http://doi.org/10.14393/ER-v31e2024-13

Conceptual approaches to statistical variability in Brazilian research: a systematic literature review[1]

Edicarlos Pereira de Sousa[2]
Irene Maurício Cazorla[3]
Carlos Eduardo Ferreira Monteiro[4]

ABSTRACT

The idea of variability is important for learning statistics because it helps understand the variables, the origin of the data, and their characteristics. Seeking to identify how Brazilian research addresses statistical variability to promote statistical education to students and teachers, we carried out a systematic literature review in the *Catálogo de Teses e Dissertações* and *Portal de Periódicos* of Capes and in the *Biblioteca Digital de Teses e Dissertações*. The results suggest little research on statistical variability in basic education. Most of the publications included focus on initial or continuing teacher education, evidencing the absence of studies about student learning. The publications also indicate little approach and conceptual limitations regarding topics such as centrality, dispersion measures, dotplot, and bloxplot. We found that activities related to comparison and representation are insufficient, implying difficulties in appropriating the investigated concept.

KEYWORDS: Statistical variability; Statistical literacy; Literature review; Brazilian research.

[1] English version by Maria Isabel de Castro Lima. *E-mail*: baulima@gmail.com.

[2] Doutorando em Educação Matemática e Tecnológica (PPGEdumatec – Universidade Federal de Pernambuco). Universidade Federal do Cariri, Brejo Santo, Ceará, Brasil. Orcid: https://orcid.org/0000-0003-4262-544X. *E-mail*: edicarlos.pereira@ufpe.br.

[3] Doutora em Educação. Universidade Estadual de Santa Cruz, Ilhéus, Bahia, Brasil. Orcid: https://orcid.org/0000-0003-3028-5513. *E-mail*: icazorla@uol.com.br.

[4] PhD in Education. Universidade Federal de Pernambuco, Recife, Pernambuco, Brasil. Orcid: https://orcid.org/0000-0003-4355-0793. *E-mail*: carlos.fmonteiro@ufpe.br.

Abordagens conceituais sobre variabilidade estatística em pesquisas brasileiras: Uma revisão sistemática de literatura

RESUMO

A ideia da variabilidade é importante para a aprendizagem de estatística porque auxilia na compreensão das variáveis, da origem dos dados e suas características típicas. Buscando identificar como as pesquisas brasileiras abordam a variabilidade estatística, no sentido de promover educação estatística a estudantes e professores, realizou-se uma Revisão Sistemática de Literatura no Catálogo de Teses e Dissertações e Portal de Periódicos da Capes, e na Biblioteca Digital de Teses e Dissertações. Os resultados sugerem que há poucas pesquisas sobre variabilidade estatística na Educação Básica. A maioria das publicações incluídas enfocam a formação inicial ou continuada de professores, evidenciando a ausência de estudos acerca da aprendizagem de estudantes. As publicações também indicam que há pouca abordagem e limitações conceituais relativamente a tópicos como centralidade, medidas de dispersão, dotplot e bloxplot. Constatou-se que atividades referentes à comparação e representação são insuficientes, implicando em dificuldades para apropriação do conceito investigado.

PALAVRAS-CHAVE: Variabilidade estatística; Letramento estatístico; Revisão de literatura; Pesquisas brasileiras.

Aproximaciones conceptuales a la variabilidad estadística en la investigación brasileña: Una revisión sistemática de la literatura

RESUMEN

La idea de variabilidad es importante para el aprendizaje de la estadística porque ayuda a comprender las variables, el origen de los datos y sus características típicas. Buscando identificar cómo las investigaciones brasileñas abordan la variabilidad estadística, con el fin de promover la educación estadística para estudiantes y profesores, se realizó una Revisión Sistemática de Literatura en el Catálogo de Tesis y Disertaciones y Portal de Periódicos de la Capes, y en la Biblioteca Digital de Tesis y Disertaciones. Los resultados sugieren que existe poca investigación sobre la variabilidad estadística en Educación Básica. La mayoría de las publicaciones

incluidas se enfocan en la formación inicial o continua de docentes, evidenciando la ausencia de estudios sobre el aprendizaje de los estudiantes. Las publicaciones también indican que existe poco enfoque y limitaciones conceptuales en temas como centralidad, medidas de dispersión, dotplot y bloxplot. Se constató que las actividades relacionadas con la comparación y la representación son insuficientes, lo que implica dificultades en la apropiación del concepto investigado.

PALABRAS CLAVE: Variabilidad estadística; Alfabetización estadística; Revisión de literatura; Investigación brasileña.

* * *

Introduction

Contemporary society demands diverse knowledge from people, which implies the need for educational institutions to think about and implement educational processes updated to everyday challenges. For example, technological advancement and the amount of information produced and disseminated socially require people to critically interpret statistical data related to various subjects. Therefore, statistics teaching in schools and universities must prioritize learning strategies that allow students to understand and analyze the world instead of reproducing calculation procedures dissociated from challenging social realities.

Cazorla and Utsumi (2010) discuss how statistics teaching was included in basic education curricula, breaking with the deterministic culture in mathematics classes, following an international trend, given the recognition of the importance of the contribution of statistics to forming critical citizens. This insertion boosted an area of research called statistical education, whose objectives are to study and understand how people teach and learn statistics, considering cognitive and affective aspects of teaching-learning, the production of statistical knowledge, and statistics didactics, with a view to statistical literacy.

Gal (2002, 2019) defines statistical literacy as the result of cognitive and dispositional components. In the cognitive component, literacy, mathematical, statistical, and contextual knowledge compete, along with the ability to elaborate critical questions. The dispositional component is formed by beliefs and attitudes, which shape the reading of the world and the critical stance before information. In this model, it is not enough to master statistical knowledge; for example, besides knowing how to calculate measures of central tendency and dispersion, we must know the origin of the data, how variables were defined, and how they are distributed. We must also evaluate whether the statistical measures used are appropriate to the context instead of others and who is interested in disclosing this information.

Aiming to identify how Brazilian research addresses statistical variability to promote statistical education for students and teachers, we carried out a systematic literature review (SLR) that analyzed articles from scientific journals, as well as dissertations and theses defended in Brazil, seeking to respond to the following question: How is statistical variability addressed in national academic research, within the scope of statistics education, at different levels and contexts of teaching and/or learning?

By answering the research question, based on the analysis of the publications included in the review, this work identifies how students and teachers have appropriated statistical variability based on situations experienced in the school space.

In the next section, we argue for the importance of the concept of variability for statistical education to promote statistical literacy. The methodology section details the research path, describing the period analyzed, the databases chosen, the search terms defined, and the exclusion and inclusion criteria for the works. Next, we present the works and a quantitative and qualitative analysis of the results. Finally, in the conclusions, we return to the research question to identify the advances and

difficulties that can be highlighted in the analyzed publications, including reflections regarding perspectives for future studies.

The importance of variability in teaching statistics

Why are studies related to variability so important to statistical education, especially when promoting statistical literacy among students and teachers? Although there is no single answer to this question, it is noteworthy that several studies show variability as fundamental to statistics (Cobb; Moore, 1997; Fife *et al.*, 2020; Watson *et al.*, 2003). Thus, they highlight the ubiquity of variability, defending the idea that there would be no data statistics if there were no variability. Furthermore, they emphasize that statistical investigation presumes variation in data, seen as essential for understanding statistics and the distinction between statistics and other areas of knowledge, such as mathematics.

In the literature, there is not always a consensus regarding the terms "*variabilidade*" [variability] or "*variação*" [variation], as some authors define the first as the noun form of the adjective "*variável*," [variable] meaning that something will probably change, while the second is seen as the noun that describes the act of varying (Reading; Shaughnessy, 2004). However, like Garfield and Ben-Zvi (2008), we chose to use them without distinction, considering all these general ideas about the data. For Shaughnessy (1997), the concept of variability is complex and difficult to define, reinforcing the need to broaden its understanding, as it is essential for students to develop statistical thinking adequately.

Silva (2007) highlights that statistical thinking can be understood as the mental strategies people use to make decisions in an investigative context. According to the author, statistical thinking and statistical literacy are directly related, as the more a subject develops their statistical thinking, the greater their chances of presenting more advanced levels of statistical literacy.

The idea of variability transcends the calculation of variation measures, such as amplitude (total), variance, standard deviation, or coefficient of variation (Isoda *et al.*, 2018). According to the authors, to understand the theoretical perspectives of variability, it is necessary to overcome the concern centered on the mathematical treatment that makes the student perform calculations that often do not understand what they actually represent. Therefore, this publication warns of the presence of mathematization of statistical knowledge to the detriment of encouraging students to question and be critical, making it difficult to perceive the phenomenon of variability as a component of data closely related to statistical literacy.

Star *et al*. (2021) address the importance of variability, reinforcing the low number of studies on its understanding and evaluation at the school level. The absence of materials that explore this concept in basic education, among other aspects, may contribute to the fact that variability does not occupy a relevant place in the curriculum worked in the classroom. In this context, Rodríguez-Alveal and Maldonado-Fuentes (2023) carried out a typology of questions regarding variability in Chilean high school books to inquire about the quality and cognitive level of the questions presented, the results of which showed a more significant presence of questions with an arithmetic approach, with little emphasis on teaching strategies based on argumentation and decision making.

Variability is inherent in the analysis of statistical data. However, understanding this concept must consider the different sources originating an investigated phenomenon. In this sense, Rodríguez-Alveal and Maldonado-Fuentes (2023) recommend approaching problems from various exploratory sources of variability: a) real or natural — inherent to the nature of the phenomenon studied: height and diameter of people's heads; b) induced — in which there is influence from third parties regarding the behavior of a phenomenon: effect of a fertilizer on plant growth; and c) sample — influenced by sample size: basic situations for statistical inference.

Recent studies that deal with teaching and learning variability, considering different perspectives with students and teachers, have shown increasing interest in investigating this concept at different levels of education, given the recurring difficulties highlighted in research on the subject (Estrella *et al.*, 2021; Garfield *et al.*, 2015). However, it becomes worrying that systematized knowledge does not show advances in the learning processes to make the student realize the interconnection between variability and other statistical ideas, such as distribution, centrality, sampling, inference, and representativeness.

Brazilian studies dealing with basic education teaching and learning of statistical concepts emerged more frequently after the National Curricular Parameters (*Parâmetros Curriculares Nacionais* - PCN) in the 1990s (Brasil, 1997, 1998). Since then, statistics topics have gained more space in the school curriculum, mainly to prepare students for situations that involved information processing because statistical learning, at that time, was focused on making students appropriate forms of data collection, organization, and systematization. However, in the following decade, academic research focused on understanding students' and teachers' thinking, reasoning, and statistical literacy and understanding that statistical education must overcome data mathematization.

The approach to the concept of statistical variability remains infrequent in Brazilian studies, justifying the contributions of this work. Therefore, in the next section, the methodological procedures adopted in identifying and analyzing national scientific production addressing the topic will be detailed to promote statistical education.

Methodological Aspects

As bibliographical research, the SLR aims to identify how scientific knowledge has been developing in a specific context. According to Lima

and Mioto (2007), it implies an ordered set of procedures for searching for solutions, attentive to the object of study, and which, therefore, cannot be random. For the authors, it is an investigation with the advantage of summarizing evidence relating to a particular topic, with clear and organized search methods, critically analyzing the synthesis of information obtained.

Petticrew and Roberts (2006) have a detailed guide for writing systematic review papers. The authors emphasize that, more than producing a scientific summary, the SLR aims to answer a question or test a specific hypothesis, identifying paths for future studies. In mathematics education, systematic reviews are used to direct investigations concerned with various objects of study.

Considering the importance of understanding how and if statistical literacy appears in Brazilian articles, dissertations, and theses that address statistical variability within the scope of educational research with students and teachers, this article set out to answer: How statistical variability is addressed in academic research national, within the scope of statistical education, at different levels and contexts of teaching and/or learning?

The searches were carried out during May 2023 in three databases: Digital Library of Theses and Dissertations (*Biblioteca Digital de Teses e Dissertações* - BDTD), Catalog of Master's Degree and Doctoral Theses (*Catálogo de Teses e Dissertações* - CTD), and the Portal of Journals (*Portal de Periódicos* - PP), the latter two being the responsibility of Coordination for the Improvement of Higher Education Personnel (*Coordenação de Aperfeiçoamento de Pessoal de Nível Superior* - CAPES). These bases are justified because they are public and freely accessible, constituting themselves as relevant for the free dissemination of national (BDTD and CTD/CAPES) or national and international (PP/CAPES) academic production. We chose not to define the period (year) of the work, making it possible to expand the number of productions and favoring the understanding of how the theme has been approached recently.

The search strategies for each database consulted were standardized, using the same five terms (descriptors or keywords): 1) statistical variability, 2) statistical variation, 3) dispersion measures, 4) variability measures, and 5) variation measures. Only terms written in Portuguese were used since the objective of this article is to investigate the concept of statistical variability in national productions. We opted for individual searches, term by term, with descriptors written in quotation marks and without Boolean operators. Finally, the corpus of this research was defined according to the following reading stages: 1) titles, 2) abstracts and keywords, 3) objectives, methodologies, and final considerations or conclusions.

With the terms used, initially, 583 results were obtained in BDTD, 171 in CTD/CAPES, and 226 in PP/CAPES. We identified repeated productions in all databases. Therefore, repetition was established as an exclusion criterion. Also discarded were works unrelated to statistics education, other types of documents (reviews, records, reports, educational products, books, or book chapters), and studies carried out outside the Brazilian educational context. In Table 1, we can observe that, of the 980 productions identified, 958 were eliminated, leaving 22 works for analysis.

In the next phase, we defined two inclusion criteria: research related, in some way, to the concept of statistical variability within the scope of statistics education and studies with a theoretical basis relevant to the problem under investigation, contributing to the understanding of how national academic production approaches the topic investigated.

TABLE 1: Search in BDTD, CTD/CAPES, and PP/CAPES, with different search terms.

Search Terms	Initial Results			Excluded Productions		
	BDTD	CTD/ CAPES	PP/ CAPES	BDTD	CTD/ CAPES	PP/ CAPES
"variabilidade estatística" [statistical variability]	10	8	4	6	7	3
"variação estatística" [statistical variation]	87	42	77	87	42	77
"medidas de dispersão" [dispersion measurements]	157	52	78	146	51	76
"medidas de variabilidade" [variability measures]	89	15	45	89	15	45
"medidas de variação" [measures of variation]	240	54	22	239	53	22
Total		980			958	

Source: The authors (2023).

For the qualitative analysis of the research, Bardin's (2016) content analysis (CA) was used, organized into three phases: 1) pre-analysis; 2) exploration of the material; 3) treatment of results and interpretation. The corpus of this research consisted of 11 works (two articles, four master's degree thesis, and five doctoral theses), chosen based on the inclusion criteria established to answer the research question.

In the material exploration phase, depending on the CA of the texts, four categories were defined: 1) Research in teachers' and undergraduates' training/collaborative contexts; 2) Academic productions with basic education students about variability; 3) Studies that involved analysis of textbooks; 4) Mobilization of statistical concepts in other higher education contexts. In the next section, the selected productions are analyzed.

Data Presentation and Analysis of Results

Based on the categories identified by the content analysis, Table 1 shows the national works that deal with statistical variability identified in this SLR:

CHART 1: National academic research that deals with statistical variability in the context of promoting statistical education.

Category	Type	Author
Research in teachers' and undergraduate students' training/collaborative contexts	Article	Oliveira Júnior and Pereira (2018)
	Master's thesis	Moreno (2010)
	Doctoral thesis	Silva (2007), Novaes (2011), Silva (2017), Santos (2020)
Investigations with basic education students about variability	Article	-
	Master's thesis	Cavalcanti (2011), Moreno (2014), Souza (2019)
	Doctoral thesis	-
Studies that involved analysis of textbooks	Article	Andrade *et al.* (2014)
	Master's thesis	-
	Doctoral thesis	-
Mobilization of statistical concepts in other higher education contexts	Article	-
	Master's thesis	-
	Doctoral thesis	Cruz (2020)

Source: The authors (2023).

Category 1: Research in teachers' and undergraduate students' training/collaborative contexts

In this category, six academic productions were identified and organized in the form of articles (1), master's degree theses (1), and doctoral theses (4). The work was centered on the learning of teachers who teach mathematics or degree students who are in the initial education process. Almost all doctoral research studies recovered in this work are in this category, comprising four of the five theses in this systematic review.

Characterization and main results of the research that constituted the first category

Silva's doctoral thesis (2007) investigated the reasoning about variation used to solve statistical problems that emerged during an investigation designed and carried out by the research subjects. Following the methodological assumptions of action research, nine basic education teachers and two mathematics degree students constituted the group of participants in this investigation. To answer the main research problem, the author was based on Garfield (2002) to evaluate the reasoning level of variation used by the mathematics teacher in different stages of the investigative cycle of statistical thinking.

Silva's research (2007) stands out in this systematic review as the first national work that addressed teachers' learning involving the concept of variation. The author noticed that the development of statistical thinking did not imply reasoning about variation in teachers and undergraduates. Participants understood the meaning of standard deviation as a measure of data variation/homogeneity, although they did not perceive it as an interval around the mean. The mean and standard deviation were not used to compare distributions. The participants confused the concept of mean, understanding it as majority, by mixing it

with the concept of mode, which is a factor of subsequent difficulty in realizing the importance of a measure of variation.

To study the characteristics of activities that favored the apprehension of variability during a formative course with seventeen mathematics teaching degree students, Moreno (2010) carried out a case study through didactic engineering to design, carry out, observe, and analyze teaching sequences. The training was organized in weekly meetings taught by the researcher.

According to Moreno (2010), pre-service mathematics teachers could realize the variation between the data but not regarding a measure of central tendency. This result increased the difficulty faced by teaching degree students in situations requiring them to interpret variability measures. Situations that involved a comparison between distributions with the same mean but with different dispersions helped research participants to understand that the mean, often, may not be recommended as a summary measure of the data, which justifies recognizing the role of measures of variation. Finally, the author highlighted that associating the idea of concentration with low variation can be a didactic obstacle to understanding variation, as there are situations in which data concentrations are located at the extremes, generating high dispersion.

Novaes (2011) developed a case study investigation on two basic education teachers interacting with other teachers during a continuing education course. The research aimed to analyze didactic and specific conceptions of descriptive statistics content mobilized by basic education teachers. The author used the theoretical model of conception, knowing, and concept (*ck¢*) proposed by Balacheff (2001) and Balacheff and Gaudin (2002) to identify statistics conceptions basic education teachers use to choose the strategies they will employ in resolving problem situations.

Novaes (2011) identified sixteen teaching conceptions for statistics teaching: three of specific knowledge and thirteen of didactic knowledge, working

interdependently. About the construction of the concept of variability, critical involvement in activities and the reproduction of mathematized procedures were not enough to promote statistical literacy. The competence of transferring deterministic thinking from mathematics to data analysis was identified in this research, offering strong resistance until finally changing. At the end of the research, the researcher observed that the characterization and possible changes in the conceptions that hinder the teaching and learning process were the result of careful monitoring of teaching activity.

Identifying and characterizing evidence of statistical knowledge that makes up the statistical literacy of undergraduates in pedagogy was Silva's (2017) primary objective. Gal's (2002) statistical literacy levels were used as a theoretical basis for this research. Through a case study, a workshop was planned to address statistical variability, whose activities focused on mean, mean deviation, median, and quartiles.

After analyzing the proposed activities and constructing conceptual maps, Silva (2017) identified signs of the development of basic notions of statistical concepts that make up the statistical literacy of prospective teachers, highlighting the variability reasoning associated with the idea of mean, in addition to concepts amplitude, dispersion, separatrix measurements, and graphical representation using the boxplot. On the other hand, problems related to the numerical scale constituted a difficult factor in promoting the participants' statistical literacy.

Oliveira Júnior and Pereira's (2018) study was carried out with sixteen participants, including teaching initiation scholarship holders and postgraduate students, to evaluate the learning of the concept of statistical variability. Using three of the seven constituent elements of the epistemological model for understanding variability by Garfield and Ben-Zvi (2005), the authors developed teaching sequences structured in two moments: manually, using pencil and graph paper, and developed using the "R" software.

As a result, Oliveira Júnior and Pereira (2018) identified two types of variability in the participants' conception: internal to a group,

recognizing the variation of data in one or more sets and between groups, characterized by the variability of statistical measures used for synthesis and comparing data sets. The authors concluded that the participants accepted the use of the software, although this result does not imply discarding using traditional resources (graph paper and pencil) since both methodologies can be applied in the classroom.

In Santos (2020), we sought to analyze the conceptions of future mathematics teachers regarding knowledge of measures of central tendency, measures of dispersion, and variability through a computerized environment from a theoretical perspective of promoting statistical education. As a methodological choice, a case study was organized into two phases: an initial survey of pre-service teachers' conceptions and a verification of their emerging conceptions when using a computerized environment. The research took as theoretical bases the *ck¢*, the technological pedagogical content knowledge (TCPK), the statistical thinking, and the statistical literacy models.

The main results pointed out by Santos (2020) suggested confusion between qualitative and quantitative variables on the part of mathematics pre-service teachers since the numerical frequency of the qualitative variable was mistaken with the variable type. In the concepts related to measures of central tendency, the following stood out: the definition of mean, mode, and median as analysis parameters; the recognition of standard deviation as a measure of dispersion; the association of variation with dispersion; understanding mean as a calculation procedure equivalent to apportionment, and the mode as the value that is most repeated. In the activity that verified the influence of data variability on the measure of central tendency and the boxplot, the subjects concluded that: the median is the second quartile in the boxplot; variability interferes with measures of central tendency; the mean and median values are close in situations that present low variability in the data. Finally, the authors reinforce the need for differentiated strategies

for the study and teaching of statistics in contexts of mathematics teachers' initial education.

Category 2: Investigations with basic education students about variability

The second category brought together three master's degree theses. The research identified in this group presented didactic proposals concerned with students' understanding of variability from the early years, covering elementary and high school. This category showed no reason for variability to be studied only after a specific age since younger children, although informally, demonstrate an understanding of it.

Characterization and main results of the academic productions that constituted the second category

Statistical variability was researched by Cavalcanti (2011). The main objective of the study was to investigate the understanding of elementary school 2nd- and 5th-grade students about this concept. Various aspects of variability (quantification, representation, description, comparison, prediction, and gradation) were investigated with forty-eight student participants, using Garfield and Ben-Zvi (2005) as a theoretical framework. The Piagetian clinical method was applied. The children were interviewed individually and encouraged to explain what they understood about the concept explored.

When grading the understanding of various aspects of statistical variability, Cavalcanti (2011) highlighted that, in descending order, those that expressed greater complexity were the comparison between data sets, the prediction based on the mode, the comparison between points with quantification of variation, and the proposition of absence of variation. According to the author, since the second grade of schooling, students can understand aspects of

variability, and this observation can be intensified if students experience teaching situations that challenge them to analyze and reflect on data.

The main objective of the study by Moreno (2014) was to investigate the contributions of dotplot and boxplot graphs in high school students' statistical variability learning. Based on teaching sequences, the author conducted qualitative research with participants who needed to collect, organize, and represent data through dotplot and boxplot, which are fundamental for understanding the concept studied. The intuitive ideas involving variability by Garfield and Ben-Zvi (2005) and Gal's (2002) statistical literacy were adopted to develop the activities. The SOLO taxonomy (Biggs; Collis, 1982) was used to categorize student responses regarding variability reasoning.

Moreno's (2014) main results highlight that students had no prior knowledge of dispersion measures but showed intuitive ideas of variability. The graph resulting from the overlay of the dotplot and the boxplot, called dot-boxplot, made evident the difficulty inherent in the median and quartiles, especially in the case of repeated values that occupy positions around such measurements. This construction allowed students to point out variability through the interquartile range. Finally, comparison activities between different variables in the same group or the same variable between other groups favored the recognition of the nature of variability.

Souza's research (2019) examined how digital technologies (DT) help elementary school students develop the concept of variability. Through intervention research, organized into three stages (pre-test, intervention, and post-test), the author analyzed the knowledge acquired throughout the application of activities and interactions between student-student and student-researcher. Using theoretical bases such as Batanero (2001) and Garfield and Ben-Zvi (2005), the research presents several reflections about perceptions of variation incorporated into the graphs constructed by the participating subjects.

The results of Souza's master's degree thesis (2019) showed strategies to help elementary school students understand variability using different representations. Students had difficulties in procedural calculation, interpreting central tendency measures, and constructing column graphs, which are essential for understanding the abovementioned concept. Another result highlighted in this research was the students' limitations in solving problem situations related to interpreting and calculating the mean. According to the author, this fact arises from mathematical operations, which students usually do not formalize. Finally, the work identified that it is easier to interpret than to construct graphs, regardless of the type considered.

Category 3: Studies involving analysis of textbooks

The third category included only one academic production (scientific article). This category included research whose results come from different contexts than the others. Therefore, the selected article was not related to the experience of training processes involving teachers/undergraduates, nor did it investigate students' understandings of statistical variability, as it arose to analyze how statistical concepts appear in textbooks.

Characterization and main results of the study of this third category

The analysis of high school mathematics books was carried out by Andrade *et al*. (2014), who sought to identify how measures of central tendency and dispersion (MCTD) appear in Brazilian and French teaching collections. The Brazilian sample was formed from seven collections from the National Textbook Program (*Programa Nacional do Livro Didático* - PNLD) from 2012. To compare with Brazilian collections, the authors selected seven collections listed on the French Ministry of National Education page. Considering the theory of conceptual fields to map the

situations in the books, the authors reinforce the importance of varied approaches for forming concepts of centrality and dispersion in students. As a result, nineteen different ways of presenting data in activities involving MCTD were identified. Some are present in both Brazilian collections. However, the boxplot appears only in the French collections. Furthermore, in most Brazilian collections, MCTDs predominate in the last year of high school.

Category 4: Mobilization of statistical concepts in other higher education contexts

The last category of analysis was also defined by a single production, referring to a doctoral thesis in cognitive psychology. Unlike the first category, which focused on training processes with teachers in initial or continuing education, the study was carried out with students enrolled in a research degree in administration at a public university. Therefore, a specific category was defined because the research was developed in a different context from previous cases.

Characterization and main results of the study of this fourth category

Cruz's work (2020) investigated the performance and reasoning involving measures of central tendency (mean, mode, and median) and measures of dispersion (amplitude, standard deviation, and coefficient of variation) of students enrolled in an administration course at a public university. The research was based on general and specific theories about development and learning to identify the impact of statistical education on professional education. As methodological procedures, students responded to a survey of attitudes toward statistics scale (SAS), answered an interview, and finally responded to an instrument with four situations covering statistical concepts. According to the author, in general terms,

the students' best results were obtained in situations related to measures of central tendency, except the median. Another important finding concerns the types of justifications for situations associated with a procedural calculation in which students performed better. Finally, the results showed that the concepts of central tendency seem better consolidated in relation to dispersion measures, which tend to present more difficulties both in the procedure and in conceptual mobilization.

Conclusion

This work aimed to answer how statistical variability has been addressed in national academic research within the scope of statistical education at different levels and contexts of teaching and/or learning, highlighting the advances in research and scientific dissemination in this area.

The results suggest a scarcity of works on variability, mainly in the first years of basic education and high school, considering we identified only two works, one from each level. Therefore, in this SLR, we observed that most studies were developed in initial or continuing teacher education courses, highlighting the absence of works focusing on student learning.

Several studies demonstrated students' and teachers' procedural understanding of statistical concepts, which contrasted with a conception of statistical literacy expected from participants. Among the main difficulties highlighted were misunderstandings regarding the numerical scale and types of variables, perception of the average only as an algebraic measure, and participants' lack of knowledge or conceptual limitations regarding dispersion measures.

Although necessary, aspects such as comparison and representation, when addressed, were insufficient for students and teachers. According to these studies, this implies perceiving the predominance of difficulties in appropriating reasoning about variability. The meanings of median, centrality, and standard deviation were more complex. These results show

that data variation is not usually seen as a measure around the mean or any other measure of central tendency.

School practices have not always contributed to the development of reasoning and, therefore, to statistical literacy regarding variability. It is necessary to expand the possibilities of representations beyond bar graphs and histograms, including using digital technologies, as point graphs and boxplots, which are not common in books, presented satisfactory didactic potential in the research that addressed them.

From the analysis of the selected works, we can underscore that didactic situations designed to help promote statistical thinking and reasoning about variability also contribute to statistically literate people, and that, therefore, reveals how statistical education, when experienced by students, teachers, and the population in general, enhances criticality, awareness, and emancipation of these subjects in society.

We conclude that knowledge of students' –and, in some cases, teachers'– statistical variability is under development in different contexts. However, the results indicate the need for further investigations covering this topic. Thus, this review contributes to reflecting on gaps in student learning or teacher education processes, enabling new studies focusing on the relationships between statistical education, statistical literacy, and statistical variability.

References

ANDRADE, V. L. V. X.; RÉGNIER, J. C.; LIMA, A. P. A. B. Apresentação dos dados em atividades que envolvem as medidas de tendência central e de dispersão nos livros didáticos do Ensino Médio no Brasil e na França. Análise das formas e das suas organizações no quadro da A.S.I. *Educação Matemática Pesquisa*, São Paulo, v. 16, n. 3, p. 1047-1067, dez. 2014.

BALACHEFF, N. Les connaissances, pluralite de conceptions: le cas des mathématiques. *Les cahiers du laboratoire Leibniz*, Grenoble, n. 19. p. 83-90, jan. 2001.

BALACHEFF, N.; GAUDIN, N. Students conceptions: an introduction to a formal characterization. *Les cahiers du laboratoire Leibniz*, Grenoble, n. 65, p. 1-21, dez. 2002.

BARDIN, L. *Análise de Conteúdo*. São Paulo: Edições 70, 2016.

BATANERO, C. *Didáctica de la Estadística*. Granada: Grupo de Investigación en Educación Estadística, Departamento de Didáctica de la Matemática, Universidad de Granada, 2001.

BIGGS, J.; COLLIS, K. *Evaluating the quality of learning*: the SOLO Taxonomy. New York: Academic Press, 1982. 296 p.

BRASIL. Ministério da Educação. *Parâmetros Curriculares Nacionais (PCN): matemática*. Brasília, DF, 1997.

BRASIL. Ministério da Educação. *Parâmetros Curriculares Nacionais (PCN): matemática*. Brasília, DF, 1998.

CAVALCANTI, E. M. S. *Para variar: compreensões de estudantes dos anos iniciais diante de aspectos da variabilidade*. 2011. Dissertação (Mestrado em Educação Matemática e Tecnológica) - Programa de Pós-Graduação em Educação Matemática e Tecnológica, Universidade Federal de Pernambuco, Recife, 2011.

CAZORLA, I.; UTSUMI, M. Reflexões sobre o ensino de Estatística na Educação Básica. In: CAZORLA, I.; SANTANA, E. (Org.). *Do tratamento da informação ao letramento estatístico*. Itabuna: Via Litterarum, 2010. p. 9-21.

COBB, G. W.; MOORE, D. S. Mathematics, statistics, and teaching. *The American Mathematical Monthly*, Washington, v. 104, n. 9, p. 801-823, 1997. DOI: https://doi.org/10.2307/2975286.

CRUZ, T. V. Q. F. *Medidas de tendência central e medidas de dispersão: um estudo acerca dos conceitos estatísticos mobilizados pelos estudantes do curso de Administração*. 2020. Tese (Doutorado em Psicologia Cognitiva) - Programa de Pós-Graduação em Psicologia Cognitiva, Universidade Federal de Pernambuco, Recife, 2020.

ESTRELLA, S.; VERGARA, A.; GONZALEZ, O. El desarrollo del sentido del dato: haciendo inferencias desde la variabilidad de los tsunamis en primaria. *Statistics Education Research Journal*, Haia, v. 20, n. 2, p. 1-14, dez. 2021. DOI: https://doi.org/10.52041/serj.v20i2.413.

FIFE, J. H.; JAMES, K.; PETERS, S. A learning progression for variability. *Research Report*, Nova Jersey, v. 2020, n. 1, p. 1-22, dez. 2020. DOI: https://doi.org/10.1002/ets2.12286.

GAL, I. Adults' Statistical Literacy: Meanings, components, responsibilities. *International Statistical Review*, Haia, v. 70, n. 1, p. 1-25, abr. 2002. DOI: https://doi.org/10.2307/1403713.

GAL, I. Understanding statistical literacy: about knowledge of contexts and models. *In*: TERCER CONGRESO INTERNACIONAL VIRTUAL DE EDUCACIÓN ESTADÍSTICA, 3., 2019, Granada. *Anais [...]*. Disponible en www.ugr.es/local/fqm126/civeest.html.

GARFIELD, J. The Challenge of Developing Statistical Reasoning. *Journal of Statistics Education*, v. 10, n. 3, p. 1-11, 2002. DOI: https://doi.org/10.1080/10691898.2002.11910676.

GARFIELD, J.; BEN-ZVI, D. A framework for teaching and assessing reasoning about variability. *Statistics Education Research Journal*, Auckland, v. 4, n. 1, p. 92-99, maio. 2005.

GARFIELD, J.; BEN-ZVI, D. Developing Students' Statistical Reasoning: Connecting Research and Teaching Practice. *Springer*, 2008.

GARFIELD, J.; LE, L.; ZIEFFLER, A.; BEN-ZVI, D. Developing students' reasoning about samples and sampling variability as a path to expert statistical thinking. *Educ Stud Math*, Dordrecht, v. 88, p. 327-342, mar. 2015. DOI: https://doi.org/10.1007/s10649-014-9541-7.

ISODA, M.; CHITMUN, S.; GONZALEZ, O. Japanese and thai senior high school mathematics teachers' knowledge of variability, *Statistics Education Research Journal*, Auckland, v. 17, n. 2, p. 196-215, nov. 2018.

LIMA, T. C. S.; MIOTO, R. C. T. Procedimentos metodológicos na construção do conhecimento científico: a pesquisa bibliográfica. *Katálysis*, Florianópolis, v. 10, n. esp. p. 37-45, maio. 2007.

MORENO, M. M. A. *Utilização do dotplot e do boxplot na aprendizagem da variabilidade estatística no Ensino Médio*. 2014. Dissertação (Mestrado em Educação Matemática) - Universidade Estadual de Santa Cruz, Ilhéus, 2014.

MORENO, M. M. B. *Ensino e aprendizagem de estatística com ênfase na variabilidade: um estudo com alunos de um curso de licenciatura em Matemática*. 2010. Dissertação (Mestrado Profissional em Ensino de Matemática) - Pontifícia Universidade Católica, São Paulo, 2010.

NOVAES, D. V. *Concepções de professores da Educação Básica sobre variabilidade estatística*. 2011. Tese (Doutorado em Educação Matemática) - Pontifícia Universidade Católica, São Paulo, 2011.

OLIVEIRA JÚNIOR, A. P.; PEREIRA, F. H. Estudo de propostas didáticas para o conceito de variabilidade estatística: utilizando "papel e lápis" e o software "R". *Revista Dynamis*, Blumenau, v. 24, n. 1, p. 20-41, jul. 2018.

PETTICREW, M.; ROBERTS, H. *Systematic Reviews in the Social Sciences*. A practical guide. Blackwell Publishing, 2006.

READING, C.; SHAUGHNESSY, J. M. (2004). Reasoning about variation. In: BEN-ZVI, D.; GARFIELD, J. (eds.). *The challenge of developing statistical literacy, reasoning and thinking*. 2004, p. 201-226. DOI: https://doi.org/10.1007/1-4020-2278-6_9.

RODRÍGUEZ-ALVEAL, F.; MALDONADO-FUENTES, A. C. Tipología de las preguntas sobre variabilidad en los textos escolares y su relación con la alfabetización y pensamiento estadístico. *Uniciencia*, Heredia, v. 37, n. 1, p. 1-19, jan-dez, 2023. DOI: http://dx.doi.org/10.15359/ru.37-1.4.

SANTOS, S. A. *Ambiente Informatizado para Letramento Estatístico-AILE: concepções de futuros professores de Educação Básica sobre as medidas de tendência central, medidas de dispersão e variabilidade*. 2020. Tese (Doutorado em Educação Matemática) - Pontifícia Universidade Católica, São Paulo, 2020.

SHAUGHNESSY, J. Missed opportunities in research on the teaching and learning of data and chance. In: BIDDULPH, F.; CARR, K. (eds.). *People in mathematics education*. 1997, p. 205-237.

SILVA, C. B. *Pensamento estatístico e raciocínio sobre variação: um estudo com professores de Matemática*. 2007. Tese (Doutorado em Educação Matemática) - Pontifícia Universidade Católica, São Paulo, 2007.

SILVA, M. F. *Estudo da aprendizagem sobre variabilidade estatística: uma experiência de formação com futuros professores dos anos iniciais da Educação Básica*. 2017. Tese (Doutorado em Educação Matemática) - Pontifícia Universidade Católica, São Paulo, 2017.

SOUZA, D. C. *Tecnologias digitais e a aprendizagem de conceitos estatísticos: a utilização do software Geogebra por estudantes do 9º ano do Ensino Fundamental*. 2019. Dissertação (Mestrado em Educação) - Programa de Pós-graduação em Educação Brasileira, Universidade Federal do Ceará, Fortaleza, 2019.

VARGAS, J. L. S.; LARA, I. C. M. Uma revisão sistemática de literatura sobre pesquisas em etnomatemática, jogos e linguagem e cultura afro-brasileira. *Revista de Educação, Ciências e Matemática*, Rio de Janeiro, v. 10, n. 1, p. 133-149, jan/abr. 2020.

VERGNAUD. G. La théorie des champs conceptuels. *Recherches en Didactique des Mathématiques*, v. 10, n. 23, p. 133-170. 1990.

WATSON, J. M.; KELLY, B. A.; CALLINGHAM, R. A.; SHAUGHNESSY, J. M. The measurement of school students' understanding of statistical variation. *International Journal of Mathematical Education in Science and Technology*, Abingdon, v. 34, n. 1, p. 1-29. 2003. DOI: https://doi.org/10.1080/0020739021000018791.

Received in June 2023.

Approved in November 2023.