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Semiotic registers in the resolution of introductory problems of Probability: analysis of productions of Pedagogy students[1]

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ABSTRACT

We present results of an investigative work carried out with the objective of identifying and understanding the registers of semiotic representation of probabilistic problems mobilized and coordinated by Pedagogy students. This is a qualitative research, a case study type, which analyzed representation registers produced by a group of fourteen students of the Degree in Pedagogy, in the period from July 27 to August 2, 2022, in the course of the subject Theoretical and Methodological Foundations of Mathematics Teaching, linked to the National Plan for Formation of Teachers for the Basic Education (PARFOR), at a public university in the State of Pará. The theoretical framework of this investigation is centered on the approaches in terms of the Registers of Semiotic Representation of the French philosopher and psychologist Raymond Duval. The didactic sequence, elaborated by the professor of the subject, presented two problems of probabilistic recognition. The results obtained point to the deepening of the understanding of the objects of knowledge investigated from the

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resolution of probabilistic problems that mobilized the varied representation and changes in the register of semiotic representation by the students.

KEYWORDS: Semiotic Registers. Teaching Probability. Theory of Registers of Semiotic Representation.

Registros semióticos na resolução de problemas introdutórios de Probabilidade: análise de produções de estudantes pedagogos(as)

RESUMO

Apresentamos resultados de um trabalho investigativo realizado com o objetivo de identificar e compreender os registros de representação semiótica de problemas probabilísticos mobilizados e coordenados por estudantes de Pedagogia. Trata-se de uma pesquisa de natureza qualitativa, do tipo estudo de caso, que analisou representações semióticas entre registros, produzidos por um grupo de 14 estudantes do curso de Licenciatura em Pedagogia, no período de 27 de julho a 02 de agosto de 2022, no curso da disciplina de Fundamentos Teóricos e Metodológica do Ensino de Matemática vinculada ao Programa Nacional de Formação de Professores da Educação Básica (PARFOR), em uma universidade pública do Estado do Pará. O referencial teórico desta investigação está centrado nas abordagens em termos dos Registros de Representação Semiótica do filósofo e psicólogo francês Raymond Duval. A sequência didática, elaborada pelo professor da disciplina, apresentava dois problemas de reconhecimento probabilístico. Os resultados obtidos apontam para o aprofundamento da compreensão dos objetos de conhecimento investigados a partir da resolução de problemas probabilísticos que mobilizaram a representação variada e mudanças de registro de representação semiótica pelos estudantes.

PALAVRAS-CHAVE: Registros Semióticos. Ensino de Probabilidade. Teoria dos Registros de Representação Semiótica.

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Registros semióticos en la resolución de problemas introductorios de Probabilidad: análisis de producciones de estudiantes de Pedagogía

RESUMEN

Presentamos resultados de un trabajo de investigación realizado con el objetivo de identificar y comprender los registros de representación semiótica de problemas probabilísticos movilizados y coordinados por estudiantes de Pedagogía. Se trata de una investigación cualitativa, del tipo estudio de caso, que analizó registros de representación producidos por un grupo de catorce estudiantes de la Licenciatura en Pedagogía, en el período del 27 de julio al 2 de agosto de 2022, en el curso de la asignatura Teórica y Metodológica, Fundamentos de la Enseñanza de las Matemáticas, vinculado al Programa Nacional de Formación de Profesores de Educación Básica (PARFOR), en una universidad pública del Estado de Pará. El marco teórico de esta investigación se centra en los enfoques en términos de los Registros de Representación Semiótica del filósofo y psicólogo francés Raymond Duval. La secuencia didáctica, elaborada por el profesor de la disciplina, presentó dos problemas de reconocimiento probabilístico. Los resultados obtenidos apuntan para la profundización de la comprensión de los objetos de conocimiento investigados a partir de la resolución de problemas probabilísticos que movilizaron la variada representación y cambios en el registro de representación semiótica por parte de los estudiantes.

PALABRAS CLAVE: Registros semióticos. Enseñanza de la probabilidad. Teoría de los Registros de Representación Semiótica.

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Introduction

The resolution of mathematical problems allows for multiple approaches and solution strategies, with a wide diversity of semiotic representations among registers, such as numeric, algebraic, figural, or in natural language. In Probability and Statistics, graphical (CURCIO, 1987;

1989; FRIEL; CURCIO; BRIGHT, 2001) and tabular (WAINER, 1995; ARTEAGA et al., 2011) forms of representation are especially important.

Duval (2012) emphasizes the importance, for students' cognitive development, of the ability not only to switch between registers of semiotic representation but also of dealing with a deeper understanding of what is gained and lost when choosing a particular type of representation. It highlights certain aspects of the object of mathematical knowledge under analysis to the detriment of others. The conscious change of register of semiotic representation reveals a deeper level of knowledge about that object.

Therefore, considering the challenges that future pedagogues will face in the teaching of Mathematics, especially in the thematic unit of Probability and Statistics, as outlined in the National Common Curriculum Base (NCCB) (BRAZIL, 2018), we conducted a qualitative investigation (LÜDKE; ANDRÉ, 1986), in the form of a case study, aiming to identify and understand the registers of semiotic representation (DUVAL, 2012) of probabilistic problems mobilized and coordinated by students of the Degree in Pedagogy, during the second semester of 2022. The participants were fourteen students of the Degree in Pedagogy affiliated with the National Plan for Formation of Teachers for the Basic Education (PARFOR), at a public university in the state of Pará, who were engaged in a problem-solving task.

In the next section, we will present our theoretical framework.

Literature review

The theoretical framework used in this research is Raymond Duval's Theory of Registers of Semiotic Representations (TRSR). Raymond Duval's Theory of Registers of Semiotic Representations (2003, 2009) focuses its studies on the learning of Mathematics, particularly on cognitive aspects related to its understanding. In this theory, the learning process requires the

mobilization of different registers so that there is no confusion between the mathematical object and its representation, as well as coordination among the different registers.

Duval (2009) asserts that it is not possible to study phenomena related to knowledge without resorting to the concept of representation, given the abstraction of mathematical object. In the relationship between the object and its representation, an underlying assumption in this theory is that "one cannot have understanding in mathematics if we do not distinguish an object from its representation" (DUVAL, 2009, p. 14).

The concept of probability as an abstract object is accessed through semiotic representations, according to the adopted probabilistic conception: classical, frequentist, axiomatic, among others. For example, in the classical approach, probability is defined as the ratio of the number of favorable outcomes to the total number of possible outcomes, assuming that all outcomes are equally likely to occur (GODINO; BATANERO; CAÑIZARES, 1996).

In the frequentist context, probability is defined based on the calculation of relative frequencies of occurrences of events resulting from repeated experiments under the same conditions. The main characteristic of this approach is that the mathematical value of probability emerges from the process of experimentation (GODINO; BATANERO; CAÑIZARES, 1996).

According Duval (2003, 2009), in order for students to succeed in mathematical activities, it is necessary for them to be able to mobilize and coordinate different semiotic representations in the resolution of the same mathematical task. For a semiotic system to be considered a register of semiotic representation, it must admit three cognitive activities: formation, treatment and conversion. In terms of formation, representations in a semiotic register are not just means to evoke a real object but also to express a mental representation. The other two fundamental cognitive activities of representation linked to semiosis (the ability to absorb or produce a semiotic representation of an object) are addressed subsequently:

> Treatments are transformations of representations within the same register, for example: performing a calculation while staying strictly within the same writing or representation system. Conversions are transformations of representation that involve changing registers while preserving the same denoted objects (DUVAL, 2003, p.16).

In the case of the study object in this research report, Probability, Canaveze (2013) and Oliveira (2014) used, in the cognitive conversion activity, the following semiotic representations among registers: natural language (content of task statements), figural (two-way table or contingency table, as well as the tree diagram), and symbolic in algebraic form (use of formulas) or numeric form (fraction to quantify probability in the classical conception).

We present below a problem from the book "Mathematics: Imenes & Lellis" for the 9th year of Elementary Education that exemplifies the cognitive activity of treatment:

> In the lottery known as Mega Sena, the probability of a bettor hitting the "sena" (that is, correctly guessing all six drawn numbers) with just one bet is approximately 1/50.000.000. If you flip a coin 20 times in a row, the chance of getting heads every time is approximately 1/1.000.000. a). a). Is it easier to win the Mega Sena with one bet or to get heads 20 times in 20-coin tosses? b). Is it true that the probability of winning the Mega Sena with just one bet is 50 times the probability of getting heads 20 times in 20-coin tosses? If it's not true, what is the relationship between the mentioned probabilities? (IMENES; LELLIS, 2012, p.114).

The answer to the two items in this problem involves a treatment, which is a transformation of representation within a register, in this case, natural language as the starting and ending register. However, in this problem, natural language is used in cognitive cooperation with the numerical register when comparing the two fractions that express the mentioned probabilities.

The authors of this textbook present the following solution to the problem:

> One of the objectives of this simple question is to show how difficult it is to win the most popular lottery in the country.
> a) It is easier to get **heads** 20 times in 20 flips.
> b) The probability of getting **heads** 20 times in 20-coin tosses is approximately 50 times that of winning the Mega Sena with just one bet (IMENES; LELLIS, 2012, p.114, emphasis added).

The probabilistic problem presented consists of a cognitive processing activity because it involves semiotic representation in the natural language register while maintaining the same semiotic system, the mother tongue.

In the following section, we will present our methodological procedures.

Method and methodological procedures

Based on the Theory of Registers of Semiotic Representation, a didactic sequence for applying this theory to the resolution of probability problems is presented and designed for classroom use. Implicitly, the intention is to highlight the potential of articulating this theory with the teaching and learning of probability in the initial training of future teachers who will work in the early years of Elementary Education. The justification for this research is supported by Duval (2009, p. 10), when stating that "what is of more practical interest to those who teach

Mathematics and to the trainers of those who teach are tools that allow for the analysis of mathematical procedures within the framework of problem solving".

This research is characterized as qualitative (LÜDKE; ANDRÉ, 1986) and is configured as a case study. In this case, the natural environment, of which the subjects were a part, that is, from the analysis of registers of representation produced by a group of fourteen students from the Degree in Pedagogy, in this sense, that we are going to consider: the training environment of the pedagogues-teachers who took part in this research.

The natural setting of this investigation was the classes of the subject 'Theoretical and Methodological Foundations of Mathematics Teaching", taught by a Mathematics professor, in a group involving the Pedagogy program of the National Plan for Formation of Teachers for the Basic Education (PARFOR)[5] at a public university located in the state of Pará.

The criterion for choosing the subject was intentionally due to professional experience, in which teaching was conducted with subjects in the field of Mathematics and Mathematics Education. The subject was taught from July 27 to August 2, 2022, the time during which the data was produced.

The didactic sequence was developed by the professor of the subject, and presented two problems of probabilistic recognition aiming to identify and comprehend the different registers of semiotic representation in probabilistic problems tackled by Pedagogy students; and instruct students in an attempt to record their understanding when reading the mathematical object probability. This instruction was presented as necessary in Duval's studies, who explains that:

[5] The Pedagogy program is offered by the National Plan for Formation of Teachers for the Basic Education (PARFOR), affiliated with the Basic Education Directorate of CAPES, in a partnership between the institution and the Municipal Government, particularly the Municipal Department of Education.

> [...] the coordination between representations highlighting different semiotic systems is nothing spontaneous. Its placement does not automatically result from classical learning very directly focused on teaching content. A specific learning effort centered around the diversity of representation systems, their unique possibilities, their comparison by aligning them, and their mutual 'translations' within each other seems necessary to promote it (DUVAL, 2009, p. 19).

Thus, this didactic sequence was designed to generate probabilistic knowledge, that is, to uncover the prior knowledge that Pedagogy students possess when they are in what is called the "zone of discomfort". Thus, there were two moments: in the first, students attempted to solve the problems individually. In the second, in the following class, they solved the same problems with intense dialogue and descriptions of each problem's applications, including interventions and observations. The students were labeled as A1, A2, A3,..., A14. The selected probabilistic problems are described below:

> PROBABILISTIC PROBLEM 01: In a pencil case there are 15 colored pencils and 6 black pencils. Answer: a) If you randomly remove a pencil from this pencil case, is there a greater chance that this pencil will be colored or black? b) What is the probability of drawing a colored pencil? c) What is the probability of drawing a black pencil?
>
> PROBABILISTIC PROBLEM 02: In a box there are 2 History books, 6 Mathematics books and 4 Portuguese books. If we take a book from this box at random, what is the probability that it will be: a) History? b) Mathematics? c) Portuguese?

In the statement of these two problems, the quantification of probability in the classical conception, given the occurrence of a certain random event, it must be considered that if there are several possibilities, all with the same chance of occurring, the probability of one or more possibilities occurring is given by the ratio between the number of favorable cases and the total number of possibilities

The analysis of these probabilistic problems, along with the students' responses and the application of our methodology by using Raymond Duval's theory, is detailed below.

In summary, this research aims to identify and understand the registers of semiotic representation of probabilistic problems mobilized by Pedagogy students.

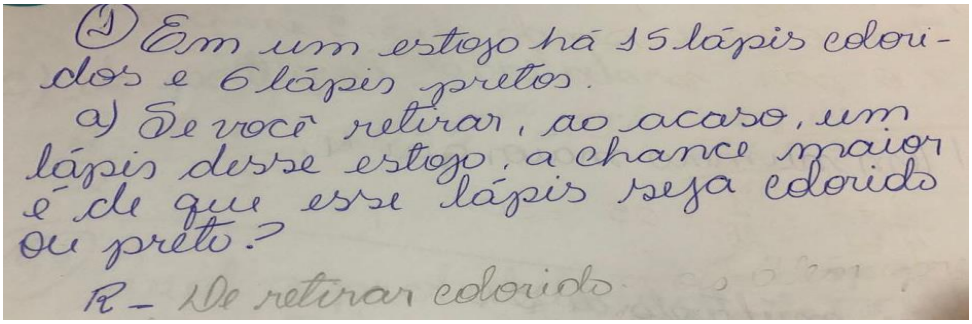
In the next section, we will present some of the results obtained through our investigation.

Results and discussion

The probabilistic problems posed to the research participants did not require the application of formulas as they could be solved through interpretation and the establishment of a strategy for resolution. It is worth noting that all participants attempted to solve or solved all the problems, even though this may have been their first experience with activities of this nature.

In this research, fourteen solutions constructed from an empirical perspective were identified. The use of the register of natural or figural language as a starting register was highlighted. Figure 1 is an excerpt from student E2's resolution, which shows the strategy of this register presented based on his/her notes in class.

FIGURE 1: Register of student E2.

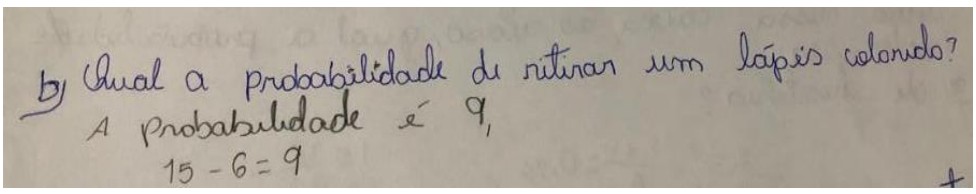


Source: Researcher's archive, in 2022.

In problem 1-a, the students did not encounter difficulties in their problem-solving strategies since this item required natural language representation, as one student mentioned, "you just had to look at the larger quantity, we have more colored pencils" (E8). In other words, even without prior experience with probability problems, all participants were able to solve this item.

Problems 1-b and 1-c involve operations with decimal fractions, decimal numbers and percentage calculations. Internal and external work on the register was requested, that is, it is a cognitive activity of treatment and conversion. These operations can be explored through the use of figural registration by considering their numerical representation, which could facilitate the student's understanding in relation to the interpretation of probabilistic thinking resulting from the operations. However, the data suggest that the majority of the class (64,29%) had difficulty solving the problem. Figure 2 shows student E14's problem-solving strategy.

FIGURE 2: Register of student E14.



Source: Researcher's archive, in 2022.

The problem-solving strategy indicates that the student had difficulty calculating the probability of simple events and representing it in fractional and decimal form. However, through the audio, we observe that student E14 said, "*I have always had difficulty with these calculations involving percentages*". And student E13 added, "*Besides involving fractions, it makes it even harder*". We noticed that the class agreed with their peers by nodding their heads. Nevertheless, these two items allowed for interaction among the students and with the lead professor. Student E14 reiterated, "*Aiming at not leaving this question blank, we wrote what we thought, even though we know it is wrong*".

When we analyze the dialogue above, as well as their protocols, we can see that they faced difficulties in converting the fractional register to the decimal register or the fractional register to the percentage register. This probabilistic problem presents a non-congruence phenomenon because the semiotic registers involved in the cognitive activity of conversion are not just a simple decoding. Duval states that:

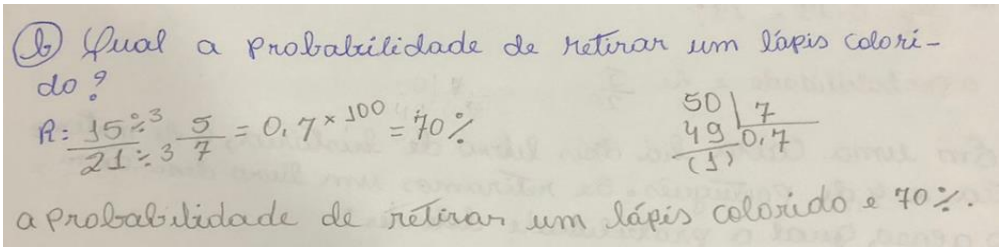
> Usually, in teaching, one direction of conversion is privileged, driven by the idea of training, with the assumption that it would automatically train the conversion in the other direction. Examples provided to students are instinctively chosen, evidently, in cases of congruence. Unfortunately, these are not the most common cases (DUVAL, 2003, p. 20).

We believe that the difficulty presented by E14 in relation to items 1-b and 1-c, as well as the majority of the class, may have been influenced by the lack of mathematical education for Elementary Education teachers in light of the new educational demands regulated by the National Common Curriculum Base (NCCB) (BRAZIL, 2018). More specifically, the NCCB regulates, over the course of the school years of Elementary Education, a mathematical education that encompasses both the deterministic view of the

aforementioned knowledge and the view of chance, whose analysis of the predictable is inherent to learning about Probability.

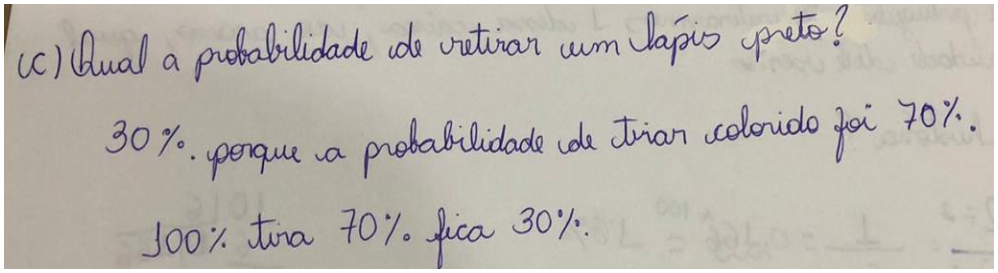
Despite this difficulty, we observed that five students (35,71%) were able to mobilize and coordinate the cognitive activity of conversion of semiotic representation among the numerical, natural language and symbolic registers. These registers aided in understanding probabilistic thinking and, consequently, the relations between mathematical content and the necessary use of these representations for calculations. In accordance with Duval's Theory of Registers of Semiotic Representation (2009), understanding only occurs when the student is able to systematize their externalization of knowledge through written protocols, as expressed in the content of the activity shown in Figures 3 and 4.

FIGURE 3: Register of student E8.



Source:Researcher's archive, in 2022.

FIGURE 4: Register of student E9.



Source: Researcher's archive, in 2022.

Duval (2011, p. 100) asserts that "all problems that present real situations [...] equally mobilize at least two registers: language, numerical writings and diagrams". In probabilistic problem 1, the

Pedagogy students mobilized and coordinated semiotic representations in natural language, numerical (decimal and fractional) and percentage, as exemplified in Figures 3 and 4. The registers used were natural language and numerical by students E8 and E9, respectively, using basic division and multiplication operations. These latter registers resulting from these mathematical operations are essential for the student to relate the definition with the representation of a decimal fraction to a decimal number and, consequently, convert it into a percentage.

Thus, the interpretation of probabilistic thinking is presented considering natural language, symbolic and figural registers. However, these students had difficulty performing the cognitive activity of conversion involving a decimal fraction, decimal number and percentage, which compromises the learning of the Probability concept. Therefore,

> [...] the semiotic representations – or, more precisely, the diversity of representation registers – play a central role in understanding. Understanding requires the coordination of different registers. However, such coordination does not occur spontaneously and is not the result of any a-semiotic "conceptualization". Most students, throughout their curriculum, fall short of this understanding. Hence, recurring difficulties and rather "narrow" limitations in their mathematical learning abilities are observed. The only successes they can achieve occur in mono-registers (monofunctional registers), which are often devoid of "meaning" and unusable outside the context of their learning (DUVAL, 2003, p. 29).

Monofunctional registers in discursive representation involve the cognitive activity of treatment, and are often associated with mathematical procedures in the form of algorithms.

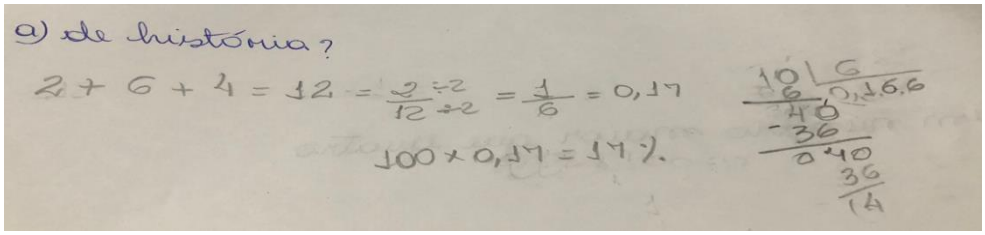
Duval's theory (2003) allowed us to identify the recurring difficulties of Pedagogy students when faced with the content of decimal numbers and percentages. Given this limitation, it was necessary to carry out specific interventions after the applicability of the first probabilistic problem, so that the subjects participating in the research were able to understand the mathematical object and use it appropriately.

We agree with Brita et al. (2017) when they argue about the role of the professor in selecting the problems to be developed in the classroom with students so that they can become familiar with the different representations of mathematical concepts to be studied, in this case, Probability. Therefore, it is necessary to understand the teaching of Mathematics at different levels of education and in different contexts, prioritizing different representations, as well as acting pedagogically by provoking situations between the content of the representation and the object represented in order to "[...] change the way in which knowledge is represented" (DUVAL, 2009, p.33).

In probabilistic problem 2, composed of three items, the task allows for the mobilization and coordination of different semiotic representations between registers. The written protocols of the 14 Pedagogy students indicated that there were no difficulties in the cognitive activities of treatment and conversion of semiotic representations in the registers in natural language (task statement) and symbolic (fraction, decimal number and percentage). Furthermore, there was no need for intervention from the professor/researcher in solving the task.

Figure 5 reinforces the assumption of the Theory of Registers of Semiotic Representation that signs of learning occur through the cognitive activity of conversion, despite the misconception in the use of the equality between 12 and the fraction 2/12.

FIGURE 5: Register of student E7.



Source: Researcher's archive, in 2022.

The problem-solving strategy presented by student E7 indicates that he/she associated, in the cognitive activity of conversion, the classical concept of Probability, expressed as the ratio between the number of favorable outcomes and the total number of possibilities.

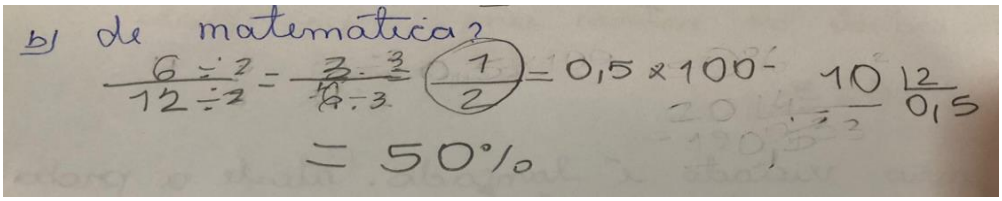
In item 2-a, the cognitive treatment activity occurred in the "transformation that produces another representation within the same register" (DUVAL, 2009, p. 54), in this case, the equivalence between the fractions 2/12 and 1/6. According to Brita et al. (2017), treatment is a cognitive activity of transformation that occurs internally within a representation while remaining within the same register.

The cognitive activity of conversion consists of "transforming the representation of an object, a situation or given information from one register into a representation of that same object, that same situation or that same information in another register" (DUVAL, 2009, p. 58). In the case of item 2-a, there was a transformation of the semiotic representation from the symbolic register in fractional form to an approximation in decimal form, as well as from decimal form to percentage form.

Despite probabilistic problem 2 requiring the mobilization of a concept that had not yet been addressed, the majority of students who worked on this question performed satisfactorily, which demonstrated a background of prior knowledge about the notion of randomness, as reported by student E12, "it is a matter of it happening or not". In the same train of thought, student E5 adds, "it is an action of chance".

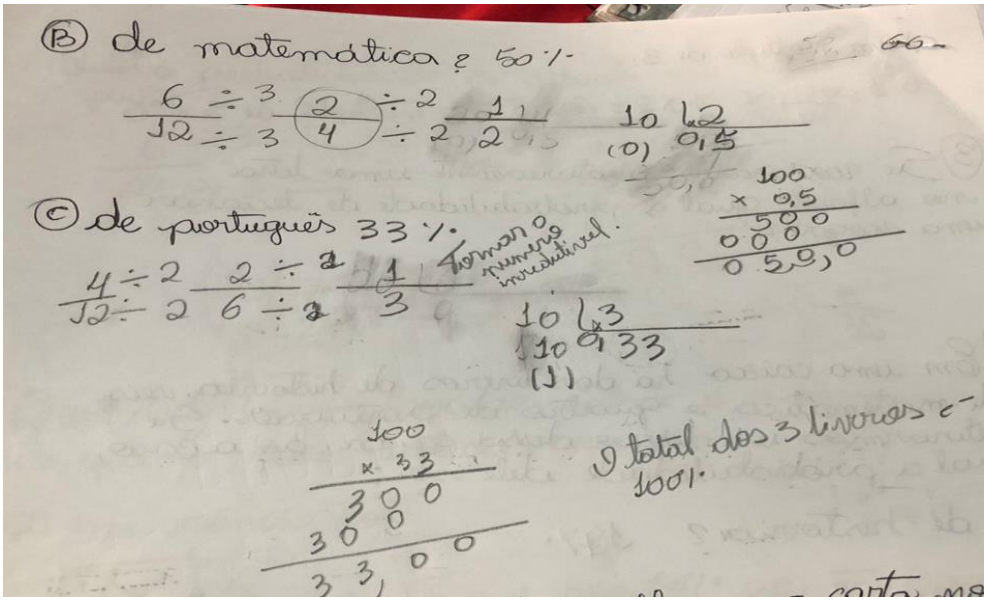
In item 2-b, the students were able to transition from different registers of representation to the concept of probability. In other words, they could engage in the treatment and conversion activity. The data indicates that students, even without the intervention of the professor/researcher, were able to carry out the procedures to answer the item, albeit with some errors. Through intense dialogues between students and the professor, the students achieved 100% of what was expected in the question. Put differently, they explored different registers: natural language, numerical, fractional, and decimal. Figures 6 and 7 below show the notes made by E14 and E1 regarding the operation in items 2-b and 2-c, respectively.

FIGURE 6: Register of student E14.



Source: Researcher's archive, in 2022.

FIGURE 7: Register of student E1.



Source: Researcher's archive, in 2022.

Another significant aspect regarding probabilistic problem 2, in both items 2-b and 2-c, is that both students mobilized fractional, percentage and decimal representations by using various semiotic registers: figural, numeric and natural language, managing to compare the two fractional quantities. Therefore, the diversity of semiotic registers allows for a diversification of representations of the same mathematical object.

This diversity "enhances the cognitive abilities of individuals and subsequently their mental representations [...] which can never be considered independent of semiotic representations" (DUVAL, 2009, p.17). In this regard, Duval (2012) emphasizes that the learning of a mathematical concept only takes place when there is an understanding of the object and its multiple representations, each with its own content and meaning.

Our analyses corroborate the ideas of Magina, Bezerra, and Spinillo (2009) in the sense that the concept of fraction is of a complex and multifaceted nature. Concerning complexity, fraction is associated with other equally complex concepts such as division, probability, percentage, ratio and proportion.

In this regard, introductory probability problems in the initial training of pedagogues require the professor to use appropriate content methodologies in order to minimize any difficulties that may arise and assist them in their learning (CARDOSO; NERES, 2021).

In the next and final section of this article, we will present some of the conclusions of our investigations.

Final considerations

We began our research by seeking to identify and characterize the nature and role of the registers of semiotic representation of probabilistic problems mobilized and coordinated by Pedagogy students. Based on the Theory of Registers of Semiotic Representation, we start from the premise that changes in representation can contribute to expanding the

understanding of such students regarding the elements inherent in the study of Probability from the perspective of Problem Solving.

Our investigations have revealed conceptual difficulties among Pedagogy students, which highlight profound shortcomings in the Brazilian Basic Education system. While the calculation of probability of events is prescribed even in the early years of Elementary Education, in the skill, considering the occurrence of a result in random events, in contexts of equiprobability, we observe that this concept is still not clear to students in undergraduate courses in Pedagogy. It is worth noting that this bias towards equiprobability predates the advent of the NCCB, which permeates the curricula guided by the National Curriculum Parameters (NCP).

The cognitive activities of treatment and conversion (as seen in Figure 5) carried out by Pedagogy students point towards an exploratory behavior aligned with the investigative practices advocated by the NCCB itself, particularly in the Probability and Statistics unit.

The choice of the methodological perspective of problem-based learning has proven to be appropriate for addressing probabilistic questions, with the potential for developing specific skills and competencies not only in problem-solving attempts but also in reworking, rewriting, or even creating problems. This approach explores a wide diversity of semiotic representations related to the same mathematical concept.

Our results indicate the need to pay closer attention to the initial training of future teachers who will teach Mathematics, especially in regard to the teaching of Probability; otherwise, the new demands outlined in the NCCB will not be adequately addressed.

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