

# Social practices and the meaning of the concept of measures<sup>1</sup>

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## ABSTRACT

In this paper, we will discuss the process of the signification of the concept of measurement based on research conducted to investigate the existing relations between the logical-historical development and the mode of appropriation of the concept of measurement based on the Cultural-Historical Theory/Activity Theory and Teaching Guided Activity. The empirical work was conducted with the collaboration of 21 teachers who teach mathematics in public schools. The analyses were based on the results of the following actions: a) Formative Experiment with theoretical and practical studies; investigation of social practices related to measurements in the fields, b) elaboration, execution, and evaluation of didactic experiments performed with elementary school students. We conclude that in the objectivation of the teaching of the concept of measurement, the process of signification reflects the mode of appropriation of its logical-historical development by the subject and depends on the sharing of cultural meanings underlying the measures acquired in the context of particular social practices of different collectivities, pointing to the need to consider, in the teaching praxis. These different mathematical and symbolic systems govern such practices in the cultural environment of the students.

**Keywords:** Mathematics teaching. Concept of Measure. Meaning Process. Logical-Historical. Social Symbolic Systems.

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*Práticas sociais e significação do conceito de medidas***RESUMO**

Neste artigo discutiremos o processo de significação do conceito de medidas a partir de uma pesquisa realizada objetivando investigar as relações existentes entre o desenvolvimento lógico-histórico e o modo de apropriação do conceito de medida em coletividades agrícolas. Baseado na Teoria Histórico-Cultural/Teoria da Atividade, o trabalho empírico teve como referência: a) investigação das práticas sociais relacionadas às medidas nas lavouras, b) realização de experimentos didáticos com estudantes do Ensino Básico e c) experimento formativo com professores. Concluímos que na objetivação do ensino do conceito de medida, o processo de significação reflete o modo de apropriação do desenvolvimento lógico-histórico dele pelo sujeito e depende do compartilhamento dos significados culturais subjacentes às medidas, adquiridos no contexto das práticas sociais das diferentes coletividades, apontando a necessidade de considerar, na praxe docente, os *sistemas simbólicos extras matemáticos* que regem tais práticas no ambiente cultural dos alunos.

**PALAVRAS-CHAVE:** Ensino de Matemática. Conceito de Medida. Processo de Significação. Lógico-Histórico. Sistemas Simbólicos Sociais.

*Prácticas sociales y significación del concepto de medidas***RESUMEN**

Discutiremos ahora de las prácticas sociales y el proceso de significación del concepto de medida, tras un estudio para investigar las relaciones entre el proceso de significación, el desarrollo lógico-histórico y el modo de apropiación del concepto de medida en comunidades campesinas. soportado en la Teoría Histórico-Cultural, el trabajo abarcó: a) investigación de las prácticas sociales con las medidas utilizadas por los campesinos b) realización de experimentos didáticos con estudiantes; c) realización de una formación con profesores. Concluimos que, en la objetivación de la enseñanza del concepto de medida, el proceso de significación es el reflejo del modo de apropiación del desarrollo lógico-histórico de la misma por parte del sujeto y de la compartición de los significados culturales de las medidas, adquiridos en sus prácticas sociales, señalando la necesidad de considerar, en la praxis docente, los sistemas simbólicos extras matemáticos que rigen dichas prácticas en la cultura de los alumnos.

**PALABRAS CLAVE:** Enseñanza de las matemáticas. Concepto de medida. Proceso de significación. Lógico-Histórico. Sistemas simbólicos sociales.

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## Introduction

In this article, we will reflect on the process of the meaning of mathematical concepts, particularly the concept of measurement in educational activity, considering the study of the history of the development of this object as a path that can lead us to the recognition of social practices and symbolic systems of traditional groups as part of collective consciousness and, from there, enable the emergence of new teaching methodologies, teaching materials, and school contents. It is part of the result of doctoral research based on a historical-epistemological analysis of the concept of measurement, according to the perspective of Cultural-Historical Theory/Activity Theory, with the following objective: to investigate the relationship between the process of meaning, the logical-historical development, and the mode of appropriation of mathematical concepts, based on the concept of measurement, in the context of mathematics teaching.

In general, the method adopted consisted in reconstructing part of the course of the logical-historical development of the concept in question, through historical episodes, until finding the general characteristics of the structure of the activity of measuring, determined by the concrete conditions subsumed in the cultural practices of small farmers. In short, the proposal was to study the historical development of the concept itself, the logical aspects that triggered its evolution, the different forms of appropriation that characterize the process of signification, and to develop analytical processes that help us explain how these elements are related to teaching. Thus, in this study, we discuss issues about the teaching of mathematics, considering a historical-epistemological analysis of measures and the appropriation of meanings.

### **The Logical-Historical and the process of signification in the context of mathematical logic**

Based on the postulates of Leontiev (1985), human consciousness consists of the following elements: the sensitive content of objective reality, social significance, and personal meaning. The first is the real natural and social world outside the subject, even if he has not yet become conscious of it. In the human relation to the

object, the object presents itself as a reflection of the subject's inner plane. Although it is only an image at first, it will later be transformed into a meaningful object as it becomes a psychological instrument for controlling actions and thoughts in conformity with symbolic representations present in the social world. However, the meaning, by itself, does not complete the process of conscientization; there is still the movement that expands its psychological content and gives it a character of individuality, namely: the movement from meaning to personal sense. The planned activity of man triggers this movement, and it is in it that each subject performs the movement of conscientization of the world.

The psychic reflection of reality regulates and channels the subject's external and internal activity. However, that which manifests itself in the objective world to the subject as motives, goals, and conditions of its activity must be in one way or another perceived, represented, understood, retained in mind, and reproduced by his memory; this refers to the processes of human activity and to the human being himself, to his state, properties, and characteristics (LEONTIEV, 1985, p. 10, our translation).

It concluded that man's activity conditions the formation of his consciousness, and his psychological connections, therefore, his humanization process. In this way, the connections of activity from the inner to the outer plane reflect the material conditions in which the activity takes place and how it is regulated. The practical activity realized in the concrete world is primary. The mental activity is secondary, initially together, only later do they separate, but from the beginning, "[...] all external material activities of a man already contain in their interior psychic components through which he regulates himself" (RUBISTEIN, 1979, p. 340). That implies that internal psychological processes do not arise only when the object world is reflected in the human mind but also when one activity is transformed into the other.

The process of signification develops from external activity with sensible objects and in interpsychic relations. First, the subject grasps the meanings present directly in the objects and dominates the logical operations themselves,

but in their clear presentation, in their more sensitive aspects, exteriorized and in a form that cannot yet be communicated. Posteriorly, in the movement of internal mental activity, the logical operations performed on the objects acquire meanings and become abstract concepts (LEONTIEV, 1978).

However, the inner activity, which takes place at the level of consciousness, which elevates the object or phenomenon to the conscious mode, is determined by how the appropriation of the logical-historical development of concepts took place and is exteriorized, impregnated by the senses processed in the subject's most intimate place: his microcosm. It is precisely this movement that makes each subject unique. In the consciousness, initially, meanings and senses appear fused, and in this condition, they are not coincident; however, this non-coincidence will become explicit later.

Meaning is, therefore, a process that begins in each individual upon arrival in the social environment and proceeds progressively upward in qualitative dependence on the forms of interactions established between individuals. It relates to the logical and historical development of objects and phenomena in the following way: Through logic, the human mind can apprehend what biological sensors cannot grasp. It can know the properties, the laws, the relations of the general to the singular, and the nexuses between the phenomena of nature and society. While the logical process triggers the development of the object or phenomenon, producing transformations in it, the new formation returns reflecting unique needs and exerting new tensions on the logical, thus constituting it as historical. The logic of the development of thought has as its fundamental law the ascension from simple to complex, from inferior to superior. This thought dynamic reflects the laws that determine the development of the phenomena of the objective world. "Logic makes known to us the form of development in its pure aspect, which thus literally does not reproduce itself in any historical process, yet the logical form of development reflects any historical process because it is indispensable to its understanding" (KOPNIN, 1978, 186).

Logic drives abstractions. The beginning point for logical movement is the appearance of real phenomena (natural or social) in mind and man's practical activity in the world. However, neither observation nor practice is logical relation.

The empirical is sensory by its content and rational by its form; the sensory content is expressed logically" (KOPNIN, p. 148). In this way, logic is the rational process by which knowledge is acquired and developed, making it historical. The consciousness is realized in the knowledge of the object, and the act of knowing is materialized in the appropriation of social meanings and the production of meanings. The subject knows something when he can attribute meaning to it when it is an internalized sign and begins to regulate his thought and activity.

From the above, there is a relationship of interdependence between the logical-historical movement of the concept, the process of signification, and the formation of consciousness (LEONTIEV, 1985). This argument is anchored on the following: a) meanings are cultural, belong to the collective consciousness, and are available to individuals in the cultural context where they develop because "man's practical activity always includes within it psychic components that reflect the conditions under which it is performed and regulate it" (RUBINSTEIN, 1979, p. 334); b) these meanings are cultural, belong to the collective consciousness, available to individuals in the cultural context where they develop. 334); b) these meanings converge to social practices, from which mathematical ideas, among other things, originate, returning to the social world in the form of new knowledge; c) in individual consciousness, forged in cultural life, social meanings are revealed and transformed by the meanings that the subject will attribute to the object, resulting from the forms of appropriation of the logical-historical movement of concepts (KOPNIN, 1978); and (d) some elements are preserved in the evolutionary process of the object: the general, which is related to the particular that are the differentiations, originating a new formation.

In this sense, in teaching, the history of concepts helps to understand the interdependence between logical-historical development, the forms of appropriation, and the process of signification. Hence, the analysis of the logical-historical-epistemological movement of concepts subsidizes the analysis of the process of elaboration and transformation of the concepts and can give them authenticity, legitimacy, and materiality, aiming at re-establishing their link with the general culture.

## **Materials and method: The research path**

Epistemologically, this study is characterized by: a) adopting as a principle for the investigation of psychological phenomena the relation of man with cultural and natural reality; b) understanding that human development occurs procedurally, in the context of the relations of all life phenomena and c) recognizing activity as an explanatory principle for the process of building human consciousness, considering activity/consciousness as a dialectic unit.

The research was carried out with the collaboration of 21 elementary school teachers who teach mathematics in public schools in the municipality of Lagoa Alegre (research field), with whom the following actions developed: (a) study about the Teaching Orienting Activity - TOA (MOURA, 1993, 2010, 2016); development of theoretical thinking (DAVIDOV, 1987); activity theory (LEONTIEV, 1983), and history of measures (KULLA, 1980; DIAS, 1998; EVES 2004; et al.); (b) study about anthropomorphic measures used by farmers in the fields and (c) application of a Didactic Experiment with elementary school students.

The Teaching Experiment was carried out in three stages: a) planning: preparation of the teaching plan, defining the LTS, the contents, the teachers' and students' actions, and the way of evaluation, according to the theoretical assumptions of TOA (Teaching Orienting Activity); b) execution - the teachers carry out the Teaching Experiment in the public schools of the Municipality; and c) evaluation: - the teachers report, presenting the data observed during the TE to their peers for analysis and collective evaluation of the results.

## **The data analysis method as a means of apprehending reality**

Supported by Vygotsky (1989), we adopted as fundamental principles of analysis: (1) analysis of the process as opposed to an analysis of the object in itself; (2) an analysis seeking to reveal the fundamental dynamic or causal relations without getting stuck in the enumeration of the external characteristics of the object or phenomenon, that is, descriptive analysis and not a descriptive one; (3) analysis of the development of the object, seeking to

reconstruct the points of qualitative change and return to the origin of a given structure. This type of analysis presumes considering the relations of the phenomenon with the totality, observing the process, which does not stop at the enumeration of the phenomenon's characteristics but seeks to apprehend the object through the explicitness of its General Law.

The analysis process was organized into three categorical levels: isolates (CARAÇA, 2010), episodes, and scenes (MOURA, 1992). The definition of each isolate expresses the initial result of a preliminary analysis of the totality. However, it is impossible to know the totality in its entirety, so the isolate is a way of partitioning the complex structure of an object and abstracting it to control the process of appropriation of it. According to Moura (1992), the episodes manifest the investigated phenomenon inside the isolates. They are abstractions that can show the transformations of the object; they are actions, operations, and sharing of meanings that lead to the objectification of activity inside each isolate. The episodes, as parts of the isolates, are dependent, but they maintain independence from each other and, therefore, cannot be overlapped. The scenes are expressions and realizations that show the dynamics of meaning in the episode's plot. They are interconnected and reflect the analysis itself.

The definition of each isolate is the result of a preliminary analysis because each one is part of a complex structure of relationships and cannot be confused with absolute isolation or the encapsulation of parts of the content. The breakdown of the totality in isolates, episodes, and scenes showed the need to organize the objective data comprehensively. However, due to the objective of this article, we will present the analysis related to scenes 7 and 11 (Table 1): which revealed indications about the process of the meaning of the measures in the pedagogical activity.

**TABLE 1** – Distribution of episodes and scenes, according to the isolates of analysis.

Isolates	Episodes	Scenes
ISO.1 Teacher learning in the formative experiment	EP. 1 – The ‘hole’ is exactly not knowing the practice	Scene 1 – The experience I enjoyed.
		Scene 2 – I always do it this way.
		Scene 3 – The problem of not knowing the teachers’ practice.
	EP. 2 – The TOA concept and theoretical thinking	Scene 4 – The mediation of the researcher.
		Scene 5 – Lightning does not strike twice.
		Scene 6 – The teaching activity and TOA.
	EP. 3 – The specific and the general: local measures	Scene 7 – Cubing the land
		Scene 8 - Measuring Grains: The Plate, The Quarter.
ISO. 2 The organization of teaching in educational experiments	EP. 4 – The development of a didactic experiment	Scene 9 – Teaching tasks: planning the actions and operations
		Scene 10 – Reflection on the action planning process: what has changed?
	EP. 5 – Teachers in teaching activity in the educational experiment	Scene 11 – The didactic experiment 01
		Scene 12 – The didactic experiment 02
		Scene 13 – The didactic experiment 03

**Source:** Elaborated by the author (2019).

Instead, the isolated is an abstraction to control the phenomenon's appropriation process. However, this does not mean that the isolated in a system of relations, which constitutes the totality, are identical. Therefore, the process of analysis that begins as the decomposition of the totality is completed with the reconstitution of these parts, generating a new synthesis and new possibilities.

### **The Meaning Process of Measures in Pedagogical Activity**

As we anticipated, due to the impossibility of presenting the fundamental research in a single article, we selected scene 7 (Table 1) to present some evidence found in this study about the significance of measures

in the pedagogical activity. In episode three, we discuss the teachers' knowledge of local measures. These are learnings of traditional content acquired by these teachers outside the school space, revealing the meaning of traditional measures.

Inspired by Vygotsky (2000), we tried to perform the analysis by dividing the subjects' speeches into parts called Discursive Thinking Units (DTU). Then, based on these units, we carried out an analysis that shows the semantic aspects of the texts and sentences extracted from the reports of the collaborating teachers.

### **Some meanings of local measures appropriated by cooperating teachers**

The development of the subject presupposes a progressive movement caused by the successive changes of phenomena in different stages of evolution. At a given stage, some characteristics are retained in the new formation (the general). They are the links with the previous formation. However, more than the general is needed to explain a higher stage with new aspects, characteristics, and properties. However, being in the different stages of the historical development of matter, the general acts as a conditioner of change. However, it is particular that distinguishes one formation from another. Thus, the study of a particular stage of the development comprises the re-establishment of the moments that culminate in the emergence of a new stage. Therefore, the correlation between the particular and the general "represents a correlation of the whole and the part, in which the particular is whole, and the general is the part (CHEPTULIN, 1982, p. 196).

Applying this assumption to the study of the measures, we point out that they were formed in different stages and in organic ways. In this sense, studying a particular stage of development of this concept requires investigating the differences and singularities of previous stages and observing it in the logical and historical context. Taking this understanding to the process of school teaching of the concept of measurement, by its nature, it is essential to study its transformation from organic to didactic content.

It is in this perspective that, in the context of the second isolate: “Teacher Learning in the Formative Experiment,” whose purpose was to produce data to evaluate the learning of mathematics teachers, the third episode entitled “The Particular and the General: local measures” was carried out. In this episode, the teachers described in detail the process of measuring and cubing land and presented particularities of the farmers' traditions and way of life. Furthermore, in this section, we tried to explain how the forms of measurement adopted by the farmers are similar and different, trying to highlight the characteristics that distinguish them from the official measures and confront this information with the teaching of this concept in schools, which constituted Scene 7, presented below in Table 21:

### **Scene 7 - Cubing the land: the fathom and the field line**

In this specific section, our goal is to analyze the teachers' knowledge about the farmers' traditional measures used in Lagoa Alegre to explain the process of the signification of these measures.

We can summarize the content of scene seven as follows:

- a) between 1301 to 1304: definition of the field line, the fathom, and its use;
- b) from 1305 to 1308 and from 1317 to 1319: the landowner and the tenant;
- c) from 1309 to 1313: “area” of the plantation (cubage) and measurement technique;
- d) from 1314 to 1316: the land measurer as a professional occupation.

**Table 2 – Episode 3 – scene 7: Cubing the land**

COD	ID <sup>4</sup>	DTU
1301	Researcher	The purpose of this action is to enable the appropriation of the social practices of local measures developed by the farmers, observing the daily life of the fields.
1302	Anjo	The plot line is a square of 25 fathoms by 25 fathoms multiplied by 625 square fathoms.
1303		For instance: If the field did not bid 25 fathoms, we could say 20. We had to divide 625 by 20 to see what the measure of the other side would be, as long as it was 625 square fathoms.
1304		The fathom could be made with a stick or “buriti” stem, the size of a man standing with his arm stretched upwards. However, one corresponded to the distance of the open arms, which was more used to measure rope, thread, cord, etc.
1305		The [land] owners sometimes did not know how to farm. They lived off the rent. The inhabitant had to pay many quarts per line of land.
1306		Sometimes there was a conflict because there was a difference in measurements.
1307		The farmer measured the land he fenced in, but the landowner was paid for the cleared (deforested) part, which was always more extensive than the fenced-in part.
1308		To measure the land was marked, in the bush, with a cord. There was a person who measured and another who took notes.
1309		Sometimes, when the boss would not personally follow up on the measure, he would send a foreman who used to pull to the patron's side, which created conflict.
1310		Researcher
1311	Derci	For a 4-sided plot, they add up the opposite sides, divide by two, and then multiply the division results.
1312		Adding the opposite sides and dividing by two allows us to approximate the area calculation by the rectangle or square, done to make the count easier.
1313		The cubing was done by two people, one to measure the land and the other to write down the data. In the end, the calculations were made.
1314		In the past, few people knew how to cover the earth. They made a living out of it, so they did not teach others.
1315		Only the large landowners had a person responsible for cubing the fields.
1316		Many knew how to evaluate, measure, etc., but only a few knew how to dig. The person who knew how to cube the ground had a unique knowledge; it was a rarity.
1317		Once, I was asked to cube a field because the owner thought the demarcated land was more extensive than agreed, and the farmer thought it was too small.
1318		I said, “measure the four firebreaks - confident that the land had not exactly four firebreaks - it had a dispute over a curve.”
1319		To solve this, we can approximate the curve by a straight line, leaving one part outside and one part inside to compensate.

**Source:** Elaborated by the author.

<sup>4</sup> The names in the tables in this article are code names given to the teachers to protect their real names.

For the teachers, the definition of the fathom is the same as that given by the farmers, i.e., it is the distance from the big toe (hallux) to the middle finger of the hand of an adult person in biped position and with the arm raised. Historical research has shown that the fathom model in Lagoa Alegre and the French toise coincide. However, this coincidence is no guarantee that the Brazilian model arose from the French toise or that they have the exact origin, although this can be considered feasible.

The toise is the predecessor of the French meter, which gradually spread throughout Europe and, consequently, to the regions colonized by Europeans. In Brazil, with the intensification of overseas travel, influenced by commercial interests, the French meter acquired an abbreviated form, received the name of 'stick' - perhaps to reduce rejection by the colonists - and became equivalent to eleven-tenths of the French meter. However, it is essential to understand that in the transition from the arm to the measuring rod and from the rod to the meter, the general principles and properties of the measurement concept are preserved, i.e., measuring depends on a comparison, a yardstick, and a magnitude. In other words, measuring depends on a comparison, a yardstick, and a magnitude. Although, in general terms, measuring with a stick or with a yardstick reflects the same operation, the difference between one and the other resides, in part, in the needs to which they are adapted and in the interests that are protected, the transition from one to the other required social changes resulting mainly from the struggle for the preservation of the economic interests of hegemonic social groups in the struggle for dominance of an official metric system, to the detriment of the causes of the poorest (DIAS, 1998).

In the case of the fathom, it can be mirrored in a stick taken from a “buritizal”, for example, that can support this standard. According to the workers, measuring a field by these means varies a lot according to the region. For example, in some regions of Piauí, the measure adopted for the plantations is the Task or Line of Plantation, as occurs in Lagoa Alegre and its surroundings. The calculation of these measures is commonly known as cubing, cubing a plot of land means finding its “area” independently of the topological conditions of the land, considering only the measures of its contours, fire breaks, or pitches.

The mathematical model applied to cubing a plot of land involves the ratio between the sizes of its four sides or plots. If the land is not a quadrilateral, a geometric adaptation is made, braiding new flights, decomposing it into quadrilaterals, making it possible to apply the formula, which is defined by the product of the arithmetic mean of the opposite sides of the quadrilateral, algebraically represented by the expression  $k = [(a+c)/2] * [(b+d)/2]$ , where a, b, c and d are the measures of the straight sides of the land. It was impossible to obtain accurate information about the origin of this formula in Lagoa Alegre. However, we recall that an identical formula was found in the Rhind<sup>5</sup> papyrus (EVES, 2004). Therefore, it reached Brazil, and possibly Lagoa Alegre, during the colonial period in Brazil. Nevertheless, we cannot reject the coincidence hypothesis if the tensions exerted by every day needs under identical conditions can generate similar solutions.

Based on the teachers' reports, the pre-metric system of Lagoa Alegre encompasses measurements of large and small distances, areas, and volumes. It is evident that, by their nature, anthropomorphic measurements present a low level of precision. However, this did not represent a problem for quite some time, as there were comprehensive forms of compensation (KULLA, 1980). Accuracy only became an actual demand when the land began to be valued by the criterion of use value, still in pre-capitalist societies. The conflicts between landowners and tenants, as shown here (Table 2, DTU 1307), need to indicate the imprecision of these measures. However, the mode of production is established between the owner of the land and the owner of the labor force. Therefore, the motivation for the disputes was linked to the forms of earnings that both wanted to earn, which partially depended on the measures found.

From the reports of the teachers, we can also infer that knowing how to measure land was a privilege, partly because, in the countryside, farmers were generally illiterate, which made it difficult to learn how to manipulate the formula, and partly because it was a way to obtain income; the “land measurer” was a paid occupation for those who provided this service to landowners. Professor Derci's description (DTU 1312) illustrates this situation. It was, therefore, an artisanal, and therefore empirical, surveying (DTU 1315).

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<sup>5</sup> Egyptian document from around 1 650 B.C., where a scribe named Ahmes writes down the solution to various problems in arithmetic, fractions, calculation of areas, volumes, geometry, and others.

Scene seven allows us to state that: (a) teachers' knowledge about traditional measures is in tune with the same conceptual basis of the farmers; (b) there is a social context in which the dynamics of collective daily life and intersubjective thinking are under the same tensions of the same symbolic system, regulating social practices in the cultural environment, of which teachers and students are participants. This represents a relationship between the organic social contents and the school contents didactically transformed in the pedagogical activity that takes part in the signification of the measurement concept.

It follows that measurement as an organic content, seen from its logical-historical development, can become a didactically organized concept. To do so, it must take as a reference the logical-historical-epistemological analysis of its meaning, which comprises, in culture, the interdependent relationship between the general and the, and between the collective and the individual, and between the mathematical contents and the mathematical extras present in the symbolic systems subsumed in each cultural field.

### **Teachers in a teaching activity: the didactic experiment 01**

In this subsection, we will discuss Didactic Experiment 01, which we had the opportunity to follow. In particular, the purpose of this experiment was (a) to find evidence that the history of measures influenced the process of organizing teaching; (b) to verify whether there were indicators of awareness on the part of teachers of the need to relate the measures used by local farmers and the decimal metric system in the teaching of mathematics; (c) to observe if the didactic experiment would evidence the relational links between the logical-historical movement of the concept of measure and its process of signification in the pedagogical activity and (d) to observe how the students would react to the teaching proposal within the TE, paying attention to the cognitive potential of anthropomorphic measures in teaching the concept of measure.

Experiment 01 was conducted in the 9th-grade class of the Alfredo da Silva Costa Municipal School, located in the urban area of Lagoa Alegre. The execution unfolded in two stages. In the first moment, the teacher discussed the general proposal of the activity with the students. From this discussion, the following

operations resulted: formation of study groups; interviews with the laborers to learn more about their work, including the issue of traditional measures; following a previously developed script; transcription and analysis of the interviews and presentation of a report to the class.

For the teacher, the results brought by the students created the opportunity for them to expand their knowledge about the content; highlight the aspects that were not addressed in their reports and promote learning of content originated in their own culture, making the connection of organic content with scientific content, transformed didactically (DTU 2501, 2502 and 2503. Table 03). Teacher Derci presented the Learning Triggering Situation (LTS) in the second moment.

In the TOA context, LTSs (Learning Triggering Situations) are designed to highlight the need for the concept. They are presented as a problem involving a social need and require discussion and understanding of the situation through hypothesis testing, understanding of language, understanding of the concept studied and its conceptual nexuses, and the search for an appropriate solution, aiming to use the solution. Araújo (2019).

In this case, the challenge was finding a solution to a specific problem posed by LTS. The group would have to establish strategies and present the possible solution to the problem and the strategy developed to the class. It was up to the teacher to talk to the groups, stimulating them and raising new questions, suggesting, and helping to understand the problem (DTU 2506, 2509, and 2512. Table 03 shows the reflections made by teacher Derci:

**TABLE 3** – Summary of reflections on Didactic Experiment 01 - Teacher Darci.

COD	DTU
2501	'I noticed that concerning field measurements, they [the students] knew almost nothing. I think they don't usually talk [about it], or they don't go to the fields with their parents.'
2502	So, together with him, we prepared a questionnaire for an interview with the farmers. Naturally, they went to their parents or neighbors to find out what a fathom, a quarter, a line of land is, but the task I kept confidential. We did not tell them what we would do in the Wednesday class; the LTS I only put in this class.
2503	This moment of the interview with farmers was a preparation for the students to understand the measurements of the field; this way, they entered the problem with the information, and the students arrived knowing something through the interviews they did.
2506	[In class], the challenge was: a) draw a rectangle for 12 lines in the figure. [...]. b) draw up a plan to build a fence around the plot with five strands of wire, indicating the amount of wire needed in meters. c) analyze the reasonableness of the results.
2509	Group 3: There was a task division in this group, and they came up with four possibilities. This means that the work tends to go faster when we divide the responsibilities.
2510	Group 4 was in the same situation as Group 1. They concluded that the 12 lines were 7500 square fathoms and the land had approximately 103 lines, but they could not envision what this rectangle would look like to form the 12 lines.
2512	In the groups that could find the solution, [their members] worked more synchronously and reasoned faster. The two groups that complained about the time either did not understand the situation or did not understand it until later, or else they did not have prior knowledge.

Source: Elaborated by the author (2019).

Our analysis shows that the task developed in this teaching experiment highlighted the cultural link between the measurement processes traditionally used by farmers and the decimal metric system, highlighting the need for the concept in question and enabling the understanding of mathematics as a historical and human construct. Moreover, the students were challenged to develop strategies to solve the problem collectively, perform multiple interactions inside and outside the school, and use appropriate meanings in their cultural environment. Specifically, the form of teaching organization adopted allowed students to: (a) understand the general foundations of the concept of measurement through the traditional measurement processes adopted by local farmers, relating them to official measures; (b) understand the motivations and interests that led to the exchange of anthropomorphic measures for artificial measures and, (c) understand the elaboration process that gave rise to the Decimal Metric System; (d) develop attitudes and procedures that promote collective work; (e) become aware that

mathematics is historical and, as such, was (is) under external tensions exercised in the field of social struggles for the prevalence of interests of one group over another and (f) the situation that triggered learning put students in activity, because it brought the need for the concept of measurement and allowed a set of interactions inside and outside the school in search of satisfaction of this standard need, imbued with the same motive, they performed different tasks with the same goal.

About possible changes in the way of organizing teaching and affectations concerning the teacher, the data showed: (a) awareness about the need to consider in teaching how the culture and social reality of students interfere with learning; (b) recognition of TOA as a way of organizing teaching that makes it possible to confront the students' cultural reality, particularly those of an empirical nature, with scientific or theoretical mathematical knowledge; (c) recognition that teaching the concept of measurement based on the historically situated relationship between the traditional knowledge of farmers and scientific knowledge promotes learning, and (d) strengthening the teacher's sense of belonging to the social context of the students themselves, to the same cultural reality, to which the teacher's conceptions, attitudes and thoughts are intended, encompassing relevant knowledge with which the teaching activity does not always dialogue and, sometimes, disregards. At this point, it is possible to infer that the General Law that governs conceptual development favors teaching based on the history of the concept, enhancing the process of its meaning by students, as we will explain in the next point.

### **The process of significance, the logical-historical and social practices in the objectification of the concept of measurement**

Based on this study, in summary, they argue that in school education, the meaning of the concept of measurement depends on how the appropriation of the logical-historical development of this concept by students occurs, encompassing the historically developed cultural meanings. This leads to a form of teaching that opposes the historical-epistemological rupture of the structural elements of the phenomena, which, by denying their historicity, also denies the historicity of man, a preponderant factor for understanding the role of shared actions in collective activities essential for the development of humanity.

Therefore, we enumerate some results of this investigation based on the dialogic relationship between the theoretical study and the analysis of the empirical data:

- a) in the process of meaning the measurement concept, ontogeny does not recapitulate phylogeny. That is, the subject does not reproduce history with all its variations but develops a complex system of abstractions capable of giving meaning to the phenomenon;
- b) empirical and scientific knowledge are always connected since knowledge is empirical by form and rational by content;
- c) the study of the logical-historical development of a specific concept is configured as an indispensable requirement to explain the origin and development of knowledge;
- d) the teaching of mathematics, based on the history of the object, enables teachers and students to understand the historical character of the concept as a result of the search for solutions to successive needs, including those related to human existence;
- e) the analysis of the logical-historical development of the measurement concept can structure several didactic actions from exploring extra-mathematical aspects of daily life, social practices, and symbolic systems of a culture of which the school is part.

In this perspective, history delivers the plot of the concept from its origin. It shows the logical form of organization of mathematical thought, for "the logical is the historical freed from the causalities that disturb it. [...]. The logician reflects not only the history of the object itself but also the history of its knowledge" (KOPININ, 1978, p. 184 and 186). Thus, in the objectification of teaching, the relationship between the logical and the historical takes us back to the relationship between the collective and the individual, between the general and the particular, and between the universal and the singular (DTUs 2502 and 2503). From these relations arise the conceptual nexuses that link mathematical knowledge to social reality, constituted by extra mathematical elements (DTU 2506) because the teaching process considers the phenomenon or object of study from its origin and reveals the crises that culminated in the

passage from one stage to another, comes into contact with the emergence of the laws themselves, the relations of causality and the nexuses that form the concept system. (Historical investigation in Mathematics)

Mainly, the knowledge about anthropomorphic measures in mathematics teaching can be used to develop didactic proposals in which the conceptual links between traditional and artificial measures can be highlighted to promote the learning of concepts in their most advanced stages.

Thus, the teaching process, which begins with the learner's appropriation of the social content and its relation to the act of measuring itself, must be designed to reach the most advanced stage of appropriation of meanings. In this way, when the student reaches the capacity to transform the appropriate into psychological instruments and understand the relationship and linkages of the object in its complexity, it becomes conscious. It will control and produce transformations in the subjects, reflected in actions, behavior, and production. From the school's point of view, only with the appropriation of theoretical concepts, which have the power to examine natural and social phenomena scientifically, is it possible to reach this level of learning. Only with empirical knowledge scientific scrutiny of phenomena is unlikely because they have limited reach, arising from their very origin, sensory perception. According to Davidov (1988), empirical knowledge separates and fixes objects in their external aspects, without links, independent and isolated, as existing by itself, which is not derived or generated from something else. In contrast, scientific knowledge understands the complexity of the conceptual system's phenomenal relations, transformations, and transit.

In the case of the objectification of the concept of measure, it can be verified:

- a) when the subject becomes capable of disregarding, on the mental plane, the other properties of the object, emptying it of all concrete matter, in order to separate and combine the comparison-statement-quantity term to produce a symbolic representation of the real;
- b) b) when the subject performs abstractions as a result of the mental effort to convert the measurement into some knowledge about measurement;

- c) when this knowledge becomes an instrument of thought and returns to the real world in the form of signs, enabling students to perform operations with measures materialized through theoretical abstractions;
- d) when measurement as a technique, expressed through a *modus operandi*, gives way to a generalized method expressed through mathematical language, synthesized in the doing-knowing-thinking scheme. (Table 3, DTUs 2509 - 2512)

It is in this sense that we defend the thesis according to which, in the objectification of the concept of measurement, in the school context, the process of signification, that is, the appropriation of meanings and attributions of meanings, are linked to two inseparable aspects: 1) how the appropriation of the logical and historical development of the concept of measurement by students takes place and 2) the sharing of cultural meanings underlying the measures, acquired in the context of social practices. Objectification, to its highest degree, depends on interweaving these two aspects.

As for the process of signification taken as a reflection of how the appropriation of the logical-historical movement by students occurs (part 1) it is characterized by:

- a) enhance the understanding of mathematics as a social content, historically developed;
- b) promote scientific thinking, since history has the quality of showing the logical aspect of the concept and the plot lived by humanity, which led to the production of knowledge;
- c) offer theoretical support for the elaboration of learning-triggering situations because, by showing the needs that have intended of the different societies and cultures where the transformations in knowledge occurred, they put the students in front of the situations that created the need for the concept and propitiated its development;
- d) enable the recognition of social practices as integral parts of the historical production of meanings and that they are governed by dynamic symbolic systems present in different cultures;

- e) demystify mathematical knowledge, favoring the recognition of mathematics as a human construction;
- f) support the epistemology of mathematics since it provides information about the origin and development of mathematical knowledge.

As Kopnin (1978, p. 168) argued, the study of the object's history creates the essential principles for understanding the essence of the concept, and this is because, “enriched by the history of the object, we must take up once again the definition of its essence, correct, complete, and develop the concepts they express.”

In the context of the interaction between the sociocultural history of measures and the elaboration of theoretical-practical knowledge (part II), a historical-epistemological analysis could make the student aware of the logic of how measures were thought.

We infer that, in teaching mathematics, it is not necessary to annul the social meanings that the student has acquired in his culture but to seek a qualitative transformation of them through the possible relations between the old and the new formation in the field of concept development toward its most advanced form. It is essential to recognize that learning is not restricted to the school environment; social conditioning factors are present in the students' consciousness and mediated by the symbolic representations they internalize as cognizing subjects.

Knowledge as a set of culturally and historically constituted processes of reflection and action [...] always has an antecedent: to think something, there is always a possibility already constituted to think it. This does not mean that we keep repeating it; it means that this knowledge has a trajectory, a history; the study of its conditions of possibilities, transformation, generalization, and refinement gives us an idea of the epistemological density of knowledge, which can be very important when designing didactic activities and interpreting what happens in the classroom (RADFORD, 2015, p. 254).

This indicates the need for the teacher in the classroom to know and recognize the different mathematical and symbolic systems (Table 2, DTUs 1311 - 1319). This implies highlighting the links between empirical and theoretical or scientific thinking that will show the connection of mathematics

with cultural expressions (art, religion, customs, and behaviors) embedded in social practices within different cultural frameworks. Practically, for teaching, it implies that such knowledge can constitute educational material, requiring a didactic approach capable of transforming and integrating it into the curricular contents.

By considering the proper mathematical knowledge of a given culture, the teacher needs to be open to sharing the meanings underlying the knowledge that govern the social practices of the students' cultural reality, needs to consider that it is not a matter of reproducing them, but of provoking a qualitative transformation in the field of psychological development. In this sense, the mathematics teacher must be attentive to the symbolic systems of social signification and, consequently, recognize students as subjects whose culture is present in consciousness, influencing behavior and attitudes in everyday life. Sharing meanings requires studying the concept within the students' social framework since knowledge develops in culture.

Accordingly, Radford (1997) argues that any effort to study a mathematical concept needs to consider aspects of the cultural framework in which that knowledge is placed. A historical-epistemological analysis of cultural practices and the symbolic systems that control them requires knowledge about culture. As far as the teaching of mathematics is concerned, the primary role of historical-epistemological analysis is to know ancient meanings or semantic fields, mathematical concepts, and procedures resulting from social practices in order to, through a process of didactic adequacy, incorporate them into current school curricula and offer subsidies for the elaboration of theories, teaching materials, teaching models, etc.

### **Final considerations**

Finally, we reaffirm that the knowledge underlying the social practices of particular collectivities constitutes school content when integrated into the concept's historical flow. Thus, the crux of the matter is to consider the logical-historical movement in objectifying the teaching of mathematical knowledge, observing the cultural structure in which students are inserted. However, the

teacher needs to observe two aspects regarding mathematical knowledge: a) the social structure surrounding the students as a contribution of the unique social practices of anonymous collectivities with their particularly valued knowledge, and b) the scientifically systematized mathematical knowledge as a universally valued historical and traditional content. This reality creates the need for the teacher to broaden his/her disciplinary and pedagogical knowledge, especially when studying non-hegemonic cultures.

In this regard, the pedagogical use of the history of concepts can influence the awareness that they develop according to a general law but not disassociated from class interests in a naive, neutral, and natural way; on the contrary, it is in the conflicting environment of asymmetrical social relations and the intricacies of politics, that knowledge originates and develops, sometimes serving the interests of certain hegemonic social groups.

Educating with mathematics implies, therefore, the formation of subjects integrated into the social dynamic. Moreover, as a historical being, he can critically understand the movement unleashed in the struggle of opposites in search of his freedom. This requires understanding that the connection between the history of the development of the object and that of the development of the subject; between the individual's cognition and the appropriation of culture; between the sociocultural environment and consciousness demonstrates the nature of individual thought, forged according to the nature of the relationship with collective thought, which under certain conditions legitimize social inequalities.

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Received in April 2022.

Approved in April 2023.