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Aspects of the professional identity of a teacher who teaches mathematics in the context of probabilistic education[1]

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RESUMO

Este trabalho tem como objetivo identificar e analisar os aspectos da Identidade Profissional (IP) de uma Professora que Ensina Matemática (PEM), desvelados a partir da participação dessa em um grupo de estudos sobre Educação Probabilística. Para tanto, olhamos para um conjunto de ações desenvolvidas no grupo, referentes ao planejamento, desenvolvimento e reflexões coletivas acerca de uma aula sobre risco probabilístico. Analisamos as narrativas da professora a partir da perspectiva metodológica da construção de Núcleos de Significação, focando os aspectos constituitivos de sua IP. Dois Núcleos foram evidenciados. Esses tematizaram importantes aspectos relativos às crenças e concepções atraladas aos conhecimentos para ensinar sobre risco probabilístico. Além disso, desvelaram aspectos de cariz pessoal, emocional e sociopolítica relacionados à sua IP como PEM.

PALAVRAS-CHAVE: Identidade Profissional; Formação Continuada; Professores que Ensinam Matemática; Risco Probabilístico; Grupo de Estudos.

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Aspects of the Professional Identity of a Teacher who teaches Mathematics in the context of Probabilistic Education

ABSTRACT

This work aims to identify and analyze aspects of the professional identity (PI) of a teacher who teaches mathematics (TTM), revealed through her participation in a study group on probabilistic education. To do so, we looked at a set of actions developed in the group, referring to planning, development and collective reflections about a class on probabilistic risk. We analyzed the teacher's narratives from the methodological perspective of constructing cores of meaning, focusing on the constituent aspects of her PI. Two cores were evidenced. These covered important aspects relating to beliefs and conceptions linked to knowledge to teach about probabilistic risk. Furthermore, they revealed personal, emotional, and sociopolitical aspects related to her PI as TTM.

KEYWORDS: Professional Identity; Continuing Training; Teachers who Teach Mathematics; Probabilistic Risk; Study Group.

Aspectos de la identidad profesional de un docente que enseña matemáticas en el contexto de la educación probabilística

RESUMEN

Este trabajo tiene como objetivo identificar y analizar aspectos de la Identidad Profesional (IP) de una Profesora de Matemáticas (PEM), revelados a través de su participación en un grupo de estudio sobre Educación Probabilística. Para ello, se analizó un conjunto de acciones desarrolladas en el grupo, referidas a la planificación, desarrollo y reflexiones colectivas en torno a una clase sobre riesgo probabilístico. Analizamos las narrativas de la docente desde la perspectiva metodológica de la construcción de Núcleos de Significado, centrándonos en los aspectos constitutivos de su IP. Se evidenciaron dos núcleos. Estos abarcaron aspectos importantes relacionados con creencias y concepciones vinculadas al conocimiento para enseñar sobre el riesgo probabilístico. Además, revelaron aspectos personales, emocionales y sociopolíticos relacionados con su IP como PEM.

PALABRAS CLAVE: Identidad Profesional; Formación Continua; Profesores que Enseñan Matemáticas; Riesgo Probabilístico; Grupo de Estudios.

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Introduction

The professional identity (PI) of teachers who teach mathematics (TTMs) has appeared as an emerging line of investigation nationally and internationally, particularly when discussing its constitution from the perspective of initial and continuing education (De Paula; Cyrino, 2021). For these researchers, this is a pressing agenda because it can promote the understanding of the (future) professional practice of the TTMs and denote moments of tension/fragility emerging from the training processes necessary for the PI constitution.

Among the various possibilities for carrying out research and training aimed at the movement of establishing the TTMs' PIs, more recently, its links with the learning of specific mathematical topics in initial and/or continuing education contexts have emerged (Graven; Heyd-Metzuyanim, 2019).

With this in mind, in this work[4], we will bring elements that help to understand the TTM's PIs based on a continuing education course focused on probabilistic education, as it is still a gaping field of investigation. Besides, we seek to situate it as a powerful space for discussion, aligned with the possibility of better seeing the different aspects that contribute to the constitution/(re)configuration of the TTM's PIs, in addition to professional knowledge, beliefs, and teaching conceptions (Cyrino, 2017).

Furthermore, we aim to identify and analyze the PI aspects of a teacher who teaches mathematics (TTM) revealed through her participation in a group and studies on probabilistic education. To this end, we focus on a set of actions developed within the group based on an experience of planning, developing, and collectively reflecting during a probabilistic risk class.

[4] This work is part of the first author's PhD research (in the final phase), supervised by the second author.

Based on a methodological process of establishing cores of meaning (CM) of the corpus under analysis, we identified and discussed some constituent aspects of the TTM's PIs from the perspective of probabilistic education, based on a characterization on which we will now reflect. Therefore, we will discuss probabilistic education, focusing on probabilistic risk and its relations with TTM's PIs. Next, we will discuss the methodological aspects, followed by the presentation of the results and the discussion. Finally, we will bring the final considerations and references of the study.

The TTM's PIs: a characterization

The constitution of a teacher's PI is exchanged between the different collective spaces of action and the construction of their professional practices. The social transformations in these manifold spaces tend to shape their PIs, mainly in terms of learning, which is constructed, revealing who they are, where they have been, and where they are going/wanting to go (Wenger, 1998).

In the case of TTM's PIs, talking it over involves the need to assume a characterization (Cyrino, 2018) of what defines them as a professional group. Developing an identity as a TTM differs from developing an identity as a teacher in other subjects because they are unique in terms of the elements that identify them (Levy; Manfredo; Gonçalves, 2012).

In our view, the studies by Cyrino (2017, 2018, 2021) are quite representative of this attempt because they assume a characterization for the TTM's PIs based on important theoretical-methodological and epistemological aspects of international literature and, above all, on results of empirical studies that have been developing in the field of initial and continuing TTM education. These studies take the TTM's PIs as an investigative motto to think, tension, and exercise research and training processes with this professional group and boost the construction and

problematization of training processes regarding contexts mediated by collective or collaborative practices in groups of studies, communities of practice, etc. Therefore, we understand that:

> The movement of the constitution of the TTM's PIs takes place from *prospective teachers' set of beliefs/conceptions interconnected to their self-knowledge, their emotions and knowledge of their profession, associated with autonomy (vulnerability and sense of agency) and political commitment* (Cyrino, 2021, p. 4, emphasis added).

This typification assumes, primarily, that the constitution/development of the TTM's PIs always takes place in a *movement* because, according to the researcher, it is constantly being negotiated throughout their lives and involved by cognitive, affective/emotional aspects and sociocultural contexts in which they participate.

Therefore, it makes no sense to think about a TTM only from their belief systems, conceptions, and professional knowledge (mathematical, didactic-pedagogical, curricular knowledge, etc.), as traditionally discussed. In addition, other dimensions/elements (or macrodomains, as we call them) are at the basis of what characterizes or identifies a TTM: self-knowledge, emotions, autonomy (vulnerability and sense of agency), and political commitments.

According to Cyrino (2018, 2021), these macro domains are closely intertwined, so when we focus on each, we inevitably mobilize all the others. Such simultaneous mobilization can be seen in the different situations corresponding to their life-education paths in (future) mathematics teaching: in the training processes in which they participate, in their classroom practices, when they retrace their (auto)biographical and professional constitution, etc.

The *beliefs and conceptions* exercised by a TTM originate from sociocultural experiences and influences throughout his life, even before taking mathematics teaching as a profession. Notably, they emerge from the various actions developed and how they conceive mathematics and professional practice as a mathematics educator. Also included in this game is the attribution of meanings to being a mathematics teacher or the type of teacher they want to be, how they should teach, their role in the mathematics teaching and learning processes, etc. (Cyrino, 2018).

Their belief systems and conceptions have repercussions on mobilizing a repertoire of *professional knowledge* beyond the disciplinary knowledge aspect (mathematical and didactic-mathematical knowledge). In addition to knowledge relating to mathematical concepts and procedures, the TTM can advance towards the construction of knowledge in the curricular field of mathematics, students, educational contexts, and the ends, values, and purposes of education (Shulman, 1987).

A practice that values moral, emotional, political, equity, and social justice issues based on this knowledge involves the construction of teachers' learning, defined as identity experiences of being, seeing oneself, and being recognized as a TTM (De Paula; Cyrino, 2021).

Amid these relationships, the professional *autonomy* (vulnerability and sense of agency) –and the *political commitment* as a TTM. Being a TTM also means exercising autonomy by practicing vulnerability and professional agency in the training contexts in which they participate, in school practice, etc. (Oliveira; Cyrino, 2011; Cyrino, 2017, 2018, 2021).

Lasky (2005) comments that vulnerability is an experience of a multidimensional nature and has a strong relationship with teachers' emotions, involving aspects of the different contexts in which they operate. However, for Oliveira and Cyrino (2011, p. 112), it takes on another meaning, as "it is not a vulnerability that weakens, makes susceptible and is paralyzing (not seeking to make the pre-service teacher "fragile")". On the other hand, they argue that it is

related to connecting to others, challenging certainties and convictions, (self)questioning with mutuality, respect, trust, involvement, and empathy.

Alongside vulnerability, the professional agency also figures as an essential element in the constitution of the TTM's PIs, meaning that it represents all types of *action* while building repertoires aimed at overcoming specific vulnerabilities and negative emotions. In other words, Cyrino (2017, p. 706) believes that agency arises in moments when the (prospective) teacher, "[...] in interaction with social structures in a mutually constitutive way, exerts influence, makes choices, makes decisions that affect their work and reveal their professional and ethical commitment, through their ideas, motivations, interests, and objectives".

In fact, political commitment can be considered as a result of the TTM's PIs professional agency, given that it is configured as a commitment to action and transformation, which considers not only the commitment to its students but also to their parents, professional colleagues, and the communities in which they are involved (Cyrino, 2017). This researcher highlights the perception of the role of mathematics and the responsibility as a TTM as essential elements of political commitment.

Lastly, and not least importantly, there is the aspect of *self-knowledge*, understood here as self-esteem, self-image, motivations for work, perception of tasks, and perspectives on their profession (Kelcthermans, 2005). For Cyrino (2018), professional teacher self-knowledge is strongly related to how teachers see themselves and how they attribute meaning to their experiences in the dynamics of interaction between what it means to be a TTM and their professional context.

Examining the PI within the scope of probabilistic education[5]: talking about probabilistic risk

We advocate that constituting a PI as a TTM involves the interlocution of personal, professional, intellectual, moral, and political aspects (Cyrino,

[5] We are conceiving the term "probabilistic education" from a perspective that expands the possibilities of work around the teaching and learning of probability beyond its conceptual and didactic-pedagogical aspect.

2017), which, in teaching and learning a given mathematical content, will influence the construction of this PI. Probabilistic education is a very representative example of this when we consider that its insertion within the mathematics curriculum contradicts the fact that everything in mathematics is exact, determined, and predictable. This is a modal context of teaching mathematics, and escaping from it seems to place the TTM in a scenario of contestation of what essentially represents mathematics.

There are other ways of teaching as a TTM and, among them, recognizing that there is a "mathematics of uncertainty" linked to situations that involve the notion of probability, chance, randomness, probabilistic risk, etc. Therefore, teaching probability makes TTMs exercise a practice different from the situations that define entirely exact mathematics. At the same time, it also tends to place it in a "risk zone" because it seems to harm the epistemological and constitutive principles of mathematical knowledge socially constructed in the school curriculum, in initial education and social life. No wonder we see teachers' skepticism and resistance to working with this content at school (Carvalho, 2017).

Therefore, probabilistic education can bring significant tensions to research on TTM education, especially from the perspective of (re)configuring their PIs. However, most studies discussing probability teaching and learning in TTM continuing education focus primarily on measuring and acquiring knowledge to teach probability (Ody; Viali, 2017).

In our opinion, this alone is insufficient to explore other aspects of the TTM's PIs, with probabilistic education as a backdrop. By engaging with the cognitive and dispositional demands (Gal, 2005), as well as the sociopolitical and cultural demands that permeate the TTM's practice, probabilistic education raises the establishment of important training contexts intensified by the PI, such as discussing scenarios in which mathematics is linked to situations of uncertainty and probabilistic risk in real-world contexts and discussing issues of a civic nature, in favor of democracy and citizen empowerment (Engel; Ridgway; Stein, 2021), etc.

We highlight here the notion of probabilistic risk as an essential aspect of probabilistic knowledge inside and outside school, which underpins the movement towards establishing the TTM's PIs. This is justified since many daily decisions depend on a good understanding of probabilities and risk, such as situations involving correlational reasoning, mainly from the association of variables in double-entry tables (Carvalho, 2017).

More recently, the literature mentions the term *risk literacy*[6], which, although it is a multifaceted construction, stands as a skill that young people and adults need to understand, critically evaluate, communicate, and engage with statistics in relation to "hot" social issues, and which can contribute to their empowerment as citizens (Gal; Nicholson; Rigdway, 2022). For researchers, this is related to how people manage, for example, financial or health risks and negative or positive costs in light of the available evidence. However, they assert that, for citizens to engage in public dialogues about broad social issues, they need to understand the ideas related to statistics and risk literacy as fused cognitive and curricular entities.

Methodological aspects

In this article, we present our analysis of aspects of the PIs of TTMs participating in a research training (Josso, 2007) developed in a study group environment. This qualitative study uses a descriptive and interpretative design of training actions relating to probabilistic education. From these actions, we analyzed what was narrated, what was said, and what was experienced by the TTMs, looking for elements that revealed signs of constitutive aspects of their PIs.

The study group[7] comprised 11 TTMs, eight students from the Mathematics Teaching Degree course at the University of Pernambuco

[6] Translation of *risk literacy*.

[7] The Study Group on the Identity and Education of Teachers who Teach Mathematics (Grupo de Estudos sobre a Identidade e a Formação do Professor que Ensina Matemática - GEFIPMAT) was created as a university extension project in partnership between UPE, Petrolina campus and teachers from municipal and state schools in the city.

(UPE), Campus Petrolina and two teacher educators (professors in that course) over 12 meetings. These meetings took place in person at the UPE campus between May and October 2022, lasting 3h30 min each.

The actions highlighted here took place over three meetings, embodied in the following actions: Action 1 – Beginning of planning a class on probabilistic risk in subgroups (9th meeting); Action 2 – Completion of planning and collective assessment (10th meeting); Action 3 – Class development at school and Action 4 – Reflections in the group after the development of the class (11th meeting). Actions 1, 2, and 4 were carried out in the study group, focusing on the TTM Ester subgroup,[8] and Action 3, in a class taught by Ester at her school and accompanied by TTM Dulce[9], the teacher educator (first author of this work) and three pre-service teachers.

The information that comprises the corpus analyzed here was obtained from audio recordings and/or video recordings of the meetings, which were transcribed in full, transforming the spoken word individually and/or collectively into text. Furthermore, we used the teacher educator's field diaries, written records and Ester's reflective diary.

In the search for understanding the meanings that constituted Ester's narratives in the group, we systematized the results and the discussion based on the construction of *Cores of Meaning (CM)* (Aguiar; Aranha; Soares, 2021) to trigger aspects of their PIs within the actions.

Initially, we outline a set of *pre-indicators*, translated as meanings substantiated in narrative excerpts (expressions or phrases with meaning) that expressed some critical incident about the identity phenomenon. Therefore, in agglutinating these pre-indicators, we constructed the *indicators* based on aspects of similarity, complementarity or even opposition between the pre-indicators. Finally, based on the indicators, we outlined and discussed the CM in light of the broader context that constituted the investigated reality and the adopted theoretical framework.

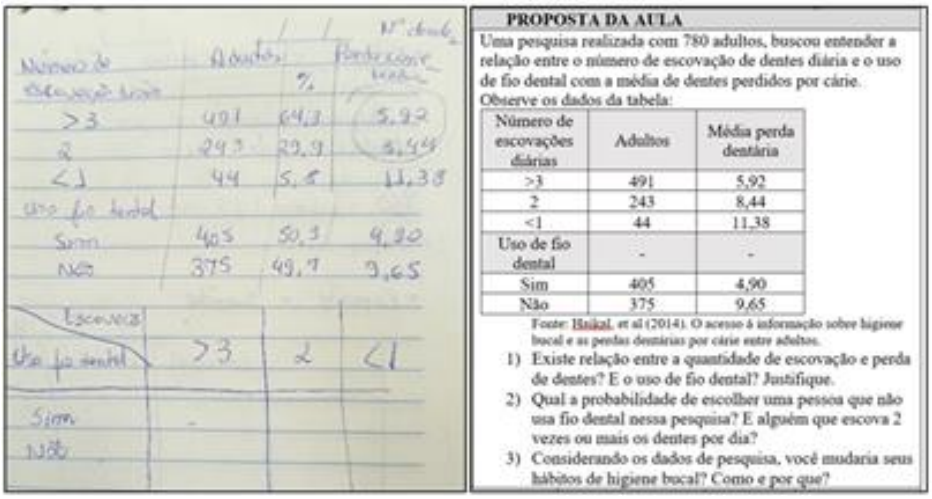
[8] Alias.
[9] Alias.

Results and discussion

The theme of the lesson planning (Action 1) was creating, replicating, or adapting some task involving probabilistic risk in double-entry tables and exploring some social context. The topic emerged in previous meetings, notably during discussions on probabilistic literacy (Gal, 2005). Initially, in subgroups, the TTMs searched for tasks in mathematics textbooks adopted in their schools on the topic. In a brief consultation, Ester's subgroup noted the scarce offer of exercises and tasks that contemplated the notion of probabilistic risk in the 6th and 9th grades of elementary school (the group's focus), also because the curriculum documents do not recommend any skill with an explicit reference to this theme for these school years.

Faced with this limitation, the group started looking for online activities. Ester, who offered to develop the class with her 6th graders, presented to her colleagues a table she outlined based on data from an article[10] she had found during planning, whose theme was about the relationships between oral hygiene and tooth loss due to cavities among adults (Figure 1).

FIGURE 1: Ester's subgroup task proposal



PROPOSTA DA AULA

Uma pesquisa realizada com 780 adultos, buscou entender a relação entre o número de escovação de dentes diária e o uso de fio dental com a média de dentes perdidos por cárie.
Observe os dados da tabela:

Número de escovações diárias	Adultos	Média perda dentária
>3	491	5,92
2	243	8,44
<1	44	11,38
Uso de fio dental	-	-
Sim	405	4,90
Não	375	9,65

Fonte: Baikal, et al (2014). O acesso à informação sobre higiene bucal e as perdas dentárias por cárie entre adultos.

1) Existe relação entre a quantidade de escovação e perda de dentes? E o uso de fio dental? Justifique.
2) Qual a probabilidade de escolher uma pessoa que não usa fio dental nessa pesquisa? E alguém que escova 2 vezes ou mais os dentes por dia?
3) Considerando os dados de pesquisa, você mudaria seus hábitos de higiene bucal? Como e por que?

Source: The authors (2023).

[10] Available at: https://www.scielo.br/j/csc/a/BDg6Wq9PtpjYyQn6Pw9HSKz/?format=pdf&lang=pt.

The numerical data presented in the article were arranged in a simple table containing many variables. However, Ester only focused on two variables (number of daily brushings and flossing) and with them, she tried to build a double-entry table correlating these two variables. Although the task Ester proposed proved to be challenging for a 6th-grade class, the subgroup, especially TTM Dulce, accepted it and, from then on, sought to resize it and construct questions related to the table (Figure 1).

The analyses of Actions 1 and 2 allowed us to arrive at a first set of pre-indicators and indicators on the aspects of the PI evoked from oral narratives and written records woven during the lesson plan construction in Ester's subgroup (Table 1).

CHART 1: Pre-indicators and indicators evoked from Actions 1 and 2

Pre-indicators	**Indicators**
Pre-service teacher: "From the idea of favorable in regarding... choosing the audience that was interviewed that is part of this data group. What is the probability of choosing... **What are the chances** of choosing someone who uses dental floss... Because that's what is required from the 6th graders: **classical probability**. **A top 405 of the total** ".	About the understanding of probabilistic language and its classical meaning: *"in the 6th grade, it's just chances"*
Ester: "In the 6th grade, it's just the **chances**!"	
Dulce: "There could be a third question to work with this **risk**, considering these data: **What would you change in your life?** It must be something aimed at **decision making**, **like, reflection**. What would be the best decision for them to avoid oral problems? But what would it be like?"	Expectations regarding decision making based on the task: *"What would you change in your life?"*
Ester: "And we hope that with this third question, **they reflect... changes**, please!"	

Ester: "Because if I asked for the probability, for example, of brushing more than three teeth per day, **491 is prime**, **and they wouldn't be able to divide**. Because I taught them to simplify, right (irreducible fraction), until possible, and, because it was a prime, they wouldn't be able to do it".	Perception regarding the challenges of the task: *"I have to teach them probability before I get here"*.
Sweet: "It will be a little **difficult** because it's a high number, but..."	
Ester: "Because they didn't do the **correlation**. There are three variables."	
Ester: "So this one is going to be kind of an **exercise class**, huh? So, **I must teach them probability before getting here**. [...] Before going to graphs, I could take a quick look at probability and go back to decimals to explain percentages".	

Source: The authors (2023).

At the 11th meeting, we did the stage of collective reflections in the group about Ester's class (Action 4), supported by observation of the development of the class at school (Action 3), by the teacher educator, TTM Dulce, the pre-service teachers and the video recording of the class. Based on the analysis of Ester's oral and written narratives, we constructed another set of pre-indicators and indicators (Chart 2).

CHART 2: Pre-indicators and indicators evoked from Actions 3 and 4

Pre-indicators	Indicators
Ester: "**I was very happy during the class** because the **theme** is the one we are trying to work with students, the issue of **oral health** because many of them didn't even brush their teeth before [...]. **Then, the science teacher had already started it, taking the** boys to the laboratory, teaching them to brush their teeth, brushing their teeth and so, **I wanted to show with scientific data** this importance, complementing the science class the why; it's not something random, you brush your teeth because you have bad breath, no, it has consequences and look how there are consequences, **and the topic came to my head quickly**. And then I decided to do something about it, and I was very happy because they already knew a lot about the matter, and the changes had already started, right? And then I think the class was kind of a way to close the issue."	Self-perception of the potential and limitations of the class: *"It was a class with mixed emotions, right?"*
Ester: "**It was a class with mixed emotions, right?** I was very **happy** when they were improving, a little **disappointed** for having worked so much on the table and couldn't interpret or know how to recognize, **but I understand that it was my fault**, I didn't work so much on table interpretation, **but I liked the class**, that's it".	
Ester: "**I really liked the guidance**...I don't think I would change the class, but **I would change** something before (in planning) so that you can get to the class the way you were, **I would have worked a little more on the issue of interpreting tables**, which was crucial there."	
Ester: "**I've always had a lot of difficulty planning, especially defining time**, because, to this day, I still can't understand, oh, I'm going to spend half an hour on this, fifty minutes on this. It's so much that, when I was commenting on the planning, I said, wow, two classes will be a long time to talk about this, and then, there, (it wasn't!), wow, five minutes, there's not enough time, we have to hurry. But it's something that would take three classes or more. **This is my biggest difficulty**, limiting the time **and seeing the challenges they may face in the classroom** ".	

Ester: "Because, in the table, I focused a lot on the structure and the structure, it was satisfactory when they answered. The problem is: When I saw the entire structure, **I could see the table, what is my conclusion faced with the results?** They only know how to say "Ah, dental floss, 375 people lost nine teeth". **They could read, but they didn't know how to interpret**, if I use dental floss, I lose fewer teeth."	
Ester: "The different part was this, **to contextualize first** and then appear, wait for them to give me... **And ask the question without ever having gone through it before**, wait for them to answer. **This was the part I was most scared of, asking about probability and no one would answer**, because no one had seen this topic, but it was cool."	From the class mediation process: *"This was the part I was most scared of"*
Ester: "**I think I fear** to bring up an issue (or not bring up an issue) but **bring up the question without having given the matter,** and then they can't do it, **and I can't mediate the class**. Because in this experience, we brought up the question of probability, and they had never seen probability in their lives, **and then, I was afraid they wouldn't be able to answer me** and not even have an idea, **and I can't mediate that without already giving the answer** ".	
Ester: "So, speaking for myself, I didn't graduate in mathematics, I don't have a degree, the teacher I am today is based on my teachers. I remember how I saw my teachers, and I tried to follow **in the same way as they did because I learned that way, and then, when this planning came where I had to reverse everything, I said, oh my God, I've never done that, I've never seen any teacher do that with me**, that's why I was so insecure, I don't know if you noticed. **I was very insecure**!"	
Ester: "I was very insecure because **if no one answered, I would say: what do I do now?**"	
Ester: "**The students had never seen the concept of probability before**. When contextualizing the question 'what is the probability of choosing an adult who doesn't use dental floss?' **I changed it to** 'What are the chances that, in a room of 780 people, you make friends with an adult who doesn't use dental floss?'"	Of reified knowledge: *"It would be more like planning a lesson with*

Ester: "For me, research and prior planning, because I'm a lot like that, I plan, I spend my Sundays planning, I plan everything I'm going to do during the week, but a lot about that, I get the book, I get the curriculum, I see, next week I will have to teach this, **I look up in the book, select exercises and just research what the concept is,** but I don't do research like we did from a practice before to after...**It would be better to plan a lesson better with contextualizations** ".	*contextualization"*
Ester: "At that moment, the teacher educator's intervention in asking them to use a calculator was valid (**I've never worked with a calculator in class, but it can be an ally**) and ask questions about the conclusion of the research and real life."	

Source: The authors (2023).

Once the indicators were established (Charts 1 and 2), we moved on to their articulation, which allowed us to infer and systematize two Cores of Meaning (Chart 3) and, thus, better understand the constitutive aspects of Ester's PI.

CHART 3: Articulation of indicators and the constitution of Cores of Meaning

Indicators	Cores of Meaning (CM)
About the understanding of probabilistic language and its classical meaning: *"in the 6th grade, it's just chances"* (Actions 1 and 2)	**CM 1 - Challenges and expectations in building a lesson plan on probabilistic risk: professional knowledge, beliefs, and conceptions and political commitment as a tuned TTM**
Expectations regarding decision making based on the task: *"What would you change in your life?"*(Actions 1 and 2)	
Perception regarding the task challenges: *"I must teach them probability before I get here"* (Actions 1 and 2)	
Of the potentials and limitations of the class: *"It was a class with mixed emotions, right?"* (Actions 3 and 4)	**CM 2 – Teaching probability presupposes expressing emotions, vulnerabilities, and the self-knowledge of being a TTM**
Of the class mediation process: *"This was the part I was most scared of"* (Actions 3 and 4)	
Of reified knowledge: *"It would be more like planning a lesson with contextualization"* (Actions 3 and 4)	

Source: The authors (2023).

From the intra and inter-core analysis, we began to discuss some constitutive aspects of Ester's PI observed throughout Actions 1, 2, 3, and 4.

Aspects of Ester's PI seized by the Cores of Meaning

CMs 1 and 2 (Chart 3) were constituted by a set of pre-indicators and indicators, which are articulated through some thematic contents, which, in our interpretation, give evidence of the mobilization of aspects of Ester's PI and, in a way, from the group itself, in terms of the macrodomains that underlie the characterization of the TTM's PI assumed here.

The meanings attributed to Ester's PI in **CM 1 –Challenges and expectations in building a lesson plan on probabilistic risk: professional knowledge, beliefs, and conceptions and political commitment as a tuned** TTM– are due to the relationship between the aspect of the *knowledge necessary to teach and learn probability in 6th grade*, combined with the *beliefs and conceptions* and the exercise of professional agency as a manifestation of *political commitment* mobilized in class planning and in presenting the proposal to the collective (Actions 1 and 2).

In the statement, "In the 6th grade, it is only **chances**!", Ester and the subgroup indicate that the task would need to be careful with using a probabilistic language compatible with this school year. Therefore, pedagogically-conceptually, it would make more sense for students to mention the term "chances" and not "probability" in the initial situations of the task. At the same time, when expressing that "Because that is what is required from the 6th-graders, **classical probability**. **Top 405 of the total**", also understand the relevance of the classic meaning of probability to the context of the situation, in this case, represented by the ratio between the number of favorable cases (405 adults who use dental floss) and the possible cases (total number of adults involved in the study).

Ester also evoked this knowledge, linked to a conception that teaching a given mathematical content presupposes the approach of previous concepts

related to it. In the case of probability, Ester and the group agreed, worrying that some data in the tables were large and prime numbers, bumping into each other in a type of division of natural numbers that her students did not yet know. Furthermore, she assumes the belief that, in order to talk about probabilistic risk, students should first learn some skill from the curriculum regarding the formalization of the concept of probability and the practice of exercises, as she told us: "So this one is going to be kind **of an exercise class**, huh? So, **I must teach them probability before getting here**".

However, although the construction of the task showed some weaknesses –mainly because Ester's table did not correlate the variables as initially desired (Figure 1)– it evoked other professional knowledge as a TTM, notably knowledge of students, educational contexts, and the purposes of education (Shulman, 1987), in this case, considering that Ester chose a socially relevant topic for her work context.

At the same time, Ester built a professional agency mediated by her participation in the group by accepting the suggestion of inserting a third question in the task (Figure 1) that would encourage students' reflection. This reinforced her political commitment to the class, as a commitment to action and transformation of her students and the community (CYRINO, 2017), thinking about the relevance of teaching about probability and risk so that her students would become aware of the importance of adopting or changes in habits regarding the number of tooth brushings and flossing.

CM 2 – Teaching probability presupposes expressing emotions, vulnerabilities, and the self-knowledge of being a TTM; it broadens the understanding of the constitutive aspects of Ester's PI, bringing to the fore the expression of *vulnerabilities*, emotions and *self-knowledge*, both in the context of class development and in post-class reflections (Actions 3 and 4). We highlight Ester's statement, which, according to our interpretation, elucidates to a certain extent the genesis of these aspects.

> *Ester*: "So, speaking for myself, I didn't graduate in mathematics, I don't have a degree, the teacher I am today is based on my teachers. I remember how I saw my teachers, and I tried to follow **their steps because I learned that way, and then, when this planning came, where I had to reverse everything, I said, oh my God, I've never done that, I've never seen any teacher do that with me**, that's why I was so insecure, I don't know if you noticed. **I was very insecure**!"

In this excerpt, it is evident that her emotions and feelings of insecurity, fear, and disappointment, at times, hovered over the development of her class. She seems to attribute this to the fact that she does not have a degree in mathematics and is a beginning teacher[11]; therefore, she did not have, until then, the experience of planning and developing a class different from what she saw her basic education teachers doing. In other words, Ester was fearful because, for the first time, she had planned a mathematics class outside the "traditional" perspective; in this case, based on the usual praxeology of first approaching the content/concept, exemplifying and, finally, carrying out fixation exercises, as it has always been exposed.

> *Ester*: "The different part was this, **to contextualize first** and then appear, wait for them to give me... **And ask the question without ever having gone through it before**, wait for them to answer. **This was the part I was most scared of, asking about probability, and no one would answer** because no one had seen this topic, but it was cool."

[11] During her participation in the study group, Ester had been working as a TTM for less than a year. She worked on a contract basis, linked to the Ensina Brasil Program, a Brazilian program that recruits and selects young talents from different careers to, with constant training and pedagogical mentoring, teach for at least two years in vulnerable schools on a paid basis. Available at: https://www.ensinabrasil.org.br/. Access: June 05 2022.

However, bringing a class on inverse praxeology, initially guided by the contextualization of the notion of probabilistic risk based on a problem situation with the topic of oral health and only then delving into the conceptual issues of probability (although her students had not seen the content before), placed her in a situation of professional vulnerability.

This vulnerability revealed a somewhat negative emotional experience for Ester (Lasky, 2005) regarding the challenge of mediating well without having to anticipate the answers to the students or the disappointment of not having had more constructive feedback from them when stating that "They only know how to say 'Ah, dental floss, 375 people lost nine teeth'. **They could read but didn't know how to interpret** it; if I use dental floss, I lose fewer teeth." Added to these negative experiences was that Ester revealed her difficulty managing her time well during the two scheduled classes and **seeing the challenges she might face in the classroom**.

However, as Oliveira and Cyrino (2011) consider, these experiences of showing herself emotionally vulnerable did not make her frail in front of the group, nor did they paralyze her before her students and class observers. On the other hand, this made her question and reflect on herself, triggering aspects of her self-knowledge (Kelcthermans, 2005), notably of her self-esteem (appreciation/self-criticism woven into her class), self-image (as she realized as a TTM at the beginning of her career), and the perception/recognition of her professional tasks, in relation, for example, to the recognition that she would need to teach more table interpretation to broaden the understanding of the notion of the underlying probabilistic risk.

As the reflections progressed, Ester also began to reveal positive emotions (joy and satisfaction) with the conduct of the class, including confirmation from the collective, mainly from TTM Dulce and the pre-service students who attended her class. The group disagreed with her revelation of feeling insecure in conducting the class, highlighting her expertise during the class, which also contributed to reinforcing her self-

esteem, as they acted with mutuality, respect, trust, involvement and empathy (Oliveira; Cyrino, 2011)

This fact corroborates the observation that the movement to establish Ester's PI did not only involve the conceptions she had of herself but, above all, the conceptions and images that her peers externalized about her (Cyrino, 2018; Kelcthermans, 2005).

Finally, when we analyze the relationships established between the thematic contents of the two CMs, we perceived more accurately the reinforcement of aspects of Ester's PI because they triggered crucial professional learning opportunities that emerged in class and others that justified her stance since the planning stage.

We highlight, among them, the sense of *professional agency* in mobilizing knowledge that denotes her didactic-pedagogical expertise by stating, for example, that "**The students had never seen the concept of probability before**. When contextualizing the question: 'What is the probability of choosing an adult who doesn't use dental floss?' **I changed it to** 'What are the chances that, in a room of 780 people, you make friends with an adult who doesn't use dental floss?'"

Furthermore, we realize that the *political commitment* is a striking feature of Ester's PI, which justifies her choosing the topic of oral health, as it was a discussion that had already taken place in Science class but which, in her case, was intended to show the risks of not brushing one's teeth and floss with scientific data. This made her rethink the teaching of probability beyond the context of games of chance, while it began to signify the importance of real contexts for teaching probability with a view to democracy and citizen empowerment (Gal, 2005; Engel; Ridgway; Stein, 2021). No less important, Ester could validate the use of teaching resources, such as the calculator. Although she had never used the calculator in her classes and it was not included in the planning, she recognized it was relevant in the investigated context.

Final considerations

The two CMs spotlighted thematic contents that expressed some aspects of Ester's PI before the characterization of the TTM's PIs assumed in this work. Such aspects were underscored based on a web of meanings established by intertwining them. The (re)construction of professional knowledge to teach probabilistic risk in the 6th grade of elementary school with the support of double-entry tables was underlined in a context which, albeit challenging, was fundamental, as it "verified" some beliefs and Ester's conceptions about the didactic-pedagogical and curriculum treatment of content, time management, and pedagogical mediation. But, beyond that, her PI was revealed as she made a political commitment to constructing class planning based on the development of students' probabilistic literacy, with a view to decision-making in real-life situations and the construction of critical citizenship.

Thus, by having the insight to address the topic of oral health linked to the notion of probabilistic risk, Ester exercised professional agency mediated by the approval of her peers to meet a demand from her school context. Although the results corroborated the finding that probabilistic risk is a multifaceted and complex construct, the group discussions about the class were quite fruitful, allowing Ester's experiences of vulnerabilities and manifestations of positive and negative emotions to (re)build her self-esteem and professional self-image.

However, the collaborative fabrics within the group shaped the (re)signification of the aspects (macrodomains) that constituted her PI through the support networks offered and received in identifying and establishing vicarious relationships. These relationships expanded learning repertoires as an identity constitution of being, perceiving oneself, and being recognized as a TTM under a different professional education and performance logic.

From the results presented, we understand the relevance of this work to guide continuing education processes in study groups with TTMs, in the sense of taking the constitution/(re)configuration of their PIs as a formative perspective and, particularly, that have as a backdrop the discussions focused on probabilistic education in probabilistic risk. This is a key aspect featured in this work, which evokes a critical future research agenda.

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