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Using R software to assist in the teaching-learning process of combinatorial analysis[1]

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ABSTRACT
Many statistical methods are based on combinatorial analysis, a topic present at all levels of education that assists in the development of statistical and probabilistic thinking. In today's highly technological and digital society, a significant emphasis is placed on the use of interactive and free computational tools. In this context, this paper proposes a decision tree, a new computational tool in R software that assists in identifying the type of combinatorial analysis to be applied to a specific problem. A pilot study conducted revealed the tool can be a viable and innovative option for teaching combinatorial analysis, according to its participants. The tool is of particular relevance for the applicability of combinatorial analysis concepts to various everyday contexts, since its understanding aids in the development of critical thinking and decision-making. The aim is to contribute to the development of some skills necessary for students' statistical and probabilistic literacy.
KEYWORDS: Combinatorial Analysis. R Software. Computational Tool. Decision Tree. Statistical literacy

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Utilização do software R para auxiliar no processo de ensino-aprendizagem da análise combinatória

RESUMO

Muitos métodos estatísticos se fundamentam na análise combinatória, um conteúdo presente em todos os níveis de ensino que auxilia na construção do pensamento estatístico e probabilístico. Na sociedade atual, altamente tecnológica e digital, há uma valorização significativa da utilização de ferramentas computacionais interativas e gratuitas. Nesse contexto, o estudo propõe uma nova ferramenta computacional no software R, uma árvore de decisão, que auxilia na identificação do tipo de análise combinatória a ser aplicada em um determinado problema. Realizou-se um estudo piloto e constatou-se que a ferramenta pode ser uma opção viável e inovadora para auxiliar no ensino da análise combinatória, pois teve aceitação por parte de seus participantes. A relevância dessa ferramenta está relacionada à aplicabilidade dos conceitos de análise combinatória em diversos contextos do cotidiano, uma vez que sua compreensão auxilia no desenvolvimento do pensamento crítico e na tomada de decisões. Dessa forma, busca-se contribuir para o desenvolvimento de algumas habilidades necessárias para o letramento estatístico e probabilístico dos alunos.

PALAVRAS-CHAVE: Análise Combinatória. Software R. Ferramenta computacional. Árvore de decisão. Letramento Estatístico.

Uso del software R para ayudar en el proceso de enseñanza-aprendizaje del análisis combinatorio

RESUMEN

Muchos métodos estadísticos se fundamentan en el análisis combinatorio, un contenido presente en todos los niveles de enseñanza que ayuda en la construcción del pensamiento estadístico y probabilístico. En la sociedad actual, altamente tecnológica y digital, se valora significativamente el uso de herramientas computacionales interactivas y gratuitas. En este contexto, el estudio propone una nueva herramienta computacional en el software R, un árbol de decisión, que ayuda a identificar el tipo de análisis combinatorio a aplicar a un problema específico. Se realizó un estudio piloto y se encontró que la herramienta podría ser una opción viable e

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innovadora para ayudar en la enseñanza del análisis combinatorio, ya que fue aceptada por sus participantes. La relevancia de esta herramienta está relacionada con la aplicabilidad de los conceptos de análisis combinatorio en diversos contextos cotidianos, ya que su comprensión ayuda al desarrollo del pensamiento crítico y la toma de decisiones. De esta manera, se busca contribuir al desarrollo de algunas habilidades necesarias para la alfabetización estadística y probabilística de los estudiantes.

PALABRAS CLAVE: Análisis combinatorio. Software R. Herramienta computacional. Árbol de decisión. Alfabetización estadística.

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Introduction

Many statistical methods are based on combinatorial analysis, a content present at all educational levels that contributes to the development of statistical and probabilistic thinking. However, in Brazil, the teaching of Statistics was officially included as a curricular content only at the end of the 1990s by the National Curricular Parameters - PCN (BRASIL, 1997, 1998, 2000). Before that period, it was considered "information processing".

Government guidelines PCN have a guiding role in national education. They are organized by discipline, thus, offering specific guidelines for the teaching of Statistics and Probability within Mathematics, which can be considered a non-deterministic part of the discipline, also known as stochastic.

The curricula adopted in Brazilian schools underwent a second reformulation through the implementation of National Common Curricular Base (BNCC) (BRASIL, 2017, 2018), a mandatory reference designed primarily for systematizing educational content, also highlighting the importance of using technologies, including software. Such teaching resources must be integrated into situations that lead to reflection (BRASIL, 2017, 2018).

In the scope of Statistics, BNCC establishes guidelines for the teaching of Probability, indicating this area of knowledge develops progressively, following a spiral perspective. The approach implies probabilistic concepts are gradually presented, revised, and deepened from Elementary School to High School, covering an above 10-year educational period. Moreover, the thematic unit of Probability and Statistics addresses uncertainty and data processing and encourages skills for dealing with everyday situations, Science, and technology.

Towards acquiring such skills, all citizens must be able to collect, organize, represent, interpret, and analyze data in different contexts and make informed decisions through the use of reasoning, concepts, representations, and statistical indices for describing, explaining and predicting phenomena (BRASIL, 2017, 2018). Students are expected to develop the ability to identify opportunities to apply Mathematics and solve problems using concepts, procedures, and results towards solutions and interpret them according to the situations.

The study of Statistics and Probability is highly relevant for students' lives, as highlighted by Gal (2005). Learning Probability transcends mere school concepts, encompassing people's social practices. However, the precariousness and frequent lack of approaches to content related to the set of knowledge of Statistics, Probability, and Combinatorics in Basic Education and even during training courses for future teachers are evident.

Lopes (2008) investigated the teaching of Statistics and Probability in an initial training of Mathematics teachers in Basic Education and observed they tended to focus only on traditional Mathematics topics, such as Numbers, Operations, Quantities, Measurements, Algebra, and Geometry, neglecting Probability and Statistics. The trend was called linearity paradigm, according to which deterministic contents are privileged and non-deterministic concepts are not taught.

Statistical Education plays a significant role in both educational process and training of Mathematics teachers for Basic Education. One of its main objectives is to promote statistical literacy, recognizing it is an immediate need among citizens. However, several challenges in teaching content, particularly combinatorial analysis, are inherent to Statistics. One of them related to the teaching of combinatorial analysis lies in the correct connection between a problem presented and the corresponding theory, often resulting in an excessive use of formulas to the detriment of understanding the nature underlying the question raised.

The proposed tool aims at encouraging the decomposition of a problem into smaller components towards a deeper understanding of its essence. It also helps teachers monitor each students' progress, identifying areas that may need more support and guidance. Additionally, it provides access to interactive learning resources, thus helping students understand concepts in a more engaging and practical way and making classes more dynamic.

The tool was designed to be in line with the guidelines established by BNCC, thus meeting some of the competencies expected in the thematic unit of Probability and Statistics, including ability to identify everyday situations. Such an ability requires choices made according to probabilistic risks, such as those related to the existence of different alternatives in a decision tree.

Through a detailed exploration of questions, students correlate combinatorial analysis concepts with practical examples of application in real-world situations, establishing a direct connection with statistical literacy. Consequently, the understanding of the usefulness of those concepts for their lives is faciliated and collaboration is encouraged, since students work together towards solving problems related to combinatorial analysis.

Due to the relevance of teaching Statistics and Probability in Basic Education and the importance of developing statistical literacy for the critical education of citizens, particularly in our technology-driven and digital society, the endorsement of interactive and cost-free computational tools becomes evident. In this context, this study provides a tool designed to help both students and teachers to improve the understanding and application of concepts related to combinatorial analysis, encouraging a progressive construction of those concepts through a more attractive teaching strategy.

Theoretical foundation

Among the several factors that influence both development and understanding of statistical and probabilistic reasoning and thinking is combinatorial reasoning. From the early years of basic education, its use, which involves the fundamental principles of combinatorial analysis through playful activities and problem solving contextualized in students' daily lives, must be encouraged.

Statistical and probabilistic literacy

According to Gal (2002), a general conception of literacy is related to reading and writing skills and, on the other hand, to statistical literacy, which involves peoples´ ability to critically evaluate, interpret, and express opinions about statistical data. The author claimed statistical literacy is an essential skill for a full exercise of citizenship in a society constantly flooded with information and is built from a critical and investigative stance, involving fundamental knowledge in Statistics and Mathematics, Reading, analysis skills, and beliefs, atitudes, and an understanding of individuals and the world that surrounds them. Such a perspective implies sociocultural actions, in which people engage in interactions and critical readings of the world.

Gal (2005) introduced the concept of probabilistic literacy, which concerns a person's ability to read, interpret, and critically analyze information on probabilities. It involves the understanding of concepts of randomness,

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event, sample space, variability, and the different meanings of probability and risk. As a result, combinatorial reasoning is valued and encouraged.

The author also asserted statistical literacy is closely associated with probabilistic literacy, which should be introduced during the early years of education and contribute to predicting various critical issues related to different situations. The present study is based on the premise that one of the essential foundations for the development and improvement of statistical literacy is probabilistic literacy, which includes combinatorial reasoning.

Combinatorial reasoning and combinatorial analysis

According to Borba (2013), combinatorial reasoning refers to a set of mathematical methods that enable the resolution of situations involving possibilities and arrangements. The methods include arrangement, combination, and permutation, which are connected by basic relationships in the field of combinatorial analysis. However, each of them has its own characteristics and requires the use of appropriate symbolic representations for accurately exploring different possibilities.

Contrarily to a common thought, combinatorial reasoning can be approached in different ways - not only through formulas - and at more advanced educational levels. As an example, students can be encouraged to think of different ways to color a drawing and dress up for a classmate's party, or even of possible routes from home to school. The perspective of introducing the teaching of those concepts during Mathematical and Statistical Literacy is supported by PCN (BRASIL, 1998) and reinforced by BNCC (BRASIL, 2017), as addressed elsewhere.

When combinatorial reasoning is approached for encouraging construction of statistical knowledge, a series of relationships are established, helping students to expand their problem-solving strategies and understand the content more completely and leading to new learning, which is fundamental for the consolidation of statistical literacy. Therefore, statistical literacy clearly plays a relevant role in the comprehensive formation of the individual as a citizen.

Combinatorial analysis enables the calculation of number of possibilities for specific events, taking into account certain conditions. Roxo et al. (1944, p. 81) claimed it is an area of Mathematics that studies the formation, counting, and properties of groupings created according to specific criteria, such as the elements of a collection. The groupings are divided into three types, namely, arrangements, permutations, and combinations, and can be formed by distinct or repeated objects.

As stated by Carvalho et al., (2016), combinatorial analysis illustrates and/or calculates all possibilities for ordering finite elements based on counting criteria and is applied to the concepts of probability and mathematical logic. Understanding such concepts is fundamental for probability studies and calculations.

Challenges and technological tools related to the teaching of combinatorial analysis

A significant number of questions on combinatorial analysis tends to be considered difficult by both students and teachers and one of the main challenges is the correct connection of the problem presented and the corresponding theory.

Many students focus on memorizing formulas instead of understanding what is being asked and how to solve a question. Their main difficulty usually lies in interpreting a problem statement and properly applying the formulas towards answering it, which may lead to confusion. On the other hand, teachers must transmit the content without overloading it with excessive formulas, going against the usual practice of presenting concepts and principles automatically followed by standardized exercises.

Aranão (2011, p. 12) reinforced teachers are the main mediators in the construction of knowledge, since they promote situations in the school environment in which students exercise the ability to think, hence, seek solutions to the proposed tasks. It should be remembered that the discussion about the gap between teaching materials and the act of teaching is an old practive. It is up to the teacher to discover the most effective way to introduce content through different methodologies towards dealing with students' questions and actions. Therefore, as emphasized by Ball and Cohen (1996) decades ago, the teacher has a unique role in the construction of knowledge.

The use of new teaching tools has recently become the subject of many debates on how teachers can fulfill the syllabus while innovating the way they teach. Spenassato and Giareta (2009) argued differentiated teaching methodologies make the teaching-learning process more enjoyable, since a traditional perspective hampers the understanding of abstract subjects, given that knowledge is an individual´s construction process.

Contents related to Statistics within the scope of Basic Education are integrated into the Mathematics syllabus. Teachers often report difficulties in covering an entire annual curriculum, resulting in a reduction or even exclusion of Statistics topics from the students' educational program. Many also detail difficulties in teaching those specific subjects and/or having complete control over them. On the other hand, students report problems in understanding them.

It is widely recognized that the teaching of Mathematics is challenging at several education levels, especially regarding contents covered by combinatorial analysis and probability. According to Mello (2017), those themes are highly important, for they enable students to expand and formalize their knowledge of combinatorial, probabilistic, and statistical reasoning.

We clearly live in a society in the digital age and, as claimed by Fantin and Rivoltella (2012), new skills and abilities are required from professionals.

As for those in the field of education, specifically teachers, a pedagogical appropriation of digital technologies is demanded for their integration to the school curriculum.

Allevato (2005) emphasized technologies make a difference when employed to creating new processes for issue resolution, since they entail activities that foster student collaboration, investigation, discussion, and exchange of ideas. Their use as facilitators for the understanding of content is a differentiator in school environments for both teachers and students. Fischbein (1994) defended ideas on the relevance of integrating intuitive, algorithmic, and formal components, considered basic in mathematical activity, and further emphasized the basic condition for the development of efficient mathematical reasoning is the exploration of the relationship between algorithmic and formal aspects.

Methodology

As addressed elsewhere, this article introduces a new tool that helps direct and understand the use of combinatorial analysis, mainly in the context of statistical probability, towards a more attractive learning process, thus, strengthening and consolidating students' critical thinking. A decision tree was developed to help identify the most appropriate combinatorial analysis for solving a problem/question.

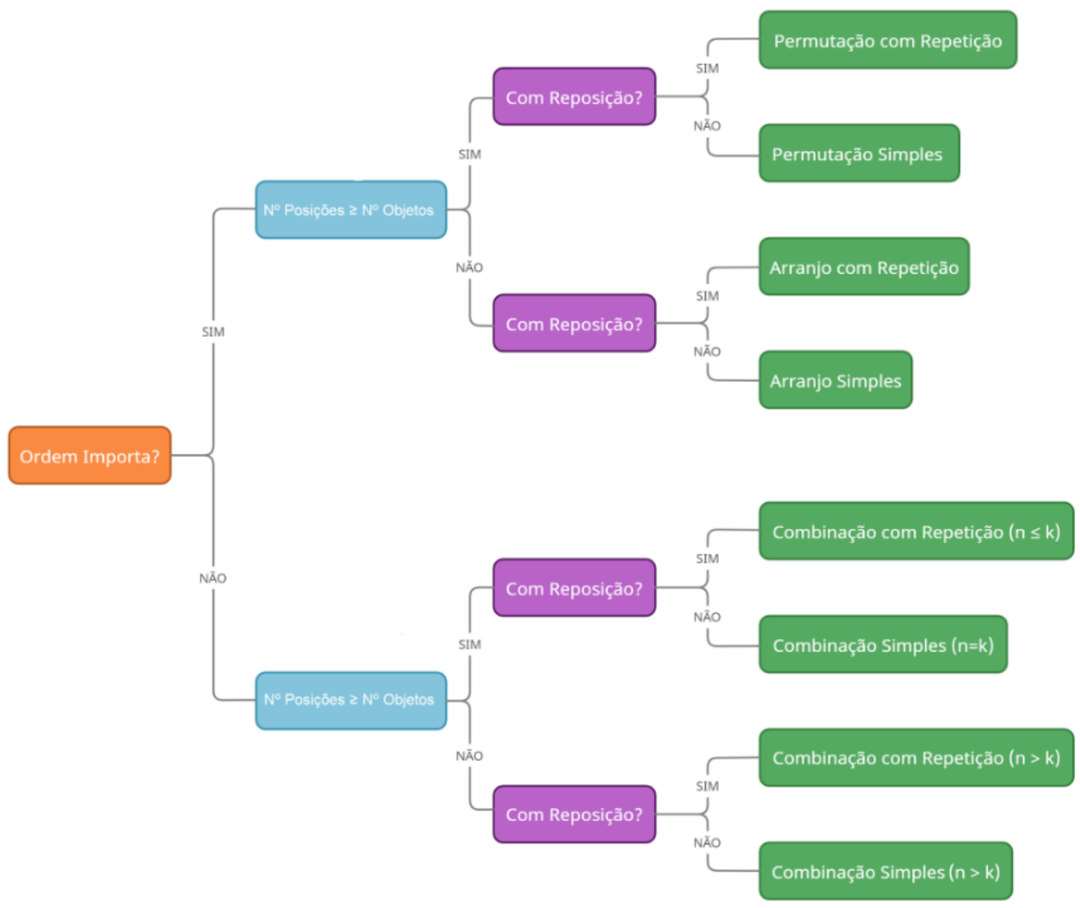
As defined by Monard et al. (2003), a decision tree is a graphical representation of a decision-making model, which shows the different options available, their possible results, and the consequences associated with each choice. It is a hierarchical structure composed of nodes and branches/branches, where each node represents a question or condition to be evaluated and branches/branches represent the different options or paths to be followed. Based on the answers to the questions or conditions, the decision tree enables the determination of the next node to be explored until a final solution (terminal or leaf node) has been reached. Each question result leads to a path to a subtree with the same structure and each result from a question leads to a path to a subtree also maintaining the same structure.

The use of a decision tree enables the representation of available options and paths to be followed, thus establishing an "algorithm" for the choices and resulting in possibilities of combinatorial analysis. It also promotes user reflection through three dichotomous questions, which help identify the appropriate combinatorial analysis technique and can be answered in any order. It encourages user investigation and questioning on the problem presented, avoiding the mechanical application of formulas.

In addition, the decision tree enables the decomposition of a problem into smaller parts for facilitating its understanding by students and encouraging their critical thinking for solving it. The tree involves three questions, namely, “Does order matter?”, “Is the number of positions greater than or equal to the number of objects?” and “Does it show repetition of objects?”. Each question is addressed individually and can be answered in alternating order, with a binary answer ("yes" or "no").

Figure 1 illustrates a decision tree in one of the possibilities that can be structured – the configurations cited in the preceding paragraph were chosen. After answering the first question, the student moves to the next branch for answering the next questions. Each answer will lead to a different path so that, at the end, a combinatorial analysis is indicated for solving the problem based on the combination of the answers to the three questions.

FIGURE 1: Decision tree implemented in Software R for the generation of the proposed tool



Source: Elaborated by the authors.

Possible answers to the dichotomous questions, namely, "Permutation with repetition", "Simple permutation", "Arrangement with repetition", "Simple arrangement", "Combination with repetition with a smaller number of objects equal to the number of positions ", "Simple combination with the same number of objects", "Combination with replacement with a number of objects greater than positions", and "Simple combination with a number of objects greater than positions"are then presented. After reading the question, the student must reflect on it, provide an answer, and follow the flowchart until reaching one of the conclusions.

As an example, let us consider the following question:

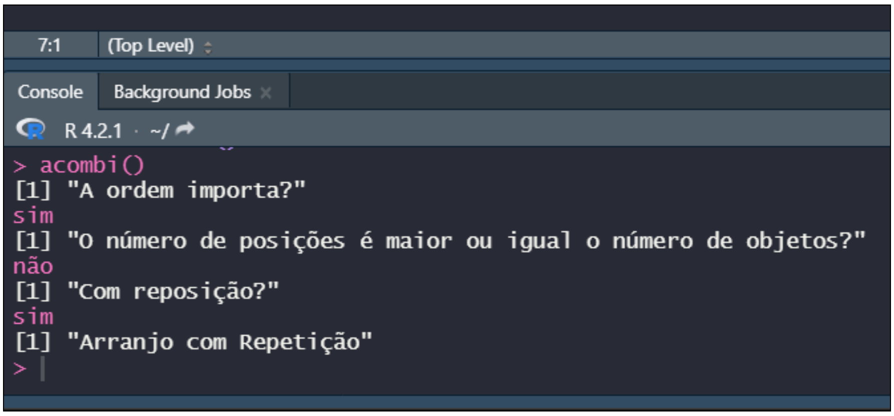
"Knowing vehicle license plates are comprised of three letters and four numbers, how many license plates are there?" In this case, the order matters, since the number of objects is not equal to the number of positions and it is a withdrawal with replacement. Therefore, the answers are “yes”, “no”, and “yes”, resulting in “Arrangement with repetition”.

After the flowchart corresponding to the decision tree had been prepared, a function called “acombi()” was developed in R programming and software language (R Core Team, 2021) . OR is a programming language often used as a tool for statistical analysis and data manipulation. Its functional capabilities range from performing simple calculations and creating tables and graphs, to more complex applications, such as building confidence intervals, testing hypotheses, linear and non-linear modeling, and creating maps.

R is currently one of the fastest growing statistical computing languages on a global scale in part due to the continued expansion of its user community, dedicated to contributing to package development. Such packages consist of sets of small programs that add new functionalities to a system. RStudio, a free software integrated development environment for the use of R, was also used in this study (RStudio Team, 2023). It is programmed in C++ and uses Qt framework for its graphical user interface.

The function created follows the same steps previously described, making it more attractive to students, since the use of technology is a great tool in education. Figure 2 illustrates the use of the tool in Example 1.

FIGURE 2: Application of the tool and output in Software R console for example 1



Source: Elaborated by the authors.

Function "acombi()" was developed for presenting the questions sequentially on the screen so that students can answer each one in a determined order. At the end of the process, the function displays the combinatorial analysis recommended for resolving the proposed question, as in a printed version. However, it does not accept variations in writing, such as "YES" or "Yes". Towards a more effective understanding of the tool, the authors made available a brief tutorial, along with the function code and a practical example, in a project hosted on RPubs (Rpubs, 2023), for facilitating its replication by readers.

The tool aims to facilitate and encourage reflection on problems proposed. If the user approaches the problem differently than the way expected, the function provides an incorrect suggestion for solving it.

The app

This study evaluated the effectiveness and potential of the proposed tool as a motivating resource for the teaching of combinatorial analysis. Specifically, the capacity of "acombi()" for helping users understand and solve problems related to combinatorial analysis through computational approaches was investigated. The level of knowledge of high school students in relation to that content was also analyzed considering the relevance of the tool at different educational levels and its recurrence in national assessments, such as National Secondary Education Exam (ENEM) and other exams for entrance to higher education in Brazil.

The experiment was conducted in an educational institution belonging to the public network in Salvador, Bahia. A class from the last grade of high school was selected, assuming the students had already been exposed to the content of combinatorial analysis, since, according to BNCC, such a topic should be introduced in the elementary cycle and deepened in high school.

The selection of the educational institution was based on a partnership previously established with the school and mediated by a student from the aforementioned class who held a scholarship under the Institutional Junior Scientific Initiation Scholarship Program (PIBIC Jr). The Mathematics teacher had already participated in other projects related to Statistics at the Institute of Mathematics and Statistics of the Federal University of Bahia (IME-UFBA), thus providing a favorable context for the study.

Initially, a study developed at IME-UFBA, in Salvador-BA, on March 2, 2018, evaluated the receptivity of "acombi()" (see Figure 2). The computer laboratories were properly equipped with R and RStudio software installed on all computers, thus facilitating the running of the application.

17 volunteer students from the 3rd grade of the Military Police College of Salvador-BA participated of the experiment. They were randomly divided into two groups, of which the first contained eight and the second encompassed nine. Each group received a set of six questions related to combinatorial analysis and probability. Figure 3 displays the class in one of IME-UFBA laboratories during the stage of solving exercises proposed on the application day.

FIGURE 3: Class in an IME-UFBA laboratory on the day of application of the tool

Source: Elaborated by the authors.

The objective of solving the exercises was to determine the type of combinatorial analysis appropriate for each problem presented with no need for numerical resolution.

Both groups were exposed to the two methods of solving questions, i.e., of the six questions resolved individually, half of them were addressed with the use of R software through "acombi()", whereas the remaining questions were resolved in a traditional way. The last method relied on the help of concrete models and low-cost tangible resources (e.g., use of a decision tree printed on a sheet and figures of the objects of the questions) for helping the students' understanding.

The dynamic occurred in two stages. Initially, the groups solved three questions - one group adopted the computational method with the help of R software and the other used concrete models. In a second moment, the groups alternated between methods and resolved three other subsequent questions.

The questions were resolved individually, since the division into groups was aimed only at interspersing methods and questions. Furthermore, for each question, the students individually received a set of concrete models as an additional tool to help visualize the possibilities of combinations.

The exercises were solved together and discussed one by one. Explanations were provided on the blackboard and interactive activities were conducted with students to help them develop the reasoning necessary to deal with the questions. A wrong answer obtained became an opportunity for students to identify which of the three dichotomous questions in the decision tree they had a divergent perspective on.

Below are three of the questions used in the experiment and their respective combinatorial analysis classifications according to the decision tree displayed in Figure 1:

1. "In a restaurant, to compose a dish, a customer must select four ingredients, of which at least one must be a vegetable. There are seven vegetable options. In how many ways can the customer compose the dish?" "Combination with Repetition (n>k)";

2. "In a chess competition, there are eight players. In how many different ways can the podium be formed (first, second, and third positions)?" "Simple Arrangement";

3. "Beatriz wants to organize ten books on a shelf - four on mathematics, three on physics, and three on chemistry. In how many ways can she organize the books by keeping those on the same subject together?" "Simple Permutation"

At the end of the experiment, a questionnaire was applied to the students as an evaluation. They were asked to give a grade from zero to ten on the ease of understanding and solving the exercises, taking into account the methods used, both with and without the help of R software.

Results and Discussion

This article proposes a new computational tool to assist in the teaching and learning of combinatorial analysis, especially in statistical probability. The results refer to its application in an experiment with high school students for evaluating the effectiveness and potential of such a new playful resource.

The application approach prioritized improvement in the knowledge of combinatorial analysis content through a playful pedagogical strategy, with the student playing an active role in the teaching-learning process for achieving the proposals of BNCC on the use of technologies, including software towards situations that lead to reflections on the issues presented.

The participants were asked to give a score from zero to ten to the ease of understanding and solving the exercises according to the method used. 9.04 and 9.44 average scores were given to the use of concrete models and "acombi()" of R software, respectively, indicating both resources were well evaluated, with very close scores in terms of average. However, the students' feedbacks, provided below, indicated the computational tool stood out:

> Participant A: "I thought it was wonderful, mainly due to the fact it didn't need internet to use it."
> Participant B: "I think it helped a lot the understanding and carrying out of the exercises."
> Participant C: "Very interesting, great idea for the tree, what was presented was very innovative. I really liked it."
> Participant D: "Very good, for it offers the opportunity for the teacher to leave the comfort zone when teaching this content, in addition to providing several discussions in the classroom with the students."
> Participant E: "I really liked the package, since it makes solving and understanding easier for students."
> Participant F: "I found the use of RACom package quite interesting, since the combinatorial analysis content is very abstract and the software significantly contributed to student learning."

The grades given to ease of understanding only reflect the students' level of appreciation for the resources and how effective they were in helping them understand the question, which does not necessarily mean they answered the questions correctly. Therefore, students' ratings for ease of understanding do not need to be directly associated with the correct answer rates for the questions.

Despite the students indicating greater ease of understanding with the use of “acombi()”, the success rates for the concrete method and “acombi()”of R software were 46% and 32%, respectively, which are low, thus highlighting a gap in the students' knowledge of combinatorial analysis. Such an indication may be directly related to the lack of an adequate approach and/or those contents in the classroom, as addressed elsewhere, which leads to a neglect of one of the fundamental bases for teaching Probability and Statistics.

It is plausible to infer that the decrease in the success rate observed in the method that uses the tool is attributable to the students' lack of familiarity with R software - it was their first contact with R and the tool. The situation is also intrinsically associated with the prevailing reality in Brazilian public schools, characterized by a lack of properly structured environments and technological teaching resources.

This study reinforces the continued importance of promoting and encouraging reflective and critical thinking at all levels of education, particularly in Basic Education. The ability to reflect and master the application of combinatorial analysis content in different contexts will be enhanced through the use of the proposed tool, resulting in more capable and literate individuals whose decision-making is likely to be more coherent in everyday situations.

The students' feedbacks revealed excitement and enthusiasm with R tool and software, reaffirming what several studies have highlighted, i.e., playful teaching provides a motivating and thought-provoking approach for students, making them more curious during the process of acquiring knowledge (Allevato, 2005; Spenassato and Giareta, 2009; Cardoso and Coutinho, 2010).

The tool has proved highly relevant due to the applicability of concepts of combinatorial analysis in different statistical contexts, thus promoting the development of individuals' critical thinking and helping them make decisions when faced with multiple possibilities. The contributions of this field of study are notable, both theoretically, enabling the construction of concepts with no excessive dependence on formulas, and in practice, offering a motivating tool for innovation in the school environment.

Some limitations that potential users (teachers and students) may face when using the tool include lack of skill or prior knowledge of the programming language used, difficulties in interpreting or understanding the information provided in the tool interface, and lack of motivation or interest in using it, especially if it is not presented in an attractive and relevant way.

The tool has theoretical limitations, since it does not cover all theoretical possibilities of combinatorial analysis, which is evidenced when examples such as circular permutation or more complex problems that require multiple approaches to combinatorial analysis for a comprehensive resolution are considered.

Challenges were faced during the experiment, especially regarding the use of R software, which is partly due to students' lack of exposure to code structuring and programming language in Basic Education. However, we believe such a reality will change soon, as the demand for improving and stimulating programming logic skills intensifies. Unfortunately, not all students have easy access to computers and some basic-level educational institutions still do not have an adequate IT infrastructure.

Finally, the pedagogical approach adopted in this study primarily aimed at making teaching more appropriate to contemporary needs, taking advantage of technological advances to improve education and, therefore, ensuring teaching is aligned with the constantly evolving challenges present in the educational landscape and society in general. The results can contribute to reflections on the importance of promoting and developing innovative and effective teaching strategies, with a view to the future, identifying emerging educational trends and considering the way education can adapt to changes, including integration of technology, for improving the classroom experience, as explored in this study.

The proposal represents a starting point, requiring improvements and adjustments to more effectively meeting the needs of students, teachers, and the school environment. The study revealed several aspects that deserve attention in future research. As an example, an introduction to computational thinking and R software for users of the tool must be included and improvements in the structure of the tool itself are required. Efforts have been devoted towards the development of other functions in R software, which will not only be limited to identification, but also assist in calculations related to combinatorial analysis.

Conclusions

The contributions of the tool stand out both theoretically, by enabling the construction of concepts with no excessive use of formulas, and in practice, providing a motivational resource to innovations in the classroom environment.

The development of combinatorial reasoning is an essential skill for students at all levels of Basic Education, due to its wide applicability in contemporary society and contribution to the formation of critical and conscious citizens. The adoption of new tools and approaches for improving combinatorial reasoning directly impacts the statistical and probabilistic literacy process, promoting a deeper understanding of such fundamental concepts.

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Although the tool is not intended to be a resource capable of meeting all students' learning needs, it was designed for guiding educational actions, being adapted according to the reality of each educational institution. As an example, a program created can be adapted through unplugged computing activities.

The programming language proved an innovative and facilitating option for students who used “acombi()” function of the decision-making tree as an auxiliary tool for selecting an appropriate method for solving combinatorial analysis exercises and probability. However, several participants showed a significant deficit in understanding the concepts of combinatorial analysis, hence, a discrepancy in relation to the syllabus expected for the respective educational level. Future studies will involve a preliminary questionnaire for the obtaining of information on the students' level of knowledge of the topic under study and a description of their socioeconomic profile and educational history.

The development of additional studies that include an introduction to R software is expected towards a better use of the designed programs. Although R software is a technological tool with growing demand, many students still do not know it or are not familiar with its use.

The use of computational tools in an educational environment provided highly satisfactory results, thus motivating adjustments and improvements to the developed program.

The project will be extended to other public and private schools in different locations through workshops for evaluations of the impacts of the proposed tool on the daily lives of students and teachers, contributing to the consolidation of skills necessary for citizens establishing foundations for their statistical literacy.

The tool has shown highly relevant due to the applicability of the concepts of combinatorial analysis in different contexts of Statistics. Their understanding helps the construction of individuals' critical thinking and decision-making when faced with multiple possibilities. The contributions are evident, both theoretically, by enabling the construction of concepts with no excessive use of formulas, and in practice, providing a motivational tool for innovations in the classroom environment.

References

ALLEVATO, N. S. G.; ONUCHIC, L. *Resolução de Problemas. Associando o Computador à Resolução de Problemas Fechados: Análise de uma Experiência.* In: Revista Educação Matemática Pesquisa, São Paulo, v. 7, n. 2, p. 370, 2005.

ARANÃO, I. V. D. *A Matemática através de brincadeiras e jogos*. 7. ed. Campinas, SP: Papirus, 2011.

BALL, D. L.; COHEN, D. *Reform by the Book: What Is - or Might Be - the Role of Curriculum Materials in Teacher Learning and Instructional Reform?* In: Educational Researcher, vol. 25, n. 9, p. 6-8, 1996. DOI: https://doi.org/10.3102/0013189X025009006

BORBA, R. *Vamos combinar, arranjar e permutar: aprendendo combinatória desde os anos iniciais de escolarização*. In: Anais do Encontro Nacional de Educação Matemática, 13, 2013, Curitiba. Anais eletrônicos... Curitiba, 2013.

BRASIL. *Parâmetros curriculares nacionais – Matemática: terceiro e quarto ciclos do Ensino Fundamental.* Brasília: MEC/SEF, 1998.

BRASIL. *Parâmetros curriculares nacionais – Matemática: terceiro e quarto ciclos do Ensino Fundamental.* Brasília: MEC/SEF, 2000.

BRASIL. *Parâmetros curriculares nacionais – Matemática: terceiro e quarto ciclos do Ensino Fundamental.* Brasília: MEC/SEF, 2017.

BRASIL. *Parâmetros curriculares nacionais – Matemática: terceiro e quarto ciclos do Ensino Fundamental.* Brasília: MEC/SEF, 2018.

CARDOSO, M.D.L; COUTINHO, C.P. *Ambientes de aprendizagem Web 2.0 no ensino profissional: um estudo sobre a utilização de uma ferramenta de colaboração online no módulo Estatística*, 2010.

CARVALHO, P. C. P; MORGADO, A. C. O; FERNANDEZ, P; PITOMBEIRA, J. B. *Análise Combinatória e Probabilidade*. 10ª ed. Rio de Janeiro: SBM, 2016.

FANTIN, M; RIVOTELLA, P. C. *Cultura digital e escola: pesquisa e formação de professores* (pp. 95-146). Campinas: Papirus, 2012.

FISCHBEIN, E. *The interaction between the formal, the algorithmic and the intuitive components in a mathematical activity*. In: Didactics of Mathematics as a Scientific Discipline, p. 231-245, 1994.

GAL, I. *Adults' Statistical Literacy: Meanings, Components, Responsibilities*. In: International Statistical Review, Voorburg, v. 70, n. 1, p. 1-25, 2002.

GAL, I. *Towards 'probability literacy' for all citizens: building blocks and instructional dilemmas*. In: Exploring Probability in School: Challenges for Teaching and Learning. Graham J. (ed.), Kluwer Academic Publishers, p. 43-70, 2005. DOI: https://doi.org/10.1007/0-387-24530-8_3.

LOPES, C. E. *O ensino da estatística e da probabilidade na educação básica e a formação dos professores*. Cadernos CEDES, Campinas, vol. 28, n. 74, p. 189-201, 2008. DOI: https://doi.org/10.1590/S0101-32622008000100005.

MELLO, H. *Desmistificando o Ensino de Análise Combinatória*. Rio de Janeiro, RJ, Brasil. 2017.

MONARD, M. C; BARANAUSKAS, J. A. *Indução de regras e árvores de decisão*. In: Sistemas Inteligentes - Fundamentos e Aplicações, v. 1, p. 115-139, 2003.

R CORE TEAM. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Viena, Áustria, 2021. Disponível em: https://www.R-project.org/. Acesso em: 28 de setembro de 2023.

ROXO, E. et al. *Matemática 2º ciclo, 2ª série. 2ª* ed. Rio de Janeiro: Livraria Francisco Alves, 1944.

RPUBS. *Utilização de uma ferramenta computacional para auxiliar no processo de ensino-aprendizagem da análise combinatória*. Disponível em: https://rpubs.com/acombi/1090144. Acesso em: 2 out. 2023.

RSTUDIO, Inc. *RPubs: Publishing in the R Community*. Disponível em: https://rpubs.com/. Acesso em: 28 de setembro de 2023.

RSTUDIO TEAM. *RStudio: Integrated Development for R*. RStudio, PBC, Boston, MA, URL: https://www.rstudio.com/. Acesso em: 28 de setembro de 2023.

SPENASSATO, D; GIARETA, M. K. *Inclusão de alunos surdos no ensino regular: Investigação das propostas didático–metodológicas desenvolvidas por professores de matemática do ensino médio da EENAV*. In: X Encontro Gaúcho de Educação Matemática, 02 a 05 de junho. Ijuí/RS, 2009.

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