and probability are similar concepts, where they do not distinguish probability as a measure of the degree of uncertainty.

Role of technology

Technology is part of the daily life of today's society, and the educational field is not exempt. In this sense, technology has been incorporated into the mathematics classroom progressively (GARCÍA, 2019), where teachers have had to implement various technological resources to promote motivation and understanding of various concepts.

In the case of Statistics and Probability, the incorporation of technology has facilitated, for example, the calculation and experimental teaching of various concepts through the use of simulators. Different authors (Biehler et al., 2013; Chance et al., 2007; Pratt et al., 2011) advise the use of technology for the study of Statistics and Probability, since it favors the visualization of different representations contributing to a better understanding of the concepts.

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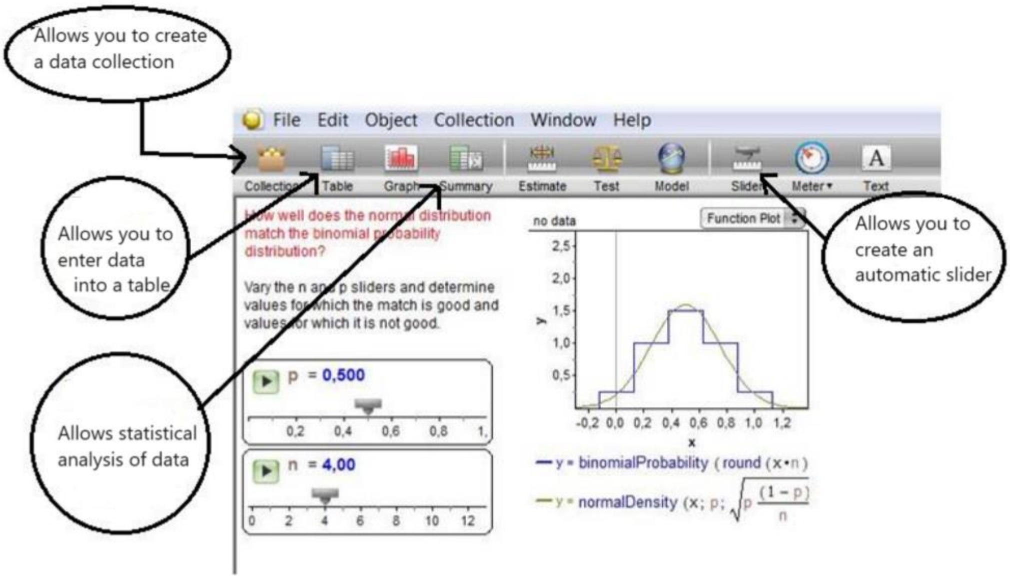
There is a wide variety of software with different characteristics used for data analysis (e.g. RStudio, Python). In this work, Fathom software was chosen, as in other research aimed at the study of probabilistic models (Cisternas et al., 2021; García et al., 2018; Landín; Sánchez, 2010; Salinas-Herrera; Salinas-Hernández, 2022).

Fathom is a paid application, which has an interface close to the user that facilitates the development of different procedures in an efficient way, contributing to the understanding of concepts of Statistics and Probability, such as, for example, sampling distributions and applications (Biehler et al., 2013; García-García et al., 2018). One of the strengths of this software is that it facilitates the performance of a large number of simulations in the context of a randomized experiment, in addition it allows to represent the data and to perform the calculation of statistics (Alpízar, 2007). Figure 1 shows the software interface with some of its functions.

FIGURE 1: Fathom Software Interface



Source: The authors.

Description of the experience

In the context of the importance of teaching statistics and probability based on promoting the capacity for reasoning in an argumentative way, together with making decisions based on data (Garfield et al., 2008), a teaching proposal is designed as described below.

Sample

The sample is intentional and by convenience (Izcara, 2014), and is composed of 35 Chilean students who finished the 3rd year of High School (15 and 17 years old) of a private subsidized school in Valparaíso. In order to carry out the teaching experience, consent was requested from the director of the establishment together with the teachers following the ethical norms. This experience was carried out in two sessions of the mathematics class (90 minutes each), and was directed by the first author together with the teacher of the subject.

The participants during the course, according to the curricular guidelines (Mineduc, 2020), had worked on aspects such as the interpretation of data in situations of uncertainty based on measures of dispersion or conditional probabilities, arguing the veracity of conjectures through the use of symbolic language and different types of representations.

Proposed task

Given the students' interest in the Pokemon game, at the time of designing the teaching proposal, it was decided to incorporate it in the context of the proposed task (Figure 2).

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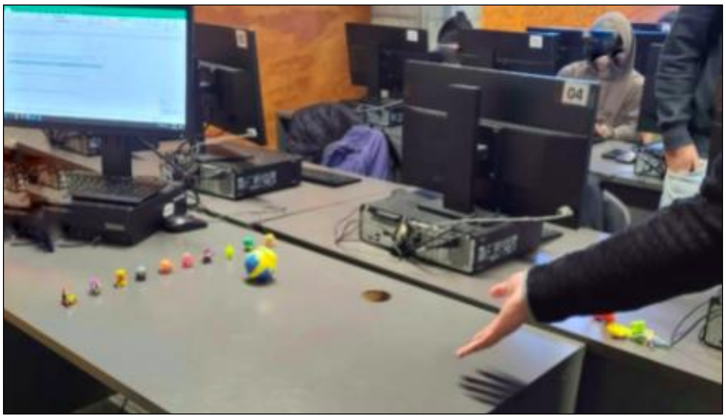
FIGURE 2: Example of materials used

Source: The authors

In the first instance, students were asked to meet in pairs by affinity, given that the number of students was odd, one of them developed the activity with the teacher of the subject. To begin with, one member of the pair carried out the experimental activity, which consisted of making 10 throws of a pokeball (tennis ball) to hit a Pokémon (small figure) (Figure 3), from a certain distance, while the other registered the results.

FIGURE 3: Experimental activity



Source: The authors.

Then, they exchanged roles performing the experimental activity again, in order to have 2 samples of launches. The students used a data table, such as the one presented in Figure 4, since this type of representation is useful for the organization of a data set (Pallauta; Arteaga, 2021). In it, for each throw (L) they recorded whether or not they had succeeded in hitting the Pokémon.

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FIGURE 4: Example of data entry

L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
✓	x	X	✓	x	x	✓	✓	✓	✓

L: launches. ✓: Hits x: not Hits

Number of hits: 6

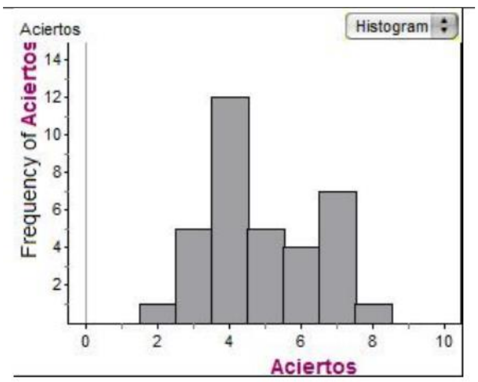
Source: The authors.

Use of software

The lead author had an institutional license of the Fathom software to carry out the experience. Through this software, the students, in pairs, performed simulations of hits in 10 throws and compared them with the data recorded in the experimental activity (Figure 5). In some cases, such simulation was similar to the one obtained in the previous stage, while others were different. Thus, some students performed the simulation again until they obtained a simulation equal to the one obtained experimentally.

Subsequently, students were asked to pool the data collected in the experimental activity by the whole class and construct a histogram in Fathom as seen in Figure 5.

FIGURE 5: Histogram with data collected in the experimental activity.



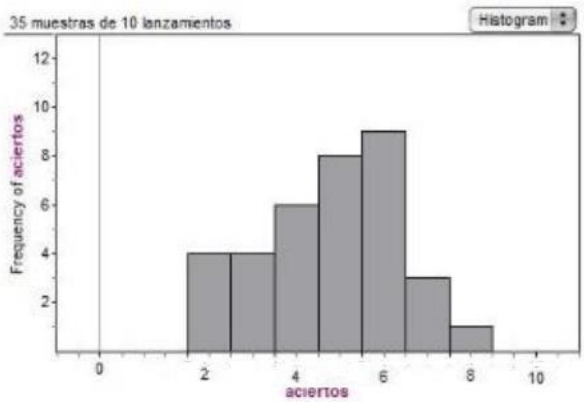
Source: The authors.

Next, students were asked to analyze the histogram, to guide this analysis, questions such as: what does it mean that the highest bar corresponds to 4 hits, how do you interpret the bar of 2 and 8 hits, and how do you interpret the bar of 2 and 8 hits?

Some examples of responses were: "The 4 means that the most repeated number of hits in all our samples is 4", "that the highest bar is 4, is because most of us hit 4 times out of the total of the 10 throws".

Afterwards, students were asked to perform a simulation in Fathom with a sample size of 35, the resulting graph is presented in Figure 6.

FIGURE 6: Histogram with simulation



Source: The authors.

The students are surprised to see that the graph (Figure 6) is different from the one obtained with the data from the experimental activity (Figure 5). From this, they run new simulations with the software, at least 20 simulations, until they obtain a graph similar to the one obtained with the experimentally collected data (Figure 5).

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Thoughts

Once the simulations were completed, the students were asked the open-ended question: What do you think is the number of times you would hit the pokemon with the pokeball in 10 throws? A class discussion ensued, with students mentioning a variety of possible outcomes with varying justification. Some indicated that it is as likely to hit as to miss, in this case they relied on the conception of randomness as synonymous with unpredictability and which has been identified in studies (Begué et al., 2021; García-García et al., 2018). Other students mentioned that they were sure that they would at least hit it 1 time, or that it was almost impossible to hit the pokeball all 10 times based on erroneous secondary intuitions (Fischbein, 1987) that are subsequently transformed into subjective beliefs that explain the occurrence of random phenomena (Batanero et al., 2016).

Also, there were students who indicated that there must be a "mathematical" way to calculate how many times a person can hit the Pokemon, since one cannot guess at the probabilities. In this sense, the idea of measuring the probability of an event occurring in randomized experiments came up.

Following the space for dialogue and reflection that took place in the classroom, students were asked to calculate probabilities experimentally and to use the information provided by the histogram to answer the following questions:

a) What is the probability that a person will hit the pokemon 4 times if he/she throws the pokeball 10 times?
b) What is the probability that a person will hit a pokemon at least 7 times if he/she throws the pokeball 10 times?

Students offered correct and incorrect answers, some of them were:

R_1: P (Hits = 4)= 6/35, correct.

R_2 :P (Hits = 4)= 4/10, incorrect.

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R_3 : P (Hits ≥7)= 34/35, incorrect.

R_4 : P (Hits ≥7)= 1/35, correct.

After the answers were given, reflection was promoted on how probabilities could be calculated without performing the experiments. For this purpose, the experience of throwing the pokeball 100 times and the time required for such activity was exemplified. The students mentioned that they were certain that there was a "shorter" and more accurate way to obtain certain results.

After sharing the answers in class, the binomial distribution was formally defined algebraically together with the formula that allows to determine the probability of different events in random experiments with dichotomous variables.

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{n}{x} p^x \cdot q^{n-x}, con\ x = \{0, 1, \ldots, n\}$$

x: corresponds to the number of times we want it to be successful.

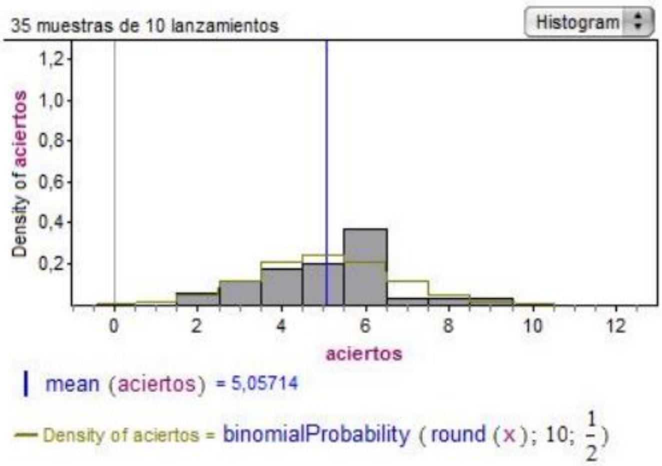
n: corresponds to the number of experiments to perform.

p: probability of success.

The students were then asked to compare the values of the probabilities obtained empirically with those obtained theoretically using the formula.

To conclude the session, simulations were performed in the software comparing the samples obtained with the binomial distribution graph. The students in Fathom generated samples of size 10, with a probability of ½ hit. They take turns entering data into the software to complete the 35 samples. From Figure 7, for example, student E2 comments: "how the rectangles are moving as they perform new simulations".

FIGURE 7: Comparison of simulations



Source: The authors.

At the end of the activity, students said they enjoyed the experience, and valued the use of the software in saving time when graphing, some of the comments were:

> E3: "How entertaining, we almost never use technology in mathematics".
> E8: "I am a repeater and last year we did not see it that way, with the software it is easier to understand, last year we spent a lot of time doing calculations".
> E11: "The simulation is super fast, it allows us to see the movement of the graph better".

Conclusions

We propose an introduction to the study of binomial distribution that incorporates the use of technology through Fathom. Taking up some of the suggestions of GAISE II (Bargagliotti et al, 2020), students were asked to generate samples of different sizes, incorporating the use of software to facilitate visualization, and thus measure the degree of uncertainty of a given event, starting from a situation framed in a close context.

In this sense, in the experience it was possible to observe that the use of Fathom contributed to the students' interest in participating in the activity, as well as in understanding some of the characteristics of the binomial distribution.

Therefore, in this experience we sought to promote the understanding of concepts instead of the algorithmic calculation of probabilities. As Batanero and Borovcnick (2016) warn, the use of simulators allows students to perform various actions such as increasing or reducing data, changing parameter ranges, significantly optimizing time. Regarding the learning of the characteristics of the binomial distribution and its application in the calculation of probabilities, it was possible to observe that the students appropriated the concepts: graphic form of the distribution, algebraic form, probability of success and failure, among others, since they were able to apply the distribution for the resolution of application problems posed later. In addition, during the time the unit lasted, they constantly made references to what they worked with the software, which allowed them to make relationships with other types of activities.

Regarding the affective aspect, the students showed a good willingness to participate in the activity, which was possible to assess through the comments made, since for some of them it was the first time they had worked with software in the mathematics class. In this way, a learning environment related to what they live daily in a digitalized environment, which the school has been slow to incorporate into the teaching dynamics, was fostered.

On the other hand, the recent health crisis generated by COVID-19, highlighted the need for citizens to have statistical and probabilistic knowledge, which involves the understanding of probabilistic models that allow adequate decision making based on data (Muñiz-Rodríguez et al., 2020). Therefore, it is essential that teachers design rich instructional processes that promote the development of statistical and probabilistic reasoning that every citizen must have to perform successfully in today's society.

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