Ensino em Re-Vista

http://doi.org/10.14393/ER-v29a2022-20

A *geometry for teaching* by José Augusto Coelho[1]

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ABSTRACT

This article show partial results of doctoral thesis of first author, that is aligned to a project extensive research. The thesis aim characterizes a *geometry for teaching* from pedagogy manuals directed to formation of teacher from first years of school. In this articles, we aim characterize the *geometry for teaching* that remained stable in production by José Augusto Coelho. This characterization took from the mentioned documents, considering the determinations about how to transform dispersed information into systematized knowledge. The results show that the *geometry for teaching* systematized by Coelho mobilizes the euclidean geometry and guides the presentation of teaching objects to happen in an intuitive way, through successive decompositions, from solids to the point. There is an indication that after this presentation makes recomposition, moving towards the systematization of forms as ideal objects. This enable us characterize the Coelho's *geometry for teaching* as *intuitive-traditional geometry for teaching*.

KEYWORDS: Pedagogy manuals. Augusto Coelho. Professional knowledge of the teacher. Teacher work tool.

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http://doi.org/10.14393/ER-v29a2022-20

Uma geometria para ensinar de José Augusto Coelho

RESUMO

Este artigo apresenta resultados parciais da tese de doutoramento da primeira autora, que está alinhada a um projeto amplo de investigação. A tese objetiva caracterizar uma *geometria para ensinar* a partir de manuais de pedagogia direcionados à formação de professores dos primeiros anos escolares. Neste artigo, objetivamos caracterizar a *geometria para ensinar* que se manteve estável na produção de José Augusto Coelho. Essa caracterização deu-se a partir dos documentos históricos mencionados, considerando as determinações sobre como transformar informações dispersas em saberes sistematizados. Os resultados apontam que a *geometria para ensinar* sistematizada por Coelho mobiliza a geometria euclidiana e orienta que a apresentação dos objetos de ensino aconteça de forma intuitiva, via decomposições sucessivas, dos sólidos ao ponto. Há a indicação de que após essa apresentação façam-se recomposições, caminhando à sistematização das formas como objetos ideais. Isso nos permite caracterizar a *geometria para ensinar* de Coelho como *geometria intuitiva-tradicional para ensinar*.

PALAVRAS-CHAVE: Manuais de pedagogia. Augusto Coelho. Saber profissional do professor. Ferramenta de trabalho do professor.

Una geometría para enseñar de José Augusto Coelho

RESUMEN

Este artículo presenta resultados parciales de la tesis doctoral de la primera autora, que se enmarca a un proyecto de investigación más amplio. La tesis tiene como objetivo caracterizar *una geometría para enseñar* a partir de manuales de pedagogía dirigidos a la formación de profesores que actúan en los primeros años de la escolaridad. En este artículo, nuestro objetivo es caracterizar la *geometría para enseñar* que se mantuvo estable en la producción de José Augusto Coelho. Esta caracterización se dio a partir de los documentos históricos mencionados, considerando las determinaciones sobre cómo transformar informaciones dispersas en saberes sistematizados. Los resultados apuntan que la *geometría para la enseñanza* sistematizada por Coelho moviliza la

geometría euclidiana y orienta la presentación de los objetos de enseñanza para que suceda de manera intuitiva, a través de descomposiciones sucesivas, desde los sólidos hasta el punto. Hay una indicación de que después de esta presentación, se hacen recomposiciones, avanzando hacia la sistematización de formas como objetos ideales. Esto nos permite caracterizar la *geometría para enseñar* de Coelho como *geometría intuitiva-tradicional para enseñar*.
PALABRAS CLAVE: Manuales de pedagogía. Augusto Coelho. Saber profesional. *Herramienta de trabajo del maestro*.

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Introduction

The professionalization of the teacher has been discussed by authors such as Machado (1995); Nóvoa (1999) and Bourdoncle (2000). These researchers point out several factors that historically have been defined as necessary to the process of recognizing the act of teaching as a profession. Among these criteria is the definition of knowledge that is proper to the teacher, that is characteristic of his profession, a concept that we want to highlight in this text, because talking about *geometry for teaching* is to discuss one of the professional knowledges of the teacher who teaches mathematics, particularly in the early school years.

When we speak of the teacher's professional knowledge, we refer to those systematized by the Research Team in the History of Educational Sciences (ERHISE), from the University of Geneva, and appropriated by Brazilian researchers, such as Bertini et al. (2017). Rofstetter and Schneuwly (2017), members of the staff mentioned, indicate the existence of specific knowledge of the professions of teaching and teacher training, which are the knowledge *to teach* and the knowledge *for teaching*, and refer, respectively, to the teacher's object and work tool.

Considering the productions about this knowledge, Bertini et al. (2017) developed the understanding of *mathematics to teach* and *mathematics for teaching* as constitutive elements of the professional knowledge of the teacher who teaches mathematics, particularly in the early school years. Thus, the *mathematics to teach* is linked to the different mathematical rubrics, related to university subjects, which are configured as the mathematics that the teacher must teach, but it is also indebted with “the purposes attributed to the school, the pedagogy that prevailed at school, the current conceptions about mathematics, amidst several other determinants” (VALENTE, 2019b, p. 53).

On the other hand, the *mathematics for teaching* is “a specific knowledge, with a professional culture, proper to the formation of the future teacher” (VALENTE, 2019c, p. 54). This mathematics is the teacher 's object of work, it is elaborated for the possession of that professional. It consists of didactic-pedagogical knowledge that mobilizes *mathematics to teach*, so that its elaboration is done by the teaching profession from its expertise. Therefore, the *mathematics for teaching*[4] is "the result of re-elaboration over time, by the teaching profession, of knowledge *for teaching* mathematics, aiming at each historical period, a tool *for teaching* mathematics" (MACIEL; VALENTE, 2018, p. 168).

This *mathematics for teaching* is what fundamentally characterizes the teacher's professional knowledge, always maintaining articulation with the *mathematics to teach*. The *geometry for teaching* is a tool of the teacher's work, knowledge specific to his profession. It is not the university subject geometry that one must know *to teach* in the first school years, or the didactic-pedagogical orientations that can be applied to it, but the geometry whose systematization puts all these aspects in articulation and mutual dependence.

[4] We emphasize that the mathematics for teaching is not linked to knowledge of action, but to the field of objective knowledge, which are those that “[...] show themselves as systematized speeches, ready to be mobilized, with the ability to circulate. They are communicable so that they can be used and appropriated in different contexts” (VALENTE, 2019a, p. 10).

At least in the last five years, research have shown that in each historical-educational period, professional knowledge of the teacher who teaches mathematics was developed. When it comes to research on professional knowledge related to geometry, we can mention the thesis of D'Esquivel (2019) and Conceição (2019).

This article presents partial results of a doctoral[5] research whose objective is to characterize a *geometry for teaching* from pedagogy manuals aimed at training teachers of the first school years in Brazil between 1870 and 1920. In the process of preparing such research, which is underway, we found, among the publications we located, that the author of the most outstanding manuals is the Portuguese José Augusto Coelho.

So, for this moment, we intend to answer the question: what *geometry for teaching* can we systematize, based on Augusto Coelho's productions dedicated to the constitution of the teacher's professional knowledge *to teach* geometry in the first school years? Hence, the elaboration of this article aims to characterize the *geometry for teaching* that it remained stable in the publications of this author, from the analysis of these documents.

Finally, we resorted to the pedagogy manuals authored by Augusto Coelho[6] that circulated in the institutions that formed teachers of the initial schooling in Brazil during the period studied in the referred thesis, 1870 to 1920. This material is considered by historians of the history of education, for example, Carlota Boto, as a means from which "the great classics of pedagogical discourse are interpreted" (BOTO, 2018, p. 160). Thus, they were responsible for taking to those teachers what the pedagogy of their time proclaimed for their institutional formation, the knowledge considered necessary for the exercise of the teaching profession.

[5] This research is in line with a broad project that historically investigates the processes and dynamics of the constitution of teachers' professional knowledge from 1890 to 1990; and it has been carried out with the support of the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Financing Code 001.

[6] José Augusto Coelho was born in Sendim, Portugal, in the middle of the 19th century. He sought to add the art of teaching aspects of the educational sciences, becoming the author who "best represented the search for the rationalization of Pedagogy" (BOTO, 2010, p. 20). For more information about the author, read Boto (2010).

How to systematize *geometry for teaching*?

The answer to this question presents several directives, based on different theoretical perspectives. Nevertheless, here we clearly state that the process of systematizing a *geometry for teaching* that we consider is based on Valente (2018) and Lima; Valente (2019), when they deal with the transformation of information dispersed in systematized knowledge.

According to Valente (2018) and Lima; Valente (2019) in the historical study on the constitution of the teacher's professional knowledge, the process of transforming knowing into knowledge[7] can be carried out based on the following steps: "recompilation of teaching experiences, comparative analysis of teachers' knowing, systematization and use of knowing as knowledge" (VALENTE, 2018, p. 380 our translation).

The procedures for the development of the first stage mentioned include "the separation of information reported in [...] pedagogical manuals [...] among other types of documentation that may reveal information about the teacher's pedagogical work" (VALENTE, 2018, p. 380). These material evidence of school culture (JULIA, 2001) compose a collection of knowledge that was dispersed in a certain historical period (LIMA; VALENTE, 2019).

Considering the sources of research that we took to prepare this article and the objective to which we propose, we can rewrite this phase of the recompilations of teaching experiences in our text as a recompilation of elements of a *geometry for teaching* in pedagogy manuals, given that this material does not express teaching experiences, but rather contains systematizations about what the teacher should do

[7] "And, here, it is important to explain the difference, in theoretical methodological terms, between knowing and knowledge, in the way we are mobilizing this conceptual duo. The first is more related to subjectivity, to the experiences lived by the subject, implicit means of action, of reasoning; the second, the result of systematization, of a more consensual character, capable of generalization and objectification, an historically institutionalized cultural product whose result is the systematization and organization of certain knowledge in order to promote its communication" (VALENTE, 2018, p. 380- 381, emphasis added).

when teaching geometry. We can say that in each of the manuals there is a particular objectification of this knowledge, which were dispersed in time and space by Brazilian normal schools from the final decades of the 19th century to the beginning of the 20th century.

This consideration may lead us to question: if in these manuals there were no reports of teaching experiences, but systematized knowledge, what will we systematize? Well then! Each manual circulated its specific objectification of *geometry for teaching*. In every place that each one of them passed, they left their own marks. So, considering that they were dispersed in different spatiotemporal contexts, we are going to systematize *geometry for teaching* that characterizes the period that the manuals understand and not just a particular case, to highlight this element of the professional knowledge of the teacher who teaches mathematics which remained stable, which leads us to the second stage of this systematization process.

The comparative analysis of teaching knowing selects new data from what was done in the previous step. The inventory prepared in this analysis is “now composed of the separation of that information about teaching experiences that are convergent from the point of view of the orientation for the teacher's work” (VALENTE, 2018, p. 381). With that, it is possible to identify trends and build consensus that represent what the teacher must know *to teach* (LIMA; VALENTE, 2019). As we have already emphasized that in this text, we are not working with teaching experiences, we understand this phase as a comparative analysis of the elements of a *geometry for teaching* in pedagogy manuals.

The last stage of the process that Valente (2018) presents in the transformation of knowing into knowledge is the systematization and analysis of the use of knowing as knowledge, which, says the author, are made simultaneously, to verify whether normative and / or didactic-pedagogical instances used what the researcher systematized. Considering the nature of the sources that we use in this text, we already understand, beforehand, that what we are going to systematize was included in such instances. Then, we will carry out, at this point, the systematization of elements of a geometry for teach in the first school years.

At this stage, as Valente (2018) points out, any element of subjectivity that may still appear from the previous stage, needs to be removed. This systematization should make the pedagogical consensuses that were established in the comparative analysis can be seen as objectified, that is, subject to generalization and use. Regardless of the book, we must show what was the *geometry for teaching* proposed by José Augusto Coelho.

José Augusto Coelho's manuals and their geometry for teach

The three pedagogy manuals by José Augusto Coelho that we managed to locate, which we will talk about next, present important contributions to the constitution of the professional knowledge of the teacher who teaches geometry. This material was aimed at teacher training and circulated in several Brazilian normal schools between the end of the 19th century and the beginning of the 20th (BOTO, 2018). In them, the author presents systematization of knowledge that the teacher must have for teaching such subject in the first school years.

The oldest manual among these is the Tome II of "Princípios de Pedagogia"[8], published in 1892, whose *geometry for teaching* that it has, is characterized in the publication of Fortaleza; Valente (2019). Between 1892 and 1907 the author published the "Manual Prático de Pedagogia". The *geometry for teaching* in this manual was discussed in an article by Fortaleza and Rocha (2020). We will use these texts and the manuals analyzed by them (COELHO, 1892; COELHO, n.d.) to extract elements that constitute this *geometry for teaching*, such as the *geometry to teach*, the knowledge *for teaching* geometry: the teaching materials indicated, the

[8] This manual was published in 4 volumes.

http://doi.org/10.14393/ER-v29a2022-20

presentation and generalization process, the teaching march, which is the order in which the contents are taught; and the reference pedagogist.

In this article, we will analytically recompile *geometry to teach* that Augusto Coelho systematized in another manual: “Noções de Pedagogia Elementar”, published in 1907. From this recompilation, we will also extract those elements, and then, we will have the necessary information to compare the *geometry for teaching* arranged by the author in each manual to culminate in the systematization of the geometry that him aimed as an element of the teacher's professional knowledge between 1892 and 1907, showing the characteristics of this geometry that remained stable in this period.

In the title “Noções de Pedagogia Elementar”, before presenting his systematization of a *geometry for teaching*, Coelho (1907) discusses the means of instruction, and states that the teaching of geometry must rely on “general collection of elementary geometric shapes” which leads us to understand that the use of representative objects should compose the didactic framework that the formation of the teacher who taught geometry provided.

Augusto Coelho (1907) defines geometry as “the science of extension forms and their relations” (COELHO, 1907, p. 81, emphasis added). So, these are the elements of *geometry to teach*[9] that integrate the systematization of *geometry for teaching* elaborated by the author to compose the training of teachers for teaching in kindergarten and primary schools.

The teaching process, i.e., "the way in which, in the view of the student, the presentation of the notions we propose to accumulate in his mind" (COELHO, 1907, p. 58), that the author uses to systematize the guidelines *for teaching* geometry is empirical and real, which is associated with intuitive teaching[10]. In this way, geometric shapes must

[9] The manuals also deal with the relations of equivalence: length, area and volume, but in this text we focus on shapes.
[10] Some of the principles of the intuitive method are “starting from the known to the unknown, from the concrete to the abstract, or from the particular to the general, from the intuitive vision to the general understanding, through a natural association with other elements and, finally, bringing together in the whole organic of each human conscience the points of view reached. The basis of this method was the idea of sensory perception. [...] it is characterized by offering sensitive data to observation, going from the particular to the general, from the concrete experienced to the rational, reaching the abstract concepts” (ZANATA, 2012, p. 107).

have their abstraction characterized in "portions of matter" that represent the abstract shape, "thus making them somehow tangible and real" (COELHO, 1907, p. 84). To achieve this process, the author indicates the use material from Froeble or other objects that have been adapted for this purpose.

The structuring of geometry in the systematization of *geometry for teaching* that Coelho (1907) proposes is perfectly pedagogical, states the author. This means that it is not around the order employed by mathematical formalization that this knowledge must be presented to students. What needs to be considered is the order that makes sense for the understanding of these children. Thus, Coelho (1907) aims at a *geometry for teaching* that begins with forms, followed by the approach of their relations.

Regarding the teaching of forms, “the solids will first be presented, then the superficial ones, then the linear ones, and finally, the points, thus advancing from the concrete to the abstract” (COELHO, 1907, p. 84-85,). Soon, we realized that the use of Froebelian materials, e.g., is a process that introduces spatial geometric notions to the students' senses. The understanding of solids starts with the use of the tangible object, but this is only the real that represents the ideal, that is, the understanding of solids in the abstract.

In the march in which Coelho (1907) disposes of geometric shapes in the systematization of *geometry for teaching*, it is observed that solid shapes are the first to be taught, they must be presented by concrete objects. Understanding the surface forms requires knowledge of the solids, the linear of the superficial and the points of the linear ones. Thus, the understanding of a geometric shape is triggered by the abstract understanding of the shape previously studied.

To guide the teacher in more detail on how to present the spatial geometric shapes, Coelho (1907) organized the study of these teaching objects in series, making it clear that the order to be followed must be respected. We have already seen that by the author, the teaching of geometry begins with spatial forms. Among these, the first to be treated in the first series of solids is the sphere, which must be shown to the student outlined in wood and of a single color, “the child must perform several children's games with it” (COELHO, 1907, p. 85). The use of the single color may emphasize to students that the sphere has a single surface. Thus, the use of color is a constitutive element of *geometry for teaching* systematized by the author.

The cube needs to be taught as a derivation of the sphere. The composition of this should be made from the insertion of “six conveniently directed sections” (COELHO, 1907, p. 85), which can only be done correctly by those who have knowledge about the properties of the cube, which shows the importance of *geometry to teach* for the constitution of *geometry for teaching*. This geometric object will, by the author's systematization, be shown to the student as a whole, compared with others in relation to equality and inequality, decomposed into smaller cubes and replaced again, when the analysis and synthesis operations are, respectively, evident.

The last shape taught in the first series of solids, cataloged by Augusto Coelho, is the rectangular parallelepiped, “which will be generated from the cube, decomposing it into solids by means of three horizontal divisions and one vertical, conveniently directed” (COELHO, 1907, p. 86). The presentation of this parallelepiped will take place in a similar way to that of the cube: shown first as a concrete whole, then compared with the cube. Its decomposition includes cubes and cobblestones and finally the primitive solid will be recomposed.

The cylinder is the solid geometric shape that starts the second series. From it, the prism must be derived, according to Coelho (1907). Different prisms are presented, so that in the decomposition process, a hexagonal prism, for example, can be transformed into triangular prisms. As in the previous cases, the presentation ends with the recomposition of the prism from which the others were derived.

The third series of solids comprises the cone and the pyramid, the first being the primitive object, whose presentation occurs, according to what Coelho (1907) indicates, "by means of a wooden cone, using the teacher all the means to the child fix the shape" (COELHO, 1907, p. 87). Thus, the representative object of the cone allows the student to know its constituent elements, which makes it easier for him to later understand the abstract properties of this concrete element that was presented to him.

The pyramid is presented as a derivation of the cone, and this is done through the orientation that is repeated for this process: "conveniently directed sections". This highlights the importance of the domain of *geometry to teach* so that the professional knowledge of the teacher is consistent, in order to show that, just being a mathematics teacher is not enough to have knowledge about it. It is also not enough to master the knowledge *for teaching* without having ownership over the mathematics that can be mobilized from them.

The presentation of the pyramid follows reasoning like that of the solids previously mentioned. The student must first know it, without yet discussing its classifications. Pyramids of the same species are compared, agreeing to Coelho (1907), according to the relations of equality and inequality. Then the comparison takes place between the pyramid and the other solids already presented.

Then, we proceed to the decomposition, which, although the species was not insisted on, must be of a pyramid of any type in triangular. Finishing the third series of solids, this geometric teaching object is recomposed. Hence, the teaching of solids ends, which, in the three series, must have for each a reference solid presented by means of tangible objects, from which others are derived through conveniently directed sections.

The study of the pyramid is followed by superficial forms, which follow an immediately higher level of abstraction. The geometric surfaces must be “concretized in small wooden surfaces - varied colored, like those used in the Froebel system” (COELHO, 1907, p. 87). In this way, Augusto Coelho's affiliation with Froebel[11] is evidenced by systematizing geometry as a teacher's work tool, highlighting Froebel's objects as constituent elements of the teacher's professional knowledge *for teaching* geometry.

Another of these elements that also stands out in the definition of what the teacher must know *for teaching* superficial shapes is order. Coelho (1907) makes it clear that the order to be followed must be: square, rectangle, triangle, and polygons in general, so that they are understood as derivations of solid forms already studied.

Performing directed sections on the cube, the square will be presented as being derived from that, as an abstract of the cube. Different squares must be associated in such a way that new concrete squares are formed from them, when it is opportune to present students with the formation of dihedral, solid angles and the relationships between the elements that constitute them. As Coelho (1907) states, with the square being presented as an abstract of the cube “the student will start to recompose the cube by means of squares and, thus becoming a clear and defined synthesis, he will be characterized by the side of the faces, the dihedral angles, solid angles, etc., etc.” (COELHO, 1907, p. 88).

According to the systematization of *geometry for teaching* carried out by Coelho (1907), the derivation of the rectangle must have the parallelepiped as its primitive solid. Its presentation, following the reasoning already mentioned for the case of the square, should highlight it as an element that makes up the parallelepiped, in order to highlight the

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[11] “For Froebel, education could not be restricted to the mere transmission of 'verbal', conceptual and abstract knowledge, which he did not despise, but considered insufficient, as practical learning was lacking”, initiated by intuitive teaching (HEILAND, 2010, p. 43).

angles as combinations of rectangles. Finally, these groupings must recompose the primitive solid.

The pyramid is the solid shape that should give rise to the triangle. The presentation of this should count on the comparison of different triangles in terms of equality or inequality, as well as with the figures already presented. The combination of triangles must be carried out in order to form dihedral or trihedral angles.

According to Coelho (1907), “then, the triangular surfaces will be characterized in the solids, already studied, that is, the faces of the pyramids, their dihedrals, their solid angles, etc., etc.” (COELHO, 1907, p. 88). Polygons in general have several regular polygons as primitive, following the same logic of presentation already described for the other cases of superficial forms.

In these processes that make up teacher education about teaching these shapes - square, rectangle, triangles and polygons in general - we realize that decomposition and recomposition are elements of the teacher's professional knowledge *for teaching* geometry that enable, respectively, the flow of concrete (solid tangible) to the abstract (surface) and from the abstract (surface) to the concrete (the solid as a geometric object), when it is possible to perceive the presentation of properties referring to the square, such as the implication of the dihedral angles and faces for its composition.

The element of order is highlighted once again when the systematization of what should build the teacher’s knowledge *for teaching* linear forms begins, which follow the superficial and are immediately abstract to them, says Coelho (1907). The presentation of linear shapes can be done with the aid of “wooden rods - when straight, and iron wires - when curved” (COELHO, 1907, p. 89).

The first to be studied, according to Coelho (1907), should be straight lines, being presented as derived from a superficial form from which they are denoted as their abstract components. The comparison with the other

http://doi.org/10.14393/ER-v29a2022-20

forms already studied must also be made, but there is a novelty in this process in relation to the previous ones, the use of the blackboard. In this, it is possible to observe combinations between straight lines in order to characterize the angles that these combinations can form, situations of parallelism, perpendicularity etc.

After this process, successive recompositions are systematized. The surfaces should be recomposed based on the juxtaposition of lines, which will be highlighted as "linear elements of that surface, such as - sides of the polygons, diagonals, [...]" COELHO, 1907, p. 89).

After the lines are juxtaposed and compose surface shapes, which already have their properties evidenced, “solid shapes will be recomposed at the expense of the superficial ones and in those the edges, diagonals, rectilinear angles, solid angles, etc., will be characterized, etc." (COELHO, 1907, p. 89, emphasis added). Regarding the guidelines for teaching curved lines, operations similar to the previous ones will be carried out, which can be derived from curved surfaces, such as the base of the circular cone.

The last geometric shape for which Coelho (1907) systematizes guidelines for teacher training is the point, the most abstract of all. This should be represented “by objects, such as shells, cork balls, etc.” (COELHO, 1907, p. 90). The process for teaching them instructs that points can be derived from the line from sections in small portions. By aligning these in series, the line can be recomposed, “with these surfaces, with these solids - featuring in the lines, surfaces and solids, the notable geometric points, as are the vertices of the angles [...]” (COELHO, 1907, p. 90).

Thus, in the systematization of *geometry for teaching* that goes from linear surfaces to the point, it becomes even more noticeable that the teacher's professional knowledge *for teaching* geometry must consider that the learning of a new geometric shape can attribute consistency to the

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understanding of the previous ones as geometric objects, being possible to ascertain the properties of each one.

Solid shapes are presented from concrete objects, superficial ones are derived from them, and when they are recomposed, new meanings are attributed to solids, when it is possible to notice that the cube has dihedral and trihedral faces and angles, for example. When the lines are understood in superficial ways, they are juxtaposed in order to recompose the primitive, and it gains new meanings, such as the understanding that it has sides and rectilinear angles. And the study of points leads them to also recompose themselves by forming lines, which make up surfaces and solids, which then have new properties highlighted.

Thus, when returning to the cube, for example, it can be noticed as a solid shape formed by faces, angles, edges, and vertices. This again shows the importance of the decomposition and recomposition process, from going from concrete to abstract and from abstract to concrete for the constitution of *geometry for teaching* that Coelho (1907) systematized to compose the professional knowledge of the teacher of the first school years.

Up to the paragraph above we made in this section an analytical compilation of *geometry for teaching* systematized by Coelho (1907). Henceforth, we will compare it with elements of this work tool of the teacher who teaches mathematics, systematized by Coelho (1892) and Coelho (n.d.).

As we have pointed out, the parameters for this comparison are the *geometry to teach*, the knowledge *for teaching* geometry: the teaching materials indicated, also the presentation and generalization process, the teaching march; and the reference pedagogist. These elements are summarized in Table 1, below, from which it is possible to compare the constituent elements of *geometry to teach*, an element of the teacher's professional knowledge *for teaching* mathematics systematized by Augusto Coelho in the three referred manuals.

http://doi.org/10.14393/ER-v29a2022-20

TABLE 1: Elements of *geometry for teaching* by Augusto Coelho

<table>
<tr><th colspan="5">Constitutive elements of geometry for teaching by Coelho</th></tr>
<tr><th colspan="2">Manual's year</th><th>1892</th><th>n.d.</th><th>1907</th></tr>
<tr><th colspan="2">Geom. to teach</th><td>Geometric Shapes</td><td>Extension forms</td><td>Extension forms</td></tr>
<tr><th rowspan="3">Knowledge for teach geometry</th><th>Materials</th><td>Wood solids: sphere, cylinder et .; wooden boards and rods; iron wires; cork balls, black board etc.</td><td>Concrete and real forms; board</td><td>Wood solids: sphere, cone etc.; wooden boards and rods, iron wires; cork balls, black board etc.</td></tr>
<tr><th>March</th><td>From concrete to the abstract; from abstract to concrete: decompositions and recompositions.</td><td>From concrete to the abstract; from abstract to concrete: decompositions and recompositions.</td><td>From concrete to the abstract; from abstract to concrete: decompositions and recompositions.</td></tr>
<tr><th>Presentation</th><td>Through concrete objects and their handling in games; from conveniently directed sections that derive one form from another already presented; homogeneous surface solids presented in a single color and for solids formed by different surfaces, each one of a color.</td><td>Gradually detaching from its own forms.</td><td>Through concrete objects and their handling in games; from conveniently directed sections that derive one form from another already presented; the sphere painted in one color.</td></tr>
</table>

	Generalization	The generalization is not mentioned explicitly, but it can be observed in the recompositions, where, although the operations are made with objects, the rules of composition of the geometric shapes are highlighted. It presents properties, even if in an intuitive way.	Regarding geometry, he uses the expression "truly systematized" that allows us to observe that generalization is closer to the intention of abstraction, without needing a concrete object that represents them (the forms). Recomposition of forms. In the presentation it detaches the forms of tangible objects and in generalization it detaches from them to the abstract level. Drawing of figures on the board.	There is no generalization cited directly, but it can be observed in the recompositions, where although the operations are made with objects, the rules of composition of geometric shapes are highlighted. It presents properties, even if in an intuitive way.
Peda-gogist		Froebel	Froebel	Froebel

Source: Elaborated by the authors

We note that the *geometry to teach* that the author mobilizes for the composition of that knowledge is the same in all works. Although they appear with some different terminologies, the three cases refer to geometric shapes. The next elements to be compared integrate what we call knowledge *for teaching* geometry. The first to be emphasized are the teaching materials indicated. When looking at the synthesis of the 1892 and 1907 manuals, we find that they speak of such materials in more detail, but the three converge to be summarized in representations of concrete geometric shapes and the picture.

As for the teaching material element, the 1892 and 1907 manuals are more detailed about the instructions to the teacher on how to present the teaching objects. Although the undated manual is not so thorough, it makes the same idea as the others understood, which instructs that the materials mentioned are used in this presentation process, so that the concrete objects are the starting point *for teaching*, to which conveniently directed sections should be applied to derive others from them.

The idea of a color for each different surface that forms a solid, such as the cylinder, for example, is the same color for homogeneous surface solids, such as the sphere, are elements that are also part of the presentation, but that are not included in the undated manual, which does not exclude the possibility that such information may have formed the constitution of the professional knowledge of the teacher whose training was also guided by this manual.

As for the teaching march, the information in Table 1, previously illustrated, makes it evident that there is a consensus on this. The guidelines for the teacher who was in training at the normal school, regarding the order of presentation of the teaching objects, did not highlight this as the only one. While they perform decompositions, they also make compositions and recompositions. The order is from solid forms to the point, but, as the decomposition happens, recompositions must be carried out.

Regarding the generalization process, we find that it is explicitly mentioned in the undated manual, which makes it clear when presenting the systematized geometry. The aforementioned manual guides the teacher to release the abstract level of concrete objects. In this too, but in more detail in the other two, the generalization occurs from the successive recomposition of the forms when the properties are underlined. Regarding the reference pedagogist element, Froebel is mentioned constantly in the three manuals, which shows the constancy in the framework adopted by Augusto Coelho in the 15 years.

Therefore, we realized that the *geometry for teaching* systematized by Augusto Coelho between 1892 and 1907 remained stable, so that there is a clear consensus among the manuals about what the teacher

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should know *for teaching* geometry, the elements: *geometry to teach*, knowledge *for teaching* geometry: teaching materials indicated, teaching march, presentation, and generalization process; and reference pedagogist. Thus, it is possible to say that these manuals compose a vulgate among themselves (CHERVEL, 1990) about *geometry for teaching*. So, we can systematize this geometry which, independently of the manual, can be denoted as Augusto Coelho's *geometry for teaching*, which shows itself as objectified knowledge, which is next.

To conclude: systematization of *geometry for teaching* by Augusto Coelho

We finish our discussions in this article by presenting the answer to the initial question we propose to reflect on: what *geometry for teaching* can we systematize José Augusto Coelho's contributions to the constitution of the teacher's professional knowledge *for teaching* geometry in the early school years? In this way, we show characteristics of *geometry for teaching* that it remained stable in the productions of this author.

José Augusto Coelho systematized a *geometry for teaching* whose geometric basis is based on elements of Euclidean geometry. Thus, the constitution of the professional knowledge of the teacher at the early school years would require consistent training in this mathematical rubric, so that having mastery over geometric shapes, relationships and properties that underlie them as a mathematical entity was the minimum of geometry that the teacher should know so that the *geometry for teaching* could be configured as an element of his professional knowledge.

Therefore, the teacher would need to know that in order *for teaching* geometry, concentrating on the forms of extension, it would be necessary for his classroom to be equipped with materials such as Froebelian, solids, boards and wooden rods, iron wires, cork balls. In addition, it was necessary to know when to use them and how to do it.

Firstly, these objects should be used in the presentation, handled in games, and the primitive forms should be sectioned conveniently to make them derive from others. Therefore, in Coelho's *geometry for teaching* the presentation of geometric shapes occurs in an intuitive way.

Regarding the solid forms of the sphere, the cube is obtained, from this, the rectangular parallelepiped; from the cylinder, the prism is generated; from the cone, the pyramid is formed. In superficial forms, the square is derived from the cube, the rectangle of the parallelepiped, the triangle of the pyramid. As for the linear shapes, the lines will be sectioned from a surface by lines parallel to each other, and the curved lines will be presented from curved surfaces as the base of the circular cone. The point must be derived from the line.

Notably, the order of presentation of geometric shapes disposes them according to their level of abstraction, from solid shapes to the point. The forms are sequenced from the concrete to the abstract. This process is done through decomposition, advancing from the whole to the parts, which characterizes the analytical march. On the other hand, from the presentation of the superficial forms there is a return from the abstract to the concrete, recomposing the solid forms, gradually going from the parts to the whole, which characterizes the synthetic march.

In this process, there is a gradual process of generalization of the properties of geometric shapes. After presenting the square as a derivative of the cube, it is already recomposed, highlighting the square as its abstract element, its faces, which together form dihedral and trihedral angles that constitute new concrete. Therefore, the return from the square to the cube gives it new meanings. Similarly, it happens with the rectangle and the parallelepiped, the triangle, and the pyramid.

As the order of presentation approaches the most abstract geometric shape, the point, plus the return of the abstract to the concrete, attributes to each type of shape studied its geometric properties. Thus, when the point is presented as a derivation of the line, successive recompositions show that

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the line is formed by a succession of points, straight lines can be juxtaposed forming rectilinear angles and characterizing the sides of the surfaces, which in turn can be combined and compose dihedral and trihedral angles forming solid shapes, like the cube.

In the last process of recomposition, thus, the teacher will be able to present, for example, the cube to the students so that they understand it as a solid shape composed of vertices, edges, dihedral, and trihedral angles and six square faces, and the congruence relations between these elements. The teaching materials that we are talking about are used in this generalization process, from the concrete to the blackboard.

In recompositions, the passage from the abstract to the concrete corresponds to the passage of the more abstract forms of order represented by tangible objects for their understanding as a geometric object itself, which composes other objects by relations between angles and faces, for example. So, this return to concrete is more associated with the understanding of the geometric properties that underlie the composition of forms than with tangible concrete, so that the empirical notions are becoming rational. According to Augusto Coelho's ideas, this recomposition by synthesis is the general decomposition of geometry according to mathematical logic.

It is important to feature the pedagogist who was a reference for Augusto Coelho in the systematization of geometry as a work tool for the teacher because this allows us to associate this author with the teaching method, intuitive in this case. The knowledge *for teaching* geometry is associated with Froebel's ideas. Although Augusto Coelho uses the empirical expression instead of the intuitive expression because he considers that intuition means to emphasize only the vision and not all the senses, the variation of colors according to the homogeneity or not of the surfaces to highlight the relationships between them in the composition of the forms, shows the importance of the exercise of vision in the work developed by the teacher.

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Therefore, the *geometry for teaching* that remained stable in Augusto Coelho's production can be characterized as intuitive-traditional *geometry for teaching*, because according to the way he guides teachers, teaching objects must be presented to students in an intuitive way, but considering the orientation for the entire teaching process, geometry as an element of professional knowledge is not limited to intuitive aspects.

From the presentations, teachers must do a kind of retrospective recomposing the shapes and showing generalizations for their composition. Augusto Coelho's *geometry for teaching* is intuitive, but it also presents properties for generalizations and systematization of geometric shapes as ideal, abstract objects. It is important to highlight that this knowledge to train the teacher *for teaching* geometry shows itself as an objectified knowledge, being capable of mobilization in any context.

Considering that this article contains partial results from the first author's doctoral thesis, these elements of *geometry for teaching* that Augusto Coelho aims in his manuals are important information so that we can systematize this work tool of the teacher who teaches mathematics over a wider period, which includes those publications.

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Received in June 2020.

Approved in November 2020.