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Intuitive understandings of arithmetic mean presented by 6th-grade students[1]

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ABSTRACT
This study aimed to analyze the response levels of 31 students attending the sixth grade of elementary school, enrolled in a private school in Porto Alegre, Rio Grande do Sul, Brazil. The research is qualitative; data collection took place through students' registers made during the activity, based on their intuitive ideas, whose situations involving media used the FIFA World Cup Qatar 2022 sticker album as context. The results indicate that for the calculation of the arithmetic mean and its understanding, most students are at the *multistructural* response level. The students' answers revealed that, although this subject has not yet been addressed at school, they already have insights about the meaning of arithmetic mean.
KEYWORDS: Arithmetic Mean. Statistical Education. Basic Education. World Cup.

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Compreensões intuitivas de média aritmética apresentadas por estudantes do 6°ano do ensino fundamental

RESUMO

Este estudo teve por objetivo analisar os níveis de resposta de 31 estudantes do sexto ano do Ensino Fundamental, matriculados em uma escola da rede privada da cidade de Porto Alegre, Rio Grande do Sul, Brasil. A pesquisa é qualitativa, e a coleta de dados se deu mediante os registros feitos pelos discentes na atividade aplicada, a partir de suas ideias intuitivas, cujas situações envolvendo média utilizaram como contexto o álbum de figurinhas da Copa do Mundo FIFA Qatar 2022. Os resultados indicam que – tanto para o cálculo da média aritmética quanto para sua compreensão de significado – a maioria dos estudantes se concentra no nível *multiestrutural*. As respostas dos discentes revelaram que, embora este assunto ainda não tenha sido abordado na escola, eles já apresentam noções intuitivas sobre o significado de média aritmética.

PALAVRAS-CHAVE: Média Aritmética. Educação Estatística. Educação Básica. Copa do Mundo.

Comprensión intuitiva de la media aritmética presentada por alumnos de 6º curso

RESUMEN

Este estudio tuvo como objetivo analizar los niveles de respuesta de 31 alumnos del sexto año de primaria, matriculados en una escuela privada de la ciudad de Porto Alegre, Rio Grande do Sul, Brasil. La investigación es cualitativa y la recogida de datos se dio a través de los registros realizados por los estudiantes en la actividad aplicada, a partir de sus ideas intuitivas, cuyas situaciones involucrando medios de comunicación utilizaron como contexto el álbum de cromos de la Copa Mundial de la FIFA Qatar 2022. Los resultados indican que, tanto para el cálculo de la media aritmética como para la comprensión de su significado, la mayoría de los estudiantes se centran en el nivel *multiestructural*. Las respuestas de los estudiantes revelaron que, aunque este tema aún no ha sido abordado en la escuela, ya tienen nociones intuitivas sobre el significado de la media aritmética.

PALABRAS CLAVE: Media Aritmética. Educación Estadística. Educación Básica. Copa Mundial.

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Introduction

In the National Common Curricular Base (BNCC), the concept of arithmetic mean is addressed from the seventh grade of elementary school onwards (BRASIL, 2018). Still, intuitive notions, or insights, of this concept are present in everyday situations before this educational stage. The National Curricular Parameters (PCN) indicate that from the 4th and 5th grades onwards, elementary school students should be able to obtain and interpret averages (BRASIL, 1997). In continuation, they advise using the arithmetic mean for the 3rd cycle (6th and 7th grades) as a tool for more in-depth data interpretation (BRASIL, 1998).

Abbagnano (2007, p. 581) defines intuition as a "direct relationship (without intermediaries) with any object; therefore, it implies the effective presence of the object". Given this, we agree with Soares (1995) when he states that intuitive notions form the initial stage of reasoning; they must be valued and encouraged. Pilati, Martins, and Kato (2017, p. 68) understand that "insight is one of the first steps through which the individual develops their perception of the world and its definitions, which are confirmed or refuted as they interact with the world that surrounds them."

Regarding the concept of average, Cazorla, Santana, and Utsumi (2019, p. 1) emphasize that, despite its relevance for statistics, its appropriation requires a "network of concepts, properties, representations, and multiplicity of situations in which it can be found." Of the basic concepts of statistics, Marques, Guimarães, and Gitirana (2011) highlight the arithmetic mean, which, according to the authors, is commonly used in schools, academia, the economy, and people's daily lives.

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Given the above, we realize the need to develop students' understanding from the beginning of basic school. For instance, in this regard, Porciúncula and Batisti (2023) emphasize that statistics is a science present in countless everyday situations, in different areas of our social environment; it should, therefore, be included from the first school years.

In this scenario, this work aims to analyze the response levels, according to Watson and Moritz (2000), of sixth-grade students in situations involving the concept of average –or arithmetic mean. Thus, we proposed situations involving the concept using as context the FIFA World Cup Qatar 2022 sticker album in a class of 31 students from a private school in Porto Alegre, Rio Grande do Sul.

We understand that the insights presented by sixth-graders about arithmetic mean allow the work of meaningful operations, approaching numbers in context. Moore (1990, p. 424, our translation) emphasizes that "meaningful data help replace the emphasis on calculation, the exercise of judgment and the emphasis on interpretation and communication of results".

Based on the analysis of contexts and their comparisons, statistical education plays a relevant role in the "development of attitudes, skills, and knowledge that allow the advancement of reflection and criticality about information conveyed through statistical content" (CARVALHO, 2006 *apud* MARQUES; GUIMARÃES; GITIRANA, 2011, p. 726).

In the following sections, we discuss the properties and meanings of the concept of average and studies that classify students' response levels at different stages of schooling. In sequence, we spotlight the methodological paths of the research, explaining the questions that served as the basis for the discussion and data analysis in the following section.

About the concept of arithmetic mean

The development of studies and research on the arithmetic mean is essential since this concept –according to Magina et al. (2010)– plays a crucial role in statistics and experimental science and is widely used in everyday life, which makes it fundamental.

Research that discusses the concept of average lists different characteristics and varied meanings and properties linked to it. Batanero (2000) specifies four meanings of average:

- Estimation of an unknown quantity in the presence of measurement errors.
- Obtaining an equitable amount to share to achieve uniform distribution.
- Application in which the average is a representative element of a set of given values whose distribution is approximately symmetric.
- The need to know the value that will be obtained with greater probability when counting on missing data in a distribution.

For Cazorla and Santana (2006, p. 18), the arithmetic mean is a measure that "summarizes and represents a data set in a single value", and to calculate it, we must divide the "whole between its components in equal parts. Thus, its algorithm consists of adding all the values of the variable and dividing by the number of data" (CAZORLA; SANTANA, 2006, p. 18).

Marques, Guimarães, and Gitirana (2011) highlight that –often– the concept of average is approached with a focus only on the procedural part to the detriment of valuing the understanding of its meanings and properties. They also say that this occurs not only with mathematics but also with statistics concepts.

When analyzing the concept of average, Strauss and Bichler (1988) structured seven properties:

a) The mean is located between the extreme values.
b) The sum of the deviations from the mean is zero.
c) The mean is influenced by each and every value.

d) The mean does not necessarily coincide with one of the values that compose it.
e) The mean can be a number that does not have a counterpart in physical reality; that is, it does not correspond to a possible value of the variable considered.
f) The average calculation considers all values, including null and negative values.
g) The mean is a representative value of the data from which it was calculated. In spatial terms, the mean is the closest value to all values.

Strauss and Bichler (1988) consider further that other properties could have been listed and tested. However, they understand that the seven properties presented are fundamental; from this, they address three aspects of their concept:

- First: the statistical aspect, present in the first three properties that are basic to the arithmetic mean.
- Second: the abstract, which is seen in properties D, E, and F. Such properties consider unobserved values.
- Third: the representativeness of a group of individual values, present in the last property, is the central aspect for the mean (MORONEY, 1956 *apud* STRAUSS; BICHLER, 1988).

Levels of understanding of the concept of average

In their studies, Watson and Moritz (1999) investigated the understanding of the comparison between data sets based on graphs built by Australian 3rd, 6th, and 9th graders in Tasmania and 3rd, 5th, 7th, and 9th graders in South Australia. In this work, the authors sought to understand how students were involved in solving problems and –in this way– classified the development levels of students' answers.

To classify such answers, Watson and Moritz (1999) initially structured the following levels: 1. *Unistructural*; 2. *Multistructural*; 3. *Relational*. In this study, the authors found the highest frequency of the *multistructural* type among research participants and an emphasis on the algorithm to the detriment of relevant aspects of data analysis. Based on this investigation, Watson and Moritz (1999) report that knowledge of the algorithm to find the average does not provide sufficient elements for students to present types of answers at the *relational* level.

Following this line of research, in a longitudinal study, Watson and Moritz (2000) investigated 94 students from the 3rd to the 9th grades, conducting follow-up interviews with two groups of students (22 students over three years and 21 after four years) to investigate the development of understanding of the concept of average. With the analyses, the authors listed six levels of response types. The first four began with conceptualizing average, starting from informal notions and explaining procedures or concepts to obtain a central dataset measure. The last two levels included the reverse-averaging process and the evaluation of weighted averages.

Given this, the first four response levels that emerged from Watson and Moritz's (2000) study on understanding the concept of average were called *prestructural*, *unistructural*, *multistructural*, and *relational*.

The answers that did not show any relationship with the notion of the arithmetic mean are at the *prestructural* level. The answers at the *unistructural* level are based solely on the relevant aspect of the task at hand. Regarding understanding, they would be those that can only present a more intuitive notion about average, indicating everyday uses for this concept. In turn, the *multistructural* level of response addresses several relevant and disjoint aspects, generally in sequence, albeit not combined. For example, the average can be represented by using the algorithm. Responses of the *relational* type are characterized by a comprehensive understanding of the interactions between several aspects of the domain, resulting in a cohesive structure and coherent meaning for the whole. At this last response level, the average is understood as a construct articulating all the information about the proposed situation.

In this investigation, Watson and Moritz (2000) found that the types of answers about understanding the concept of average produced by students were mostly at *uni* and *multistructural* levels, presenting little responses at the *relational* level.

Following this line of investigation, Watson (2006) discussed the understanding of the average of students in the 3rd, 6th, and 9th grades in a longitudinal study based on questions that relate this concept to family and media contexts. As a result, the study argues that students emphasized memorizing the algorithm, which suggests their recent participation in a class about averages. However, these students could not give more comprehensive or systematic answers.

It is worth noting that, according to Watson (2006), the development of average understanding occurs gradually throughout schooling since students present different response levels as they progress. The author points out that the arithmetic mean is widely used to illustrate statistical data, and its calculation procedure for obtaining the arithmetic mean can be understood by students aged between 9 and 12 years. This circumstance results in an overexposure of this construct since it can be used in tests with correct or incorrect answers, dispensing with a detailed verbal explanation.

In this sense, it is relevant to look at the proposition of activities to discuss various aspects of the concept of average (meanings, properties) that can explore from an intuitive approach that highlights students' experiences to those that establish relationships between data from problems and the contexts in which they are inserted, encouraging different levels of responses to emerge.

In his investigation, Eugênio (2013) discusses the interpretation of graphics in the software TinkerPlots, presented by students in the 5th and

9th grades of elementary school, examining their explorations to understand the concept of average.

In his study, the author found that most 5th graders had a *prestructural* response level, not presenting previous conceptual notions about the average. Concerning the 9th-grade students, the author observed more *unistructural* responses. In some cases, students' answers indicated thoughts about measures of central tendency, highlighting the *multistructural* response level.

From the above, we can observe the complexity of the concept of average about the meanings and properties that should be part of basic education classroom repertoire. We understand that the response level categories presented allow an investigation into elementary school students' intuitive perception of the concept of average, particularly in the sixth grade, covered in the present study.

Methodology

This qualitative study is characterized by the investigation of phenomena in their own context, considering the knowledge and practices of the subjects involved (BOGDAN; BIKLEN, 1994).

Realizing that the World Cup event mobilized students, who engaged in completing their sticker albums, we realized that the information about the players could spark discussions related to statistical concepts during mathematics classes. In line with the BNCC (BRASIL, 2018) and the response levels on the meaning of average by Watson and Moritz (2000), we structured a set of questions involving the concept of average. To this end, we chose to use the FIFA World Cup Qatar 2022 sticker album as context for a class composed of 31 students attending the 6th grade of elementary school at a private school in Porto Alegre, Rio Grande do Sul.

Students who had the album –completed or not– were asked to take it to school for the activity. Initially, to contextualize the subject, we mediated a discussion with the class about general aspects of the World Cup, such as the history of the tournament, curiosities, participating countries, location, biggest winners and top global players of this event.

Next, each student had to raffle a team participating in the 2022 World Cup to answer the questions that would be asked. At this point, we commented briefly on the probability that each would have of taking the country they had in mind, considering that such a raffle did not involve replacement teams.

Some students were concerned that not the entire album was filled with stickers from the raffled selection. Others who did not have the album were apprehensive because they said they might be unable to do the activities. Therefore, we asked them to sit in pairs or trios with colleagues who had the necessary data. In this regard, it is worth noting that working together enabled them to share information, exchange ideas and explanations, and collectively engage in the proposed activities.

At the end of this meeting, each person had to deliver their answers to the class teacher, one of the authors of this article.

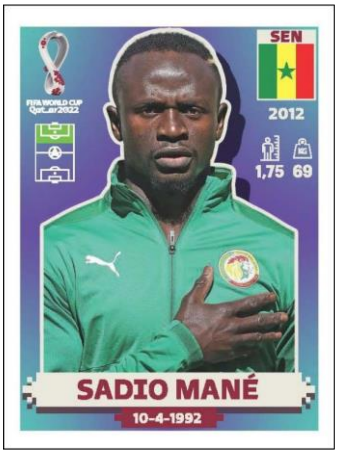
Figure 1 exemplifies the album pages referring to the Senegal National Team, with all the stickers glued.

FIGURE 1: Senegal National Team – FIFA World Cup Qatar 2022.

Source: FIFA World Cup Qatar 2022 sticker album (PANINI, 2022, p. 12-13).

Figure 2 shows one of the stickers in the album depicting information about the player (height, weight, date of birth, and position).

FIGURE 2: Information about the player Sadio Mané



Source: FIFA World Cup Qatar 2022 sticker album (PANINI, 2022, p. 13).

In this scenario, we must underscore that these students had never formally studied the concept of average, which, in this school, is worked on the Mathematics subject in the 7th grade. Therefore, data collection, which took place through students' registers during the activity, aimed to analyze perceptions based on their intuitive notions.

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Below, we present the five questions proposed to students to be answered after the raffle:

a) Regarding your raffled selection, what are the most frequent ages of the players?
b) Regarding your raffled selection, what is the average height of the players?
c) Explain how you calculated the previous question.
d) Regarding your raffled selection, what is the average weight of the players?
e) Explain, in your own words, what it means to “find the average of a data set” (note: this is not to explain how the calculation is done, but rather the meaning of average).

Due to the limited space in this study, the analyses focus on questions “c” and “e” presented above.

The research follows the disposed in Resolution N. 510 of April 7, 2016, article 1, item VII, about the standards of the Research Ethics Committee in the area of Human and Social Sciences (BRASIL, 2016). To preserve the students’ identity, when necessary, they will be identified by E1 through E31.

In this study, the term “average” will be used as a synonym for simple arithmetic mean, according to the context already explained.

Discussion and analysis of results

This section describes the steps of the proposed activity and the analysis based on Watson and Moritz (2000), adopting the *prestructural*, *unistructural*, *multistructural*, and *relational* response levels on the meaning of average. To this end, we use Strauss and Bichler’s (1988) notions of properties and aspects of the average.

This practice with the sixth-grade class was not intended to be an in-depth study of arithmetic mean but rather to encourage an introductory reflection on the subject, using as motivation the sticker album, something present in students' daily lives.

To visualize students' words evoked in the two analyzed questions, we produced word clouds based on the responses collected with the free app Wordclouds, as seen in Figure 3.

FIGURE 3: Word clouds representing the terms used in students' answers to questions *c* and *e*

Source: Research material.

The first word cloud (on the left) shows a relationship with students' operations to calculate the players' average height (divide, add, number, numbers, quantity, round, result, and calculation) and words that identify the context of question *c* (height, heights, players, album, and Senegal).

In the second cloud (on the right), referring to the question that asks students to explain their understanding of average (question *e*), words such as number, numbers, divide, add, result, multiples, six, quantity, and calculation are noted. Such words refer to the notion of arithmetic operations. Other words such as average, data, center, equivalent, between, mean, half, common, statistics, and standard were evoked, highlighting the list of words that intuitively reflect the notion of arithmetic mean for these students.

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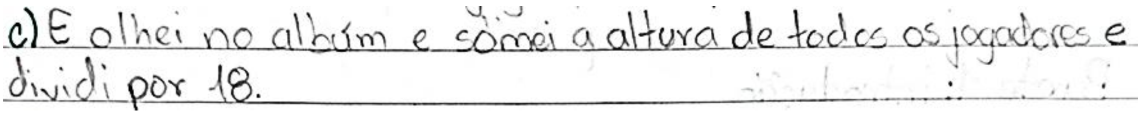
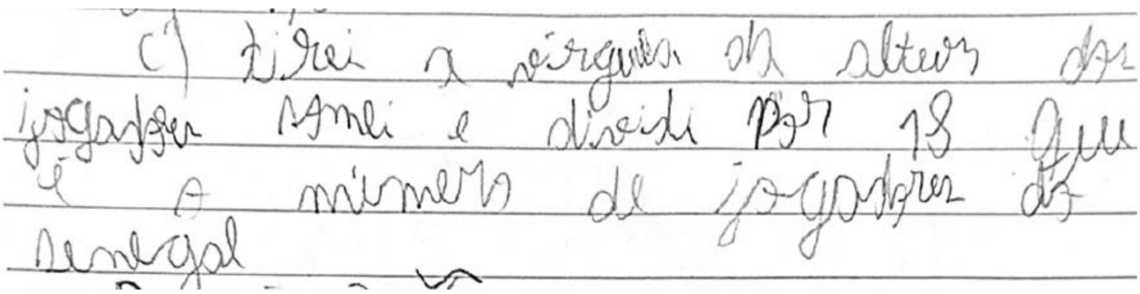
Regarding question c, which asked for an explanation of the calculation of the arithmetic mean of the players' height, we found that most students knew how to calculate it, given that, of the 31 answers, 22 were correct. Figures 4 and 5 exemplify two of these responses.

FIGURE 4: Answer given to the question *c* by student E23



Source: Research material.

FIGURE 5: Answer given to the question *c* by student E4



Source: Research material.

Number 18, cited by the students (Figures 4 and 5), refers to the total number of players from each team, as exemplified in Figure 1. According to Watson and Moritz's (2000) classification, these responses belong to the *multistructural* level, as they elucidate the procedure used for the averaging algorithm.

Although the students had not formally studied the arithmetic mean, a good part of the class was expected to have an idea of how to do the calculus. In this regard, Melo's (2010, p. 76) study with 3rd and 5th-grade students on invariants and meanings of the concept of average found that "students have some type of idea about the average, using it in some everyday situations, but without understanding what it really means".

It is pertinent to reiterate that the students participating in this research did not receive any previous explanations about the meaning of

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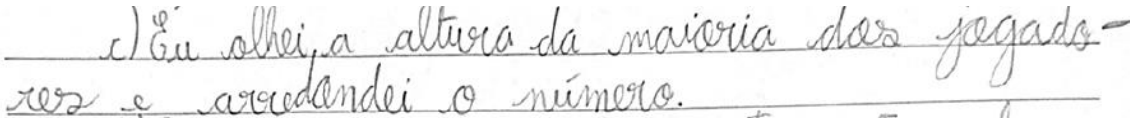
average. Therefore, as Melo (2010) states, we believe such students probably already had insights of average.

The registers considered incorrect regarding the question that requested an explanation of the calculation of the arithmetic average of the players' height were separated into groups, using the similarities in the ideas presented as a criterion. We considered the ideas of rounding, maximum and minimum values, answers without explanation, and answers without apparent meaning. Figure 6 illustrates a response that addresses the notion of rounding.

FIGURE 6: Answer given to question *c* by student E17



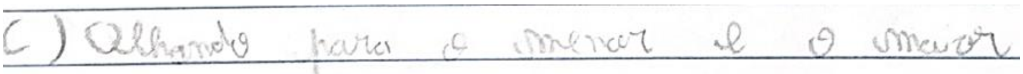
Source: Research material.

The rounding of the players' heights does not represent the average of the data set. In this sense, the intuition presented by student E17 seems to be justified in arithmetic ideas, such as rounding, without mentioning the arithmetic mean. Therefore, this level of response was classified as pre-structural, as it is not related to the concept of average.

When observing the players' heights, the student possibly realized that most were close to a specific value and rounded off the verified heights so that they were all equal to the observed value. When acting this way, the student seems to have an insight, based on more frequent values, that this would be the average.

Figure 7 shows an example in which the student considered the maximum and minimum values of the heights to determine the average value.

FIGURE 7: Answer given to question *c* by student E15

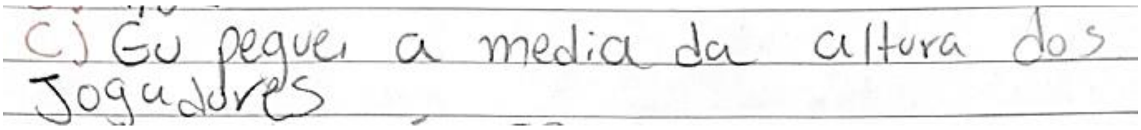


Source: Research material.

Student E15's answer, shown in Figure 7, refers to the first property of the concept of average listed by Strauss and Bichler (1988). This student suggests understanding that the average is located between the extreme values. However, we cannot perceive that his understanding is about the other values, for example, that each of these values and that they all influence the average value. This answer, as stated, can be characterized as belonging to the *unistructural* level, as it only presents a brief observation about the maximum and minimum values of the players' heights, a specific aspect of the proposed question.

Among the answers without explanation, we present in Figure 8 the one constructed by student E20.

FIGURE 8: Student E20's answer to question *c*



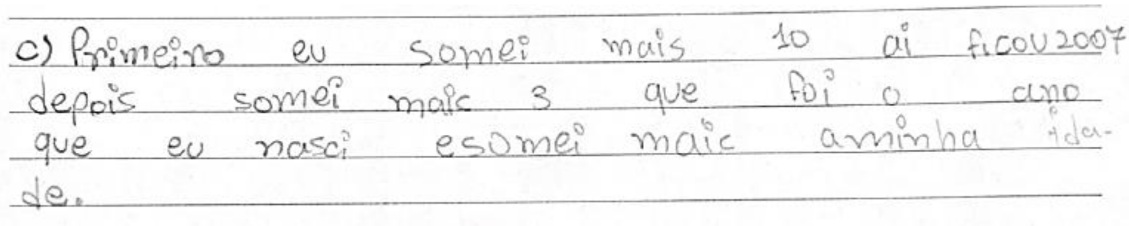
Source: Research material.

When stating that he took "the average height of the players", student E20 did not specify what was done; however, his answer presents an intuitive notion about average and uses colloquial language (*I took the average*). Thus, based on Watson and Moritz (2000), E20 seems to be at the *unistructural* level.

Eugênio, Carvalho, and Monteiro (2016, p. 1184) emphasize that average is linked to various contexts in everyday life, including "contexts of colloquial language use". Therefore, in the classroom, it is pertinent to address the possible meanings of this concept to contextualize and clarify when to use it.

To exemplify the group of registers considered to have no apparent meaning, Figure 9 shows the answer given by student E19.

FIGURE 9: Answer given to question *c* by student E19



Source: Research material.

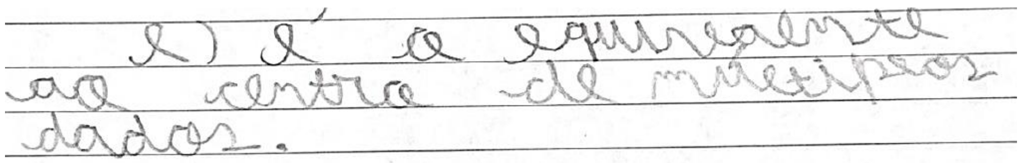
E19's answer was classified as belonging to the pre-structural level, as it does not clearly convey the idea of having a relationship with the arithmetic mean.

Regarding question *e*, which requested an explanation about the meaning of "finding the average of a data set", several doubts arose, and the answers were varied during the class. Many turned to the teacher, seeking to argue their justifications based on the average calculation since they claimed that it was difficult to explain the arithmetic mean.

In this scenario, the teacher said that the objective was not to explain the calculation but to reflect on the meaning of the average. Even so, of the 31 registers, three ended up just mentioning the algorithm involved. About this, Marques, Guimarães, and Gitirana (2011) discuss the overvaluation of procedures instead of understanding meanings and properties. Watson and Moritz (2000) and Eugênio *et al*. (2016) state that the answers –when they use the algorithm to solve problems based on averages– are characterized as *multistructural*. Even though they knew how to use the algorithm, these students just explained it, which indicates the absence of context elements in their explanations, presenting an insight based on the arithmetic of the concept of average. This fact reinforces results found by Watson and Moritz (1999; 2000) in which the use of the average algorithm by students is insufficient to understand the properties and situations presented by the concept of average.

It is worth highlighting that nine registers related the meaning of the average to the centrality of the data, listing terms such as “center”, “middle”, and “between”, like student E16, who related his explanation to the idea of the median, when explaining that “*The average means finding out which number is in the middle*”. Another answer is presented in Figure 10.

FIGURE 10: Answer given to the question *e* by student E31



Source: Research material.

Batanero (2000, p. 3), in a discussion about the meanings of average, shows that “to represent a data set, we use the average for its properties of a central location, as it is the ‘center of gravity’ of the space of sample or populational values”. In this way, E31’s reflective, intuitive answer suggests the search for this central location as presented by the author. Therefore, this type of answer was classified as *multistructural* since it comes close to the notion of average.

However, we know that the mean is a good representation of the center of the data distribution when these distributions are symmetric. For cases of asymmetric distribution, the mode or median could better represent a data set (BATANERO, 2000).

It is important to emphasize that five records alluded to the concept of fashion, as exemplified in Figures 11 and 12.

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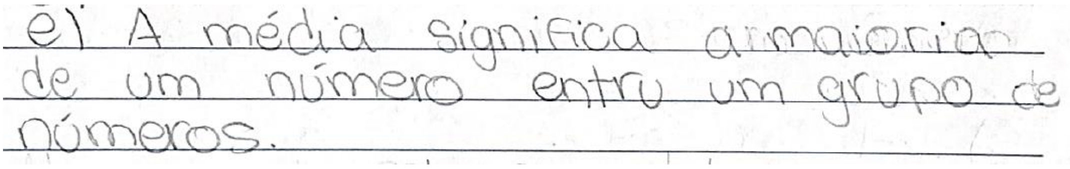
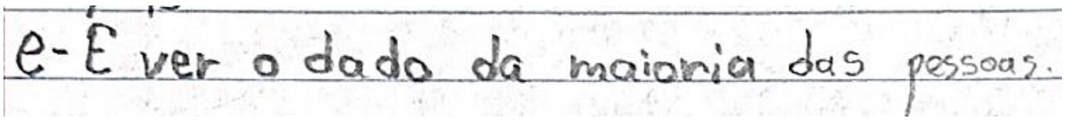
FIGURE 11: Answer given to the question *e* by student E2


e) A média significa a maioria de um número entru um grupo de números.

Source: Research material.

FIGURE 12: Answer given to the question *e* by student E13


e- É ver o dado da maioria das pessoas.

Source: Research material.

The responses displayed above E31, E2, and E13 – in Figures 10, 11 and 12, respectively – were classified as belonging to the *multistructural* level, since they mention the concepts of median or mode. In this regard, Watson and Moritz (2000) indicate that the use of expressions *middle* or *between a data set* may represent an insight into the concept of median, while the use of the expression *the same as most others* can be associated with the concept of mode.

The authors think that developing an understanding of mode, median, and mean during the schooling process is needed. Therefore, schools should emphasize conceptual and procedural aspects and encourage discussions about the relevance of using each measure to represent different situations (WATSON; MORITZ, 2000).

We recognize that some students may be confused about the concepts and applications of these three measures of central tendency. In this scenario, it is up to the teacher to clarify the meanings and their uses in the most diverse everyday situations.

Three registers explained the meaning of average by associating it with approximation. In Figure 13, there is an example of this situation.

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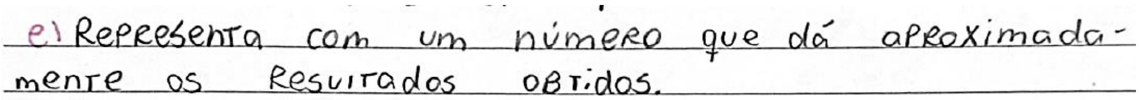
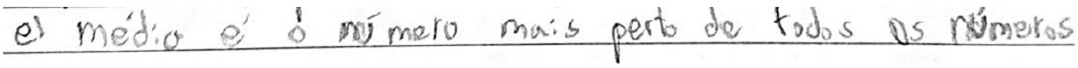
FIGURE 13: Answer given to the question *e* by student E28


e) Representa com um número que dá aproximada-
mente os resultados obtidos.

Source: Research material.

One student associated the average with approximation, as shown in Figure 14.

FIGURE 14: Answer given to the question *e* by student E18


e) média é o número mais perto de todos os números

Source: Research material.

Response levels that consider the average as an approximate number or as an idea of proximity suggest the insight of the property that the average is a value closest to all values. In this way, they can be classified as *multistructural*, given that they mention one of the properties of the very concept of average.

One of the registers linked the average to the definition of a standard (E17), while another stated that it is "*see beyond the data*" (E19), being considered at the *unistructural* level because, although they do not present a closer relationship to the concept of average, they indicate the connection between a piece of data and the data set. It is worth highlighting the presence of four answers that related the average to half of a calculation. We considered these to be at the *prestructural* level because, although they allude to calculations, they refer to situations unrelated to contexts.

Student E15's register stood out due to the example he created when he stated that finding the average of a data set "*means obtaining data from a survey; for example, you want to know if the average height of your family of six, you take the heights of your family and add them and divide them; that is, the average is more or less the number that if your family were one person, they would be this tall*." From his reflection, we infer that some of the average properties are present, such as: it is located between the extreme values since the person created must have a height between the shortest and the tallest; it does not necessarily coincide with one of the values that compose it, as it does not need to be the same height as anyone in the family; it is a representative value of the data that was calculated, since the estimated height is obtained from family height data.

Therefore, we believe that this last answer is at a *relational* level. We recognize that the student gives an example of a context that goes beyond the average algorithm, presenting a reflection on the meaning of average as a representative element of a set of values.

Of the remaining responses, five students left it blank, and one wrote, "*I don't know*". We understand that, in fact, such questioning about the meaning of average can prove to be quite challenging and uncommon, being rarely explored during basic schooling since the emphasis is, most of the time, on the algorithmic procedure, as already discussed here.

In this scenario, while in the first question –calculating the average height of the players– most students answered straightforwardly and correctly, the reflections requested about the meaning of the average raised concerns, and they felt prompted to answer it. We expected this anxiety mainly because this group had never been formally introduced to the topic at school. In this way, students' notions relate to their insights on the subject. Furthermore, it is pertinent to underscore that understanding the meaning of the arithmetic mean is not trivial since its domain is linked to understanding the properties listed by Strauss and Bichler (1988).

Final considerations

This qualitative study analyzed the response levels of a class of 31 students attending the sixth grade of elementary school, based on their insights about the meaning of average. To this end, we proposed situations involving the arithmetic mean from the context of the FIFA World Cup Qatar 2022 sticker album.

Regarding the question asked to students about calculating the average, we found that 22 of them answered correctly, demonstrating knowledge of the arithmetic operation involved. Thus, the answers given were classified as belonging to the *multistructural* level, according to Watson and Moritz's (2000) response levels on the meaning of average.

Regarding the reflection requested about what it means to "find the average of a data set", we found that –again– most registers (20 of 31) indicated that students' insights could be classified, according to Watson and Moritz (2000), at the *multistructural* level. The students' answers revealed that, although this subject has not yet been covered at school, they already have insights into the meaning of arithmetic mean.

We consider that statistical concepts should be explored early in basic education mathematics classes so that they do not focus solely on memorizing formulas and applying algorithms in repetitive exercises. Therefore, it must be possible to experience situations that require reflection and provide an understanding of ideas, relating them to everyday scenarios that are part of the students' reality that make sense to them.

From this perspective, diagnostic didactic situations, such as the one in this study, support analyzing students' insights on a given subject, with which they contribute to planning more effective actions in the classroom.

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