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Thereza Pereira Rocha's Writings: from Pestalozzi to Thorndike, knowledge to teach arithmetic problems[1]

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ABSTRACT

This article proposes to observe the systematizations in the constitution of professional knowledge, based on appropriations made by Thereza Pereira da Rocha, in her notebook dated 1958, in which she recorded notes of an improvement course. The following concepts were used as methodology: *processes* (BURKE, 2016), *professional knowledge* (HOFSTETTER; SCHENEUWLY, 2017) and, closer to this theoretical objective, the concepts of *Mathematics for teaching* and *Mathematics to be taught* (BERTINI; MORAIS; VALENTE, 2017). It is understood that the knowledge recorded in the notebook refers to precepts prior to the production of this document, more specifically to two authors: Pestalozzi and Thorndike. The first author marks the dissemination of the intuitive method in the final years of the 19th century in Brazil. The second, based on advances in child development, proposes changes in relation to the teaching of arithmetic in the early 20th century. As conclusions, it is indicated that in the notebook there are precepts spread by these authors, which means that the teaching of arithmetic did not simply discard previous ideas, but incorporated them into the so–called "modern" teaching.

KEYWORDS: School notebook. History of Mathematics Education. Professional Knowledge. Arithmetic Problems.

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Escritos de Thereza Pereira Rocha: de Pestalozzi à Thorndike, saberes para ensinar problemas aritméticos

RESUMO

Esse artigo propõe observar as sistematizações na constituição do saber profissional, com base nas de apropriações feitas por Thereza Pereira da Rocha, em seu caderno datado do ano de 1958, no qual registrou anotações de um curso de aperfeiçoamento. Utilizaram–se como metodologia os seguintes conceitos: *processos* (BURKE, 2016), *saberes profissionais* (HOFSTETTER; SCHENEUWLY, 2017) e, mais próximo desse objetivo teórico, os conceitos de *matemática a ensinar* e *matemática para ensinar* (BERTINI; MORAIS; VALENTE, 2017). Entende–se que os saberes registrados no caderno remetem a preceitos anteriores à produção desse documento, mais especificamente a dois autores: Pestalozzi e Thorndike. O primeiro autor marca a disseminação do método intuitivo nos anos finais do século XIX, no Brasil. O segundo, a partir dos avanços a respeito do desenvolvimento infantil, propõe mudanças em relação ao ensino de aritmética no início do século XX. Como conclusões, indica–se que no caderno há preceitos difundidos por esses autores, o que representa que o ensino de aritmética não descartou simplesmente ideias anteriores e, sim, incorporou–as no ensino dito "moderno".

PALAVRAS–CHAVE: Caderno escolar. História da Educação Matemática. Saber Profissional. Problemas Aritméticos.

Los escritos de Thereza Pereira Rocha: de Pestalozzi a Thorndike, saberes para enseñar problemas aritméticos

RESUMEN

Este artículo propone observar las sistematizaciones en la constitución del saber profesional, a partir de las apropiaciones realizadas por Thereza Pereira da Rocha, en su cuaderno de 1958, en el que registró notas de un curso de perfeccionamiento. Se utilizaron como metodología los siguientes conceptos: *procesos* (BURKE, 2016), *conocimiento profesional* (HOFSTETTER; SCHENEUWLY, 2017) y, más cerca de este objetivo teórico, los conceptos de *matemáticas para enseñar* y

matemáticas a enseñar (BERTINI; MORAIS; VALENTE, 2017). Se entiende que los conocimientos registrados en el cuaderno se refieren a preceptos anteriores a la producción de este documento, más específicamente a dos autores: Pestalozzi y Thorndike. El primer autor marca la difusión del método intuitivo en los últimos años del siglo XIX en Brasil. El segundo, basado en los avances en el desarrollo infantil, propone cambios en relación con la enseñanza de la aritmética a principios del siglo XX. Como conclusiones se indica que en el cuaderno hay preceptos difundidos por estos autores, lo que significa que la enseñanza de la aritmética no simplemente descartó ideas anteriores, sino que las incorporó a la enseñanza llamada "moderna".

PALABRAS CLAVE: Cuaderno escolar. Historia de la educación matemática. Saber profesional. Problemas aritméticos.

* * *

Introduction

Search, preserve, sanitize, catalog, submit to the Digital Content Repository (RCD)[4], read and analyze documents are some guiding steps of the research work of the History of Mathematics Education Group in Brazil (GHEMAT). Through a Thematic Project[5] of national level, GHEMAT proposes to investigate professional knowledge, researching school documents.

Therefore, from these documents[6], inserted in the RCD, used as sources in academic research, this article seeks to emphasize how the teaching of arithmetic problems for the primary course in the late 1950s may have been based on authors of pedagogical manuals from the early 20th century.

[4] The RCD is constituted as a database, allocated in the virtual servers of the Federal University of Santa Catarina (UFSC). According to Giusti, Godoi and Costa (2020), it is a digital platform, with free access to the public and researchers, managed and fed by members of the History of Mathematics Education Group in Brazil (GHEMAT). "It is an online database that gathers, in an organized manner, historical documentation obtained from Brazilian collections, as well as the productions of the members of GHEMAT (articles, dissertations and theses)" (2020, p. 3). The access can be made through the link: https://repositorio.ufsc.br/handle/123456789/160300.

[5] The Thematic Project entitled "Mathematics in teacher training and teaching: processes and dynamics of production of professional knowledge, 1890–1990", authored by Valente et al. (2017), has funding from the Fundação de Amparo a Pesquisa do Estado de São Paulo (FAPESP). Process n. 17/15751–2, in effect from December 1, 2017 to November 30, 2022.

[6] Among these documents are: pedagogical manuals, textbooks, pedagogical magazines, school legislation, teaching programs, school notebooks, tests, exams, evaluations, several materials coming from personal collections, among others.

The Thematic Project has several academic researches[7] at the national level, which have in common the general objective: "to investigate the processes and dynamics of constitution of the teacher's professional knowledge who teaches Mathematics in the period between 1890–1990" (VALENTE et al., 2017, p. 30) which seek to answer the questions that guide this broad investigation. They are: "How are professional knowledge of the teacher who teaches Mathematics produced, systematized and institutionalized? How to characterize Mathematics as professional teaching knowledge". (VALENTE et al., 2017, p. 10).

To answer these questions, and characterize the professional knowledge of the professor who teaches Mathematics in these hundred years, we highlight the research of the authors of this article, the doctorate concluded by Bruna Lima Ramos Giusti; and, the ongoing doctorate of Andréia Fernandes de Souza. Both aim to characterize the professional knowledge of the teacher who teaches Mathematics in the early school years.

In order to analyze the constitution of this knowledge, as proposed in the Thematic Project, the following were elected as theoretical–methodological tools: concepts derived from Cultural History, from Socio–Historical studies and from the History of Education, as well as the History of Mathematics Education. From this theoretical basis we can highlight: *processes* (BURKE, 2016), *professional knowledge* (HOFSTETTER; SCHENEUWLY, 2017) and, closer to this theoretical objective, the concepts of *Mathematics to be taught and Mathematics for teaching* (BERTINI; MORAIS; VALENTE, 2017).

In this context, to characterize a Mathematics *to teach* and *for teaching*, the analysis of school documents is the initial framework of this research. Giusti's thesis (2020) presents the analysis of some notebooks of normalists of Practice, more specifically the Methodology of Arithmetic teaching, from the 1950s. The author seeks to understand the professional knowledge they acquired during the course of the normal school and that

[7] Among scientific initiation, master's, doctorate and post–doctorate.

could be systematized to use it when they became primary school teachers. The article by Souza (2020), which contains partial results of the author's thesis, analyzes articles from pedagogical journals between the 1890s and 1960s that approached the teaching of arithmetic problems. The intersection point of the studies carried out by both authors is the constitution of the professional knowledge of the teacher who teaches Mathematics, particularly the arithmetic problems of the primary course, as can be seen in a first meeting movement in Souza and Giusti (2019).

For this article, the Practice notebook, by Thereza Pereira da Rocha, produced in 1958, during a course of improvement, is resumed. This notebook was the target of Souza and Giusti's study (2019), when the authors intended not only to relate the teaching of problems in it, with the guidelines of teaching programs and pedagogical magazines of the 1940s and 1950s, but also to find elements that characterized the *Mathematics to be taught* and the Mathematics *for teaching* the arithmetic problems. In other words, it sought elements that could make professional knowledge noteworthy for the teaching of problems, put into practice in the 1950s.

To fulfill these objectives – to characterize these *professional knowledge*s – the study is based on the research methodology, conceptualized by Peter Burke (2016, p. 74, emphasis added), which states that historiographic research can be divided into "[...] four main stages of the sequence from obtaining to using information: *collection, analysis, dissemination and use* [...]". These stages, described by Peter Burke (2016), are methodological steps, often difficult to be inserted in qualitative research. Because of this, initially Valente (2018) and then Lima and Valente (2019) appropriated these phases and brought them into the history of Mathematics education, proposing methodological procedures that best fit the research that deals with professional knowledge.

Thus, with the objective of characterizing the processes and dynamics of building professional knowledge, Valente (2018) indicates the following stages of work: *recompilation of teaching experiences,*

comparative analysis of teaching knowledge, systematization and analysis of the use of expertise as knowledge (VALENTE, 2018).

The *recompilation of teaching experiences* is when the researcher will make "selection and separation of information" in documents, in order to "highlight information about the teachers pedagogical work" (LIMA; VALENTE, 2019, p. 940), and to gather it "dispersed knowledge in a given historical time" (2019, p. 940).

The *comparative analysis of teaching knowledge* is when the researcher can make a new selection from what was done previously. This new inventory can indicate trends in teachers' experiences at a certain time, which will allow the construction of pedagogical consensus on the teaching profession (LIMA; VALENTE, 2019, p. 941).

Finally, the *systematization and analysis of the use of expertise* as *knowledge* is when the researcher will gather all this expertise used by teachers, which can be generalized, i.e., capable of being systematized at a given time, and characterize it as professional knowledge of the teacher who teaches Mathematics. According to Lima and Valente (2019, p. 941), this stage also includes the "verification in normative and/or didactic–pedagogical instances of the occurrence of the use of the elements systematized by the researcher".

In any case, the use of these three stages or procedures will help characterize the professional knowledge of the teacher who teaches Mathematics, which is established from the relationship between *the Mathematics to be taught and the Mathematics for teaching*.

This analysis methodology has already been used in some concluded GHEMAT researches[8], as in the analysis of the notebooks of normalists made by Giusti (2020), including the notebook of Rocha (1958). From these researches it was possible to systematize, characterize and discuss the professional knowledge of the teacher who teaches mathematics in the early school years. For this article, the focus will be on the teaching problems, present in this notebook from the 1950s, based on authors from the late 19th and 20th centuries.

[8] As can be seen in the studies of Maciel (2019) and Giusti (2020).

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From Pestalozzi to Thorndike: brief history

For a long time teaching was marked by some defined characteristics, such as: elitization of contents, encyclopedism, memorization, catechetics, triggering of ideas through the organization of the discipline itself, among others. These characteristics come from a model, said and recognized as being traditional (SAVIANI, 2011).

In this way, teaching had the appearance of those molds used in catechizing the indigenous people, which consisted of being able to teach and learn Christian principles through questions and answers. These characteristics remain and coexist with others until the present day.

At a certain moment, the State took for itself the responsibility of offering education as a right, and society began to glimpse education as a form of social ascension and improvement of living conditions. At the same time, a movement for change in the ways of teaching was emerging.

In the middle of the 19th century, there was a worldwide need to create a new teaching method that could overcome traditional pedagogy, as Valente points out (2008, p.1):

> A new teaching method was sought. It would come accompanied by new materials, the creation of Pedagogical Museums, pedagogical excursions among other activities. This new method was exhibited in the Universal Exhibitions, organized for the diffusion of renewed pedagogical practices, didactic materials and their applications. The new method was characterized by the proposal of a concrete, active teaching, to be called intuitive teaching.

The intuitive method, with Swiss educator Johann Heinrich Pestalozzi (1746–1827) as its greatest promoter, seemed an alternative and a great bet for the improvement of teaching.

This method had as main foundation the valuation of intuition, privileging the senses, the observation for the acquisition of knowledge, the experimentation, the concrete, opposing to the so called traditional teaching.

> The so–called "intuitive method" owes this name to the accentuated importance that its advocates gave to intuition, to observation, as the first and irreplaceable moment of human learning. [...] the defenders of the intuitive method called attention to the importance of the observation of things, objects, nature, phenomena, and the need for the education of the senses as fundamental moments in the process of school instruction. (FARIA FILHO, 2003, p. 143, author's emphasis)

For Pestalozzi, intuition was not merely a passive observation of objects, but it also included intellectual activity, with questions that led to the integral development of children.

Soetard (2010, p. 89) states that "every philosophy carries with it a pedagogy; and the opposite, every pedagogy supposes a philosophy". With this quote, he justifies that the pedagogy proposed by Pestalozzi had a great relationship with the thought of Immanuel Kant[9].

Since the intuitive method presented Pestalozzi's aphorisms as its presuppositions, which prioritize learning, starting "from the concrete to the abstract," "from the easy to the difficult," "from the known to the unknown," "from the perception of things through direct contact" (ZANATA, 2012, p. 107), education could not be repressive and was the means for cognitive, affective, and character abilities to be developed.

Pestalozzi thought about education in a broad way, and the intuitive method could be used in all disciplines and for all students. And because of this, his ideas circulated in Brazil since the end of the 19th century through teaching programs, school books, articles in pedagogical magazines and in courses for teacher training. He changed the precepts of traditional pedagogy, seeking to look at children and educate them from the senses.

Edward Lee Thorndike (1874–1949), American, who was born after the end of the American Civil War in Massachusetts. Thorndike

[9] Immanuel Kant (1724–1804) is considered the main philosopher of the modern era who synthesized rationalism and English empirical tradition (which was based on induction), according to Soetard (2010).

"graduated from *Wesleyan University* in 1895; completed his master's degree at Harvard in 1897; and his doctorate at Columbia in 1898" (RABELO, 2016, p. 49).

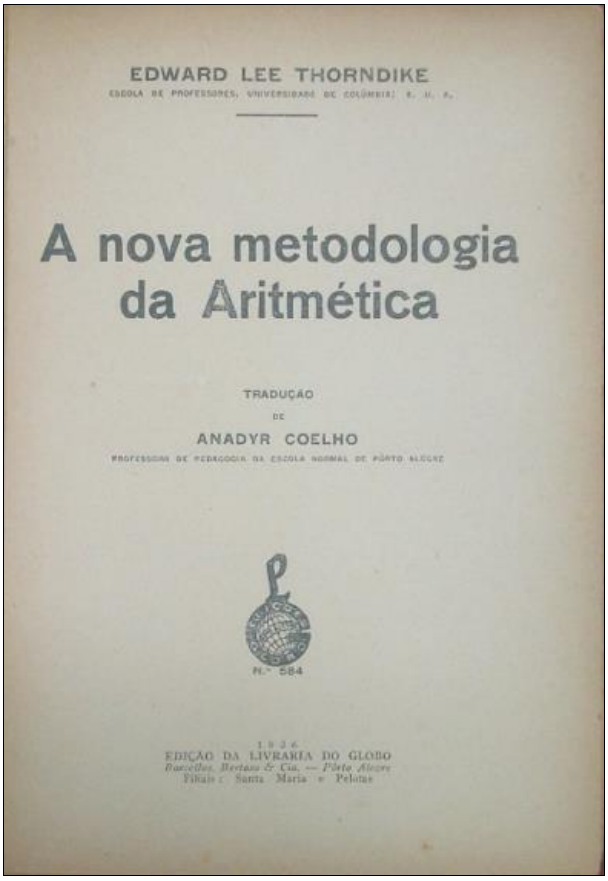
In 1917 he published *The Thorndike Arithmetics*, but there is evidence of his appreciation for the subject in previous years, as Rabelo points out (2016, p. 74), "it is possible to go back at least to 1909, when he published the booklet *Exercises in arithmetic, selected, graded and arranged to meet the requirements of the hygiene of the eye and neuro–muscular apparatus*". And Thorndike's interest in Mathematics continued over the years, publishing the books *New methods in arithmetic*, in 1921, and, in 1922, *Psychology of arithmetic.*

Thorndike was hired as a professor at *Teachers College*, *Columbia University*, in 1899. In 1920, in recognition of his published works, he was invited to assist in the implementation of policies regarding school curricula in the United States. His ideas, arising from experimental and connectionist psychology[10], seen as a new psychology, were focused on mother tongue teaching and Mathematics.

In Brazil, it is possible to perceive the appropriations and circulations of this author[11]. One of the signs of this interest in Thorndike's writings is revealed by the translation of his work, published in 1936, *The new methodology of arithmetic*, by Anadyr Coelho. This book was widely quoted in articles in pedagogical magazines at the time and appeared as a reference in Teaching Programs until the mid–1960s.

[10] These studies were based on the stimulus–response relationship, use and disuse, reactions from the effects of learning (RABELO, 2016).
[11] See Rezende (2016).

FIGURE 1: Cover sheet of the book *The New Methodology of Arithmetic*, by American author Edward Lee Thorndike



Source: Thorndike (1936). Available in: https://repositorio.ufsc.br/handle/123456789/134890.

Thorndike presents several interesting lines in this book and it seems strange that it shows the reader two facets of school methods at all times: the old and the new.

His criticisms are around old methods used for teaching arithmetic. He also makes comments about the teaching of the mother tongue, alluding to the synthetic method in which the letters are learned, then syllables, words, phrases and texts, indicating this same logic for arithmetic. He criticizes the repetitions and the boring exercises, as follows:

> The Law of Exercise, thus enunciated: use fortifies and disuse weakens mental connections. And the Law of Effect, thus enunciated: The connections accompanied or followed

> by states of satisfaction tend to strengthen; the connections accompanied or followed by states of boredom tend to weaken. (THORNDIKE, 1936, p. 78.)

Unlike Pestalozzi, Thorndike appropriated a referential of conexionist psychology, and built a way of teaching arithmetic. In other words, he elaborated a knowledge for teaching arithmetic that would be used in schools during a period of the 20th century.

Having as a parameter records made in a notebook, produced in a normal school, it can be affirmed that Pestalozzi, born in the 18th century, and Thorndike, born in the 19th century, would live together, in the writings of Thereza Pereira Rocha in the 20th century.

Writings of Thereza Pereira Rocha: Practice Notebook, Arithmetic Methodology, São Paulo, 1958

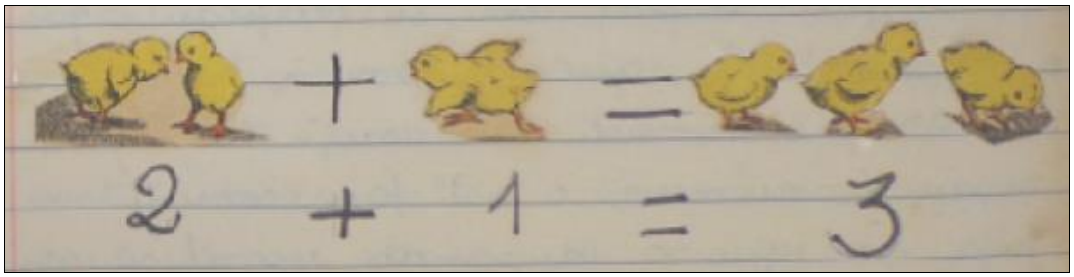
Thereza Pereira Rocha's notebook (1958)[12] was produced in a Improvement course, held in São Paulo. She was a normalist primary school teacher, and this improvement course was specific for teacher training. The notebook analyzed is from the curricular component of Practice, that is, it has records of several subjects, among them, there is the "Methodology of Arithmetic". At the end of this notebook, it is possible to see a part destined to a class diary, probably used in class with students of the primary course.

Because it is a Practice notebook, it can be seen how the normalist intended to teach some contents of arithmetic, among which: unit, ten, hundred, the four operations (sum, subtraction, multiplication and division), footage, fractions and problems. In addition, it is also possible to verify some considerations regarding the method to be used, as well as indications of pedagogical manuals and teaching programs. It seems that this notebook was built based on the instructions provided by that time.

[12] This notebook is scanned in the Digital Content Repository (RCD) and can be accessed through the link: <https://repositorio.ufsc.br/handle/123456789/163509>.

Throughout this notebook there are several colorful illustrations, drawn in crayons or collages.

FIGURE 2: Drawings representing quantity for arithmetic acquisition



Source: Rocha (1958, p. 8)

The normalist states that using illustrations serves to make the child relate the quantities to the corresponding numbers. This would be a process of initiation of arithmetic abstraction. According to Rocha (1958), the method used previously, since the 1920s, to teach arithmetic to children was the deductive.

> In primitive times, it was thought that the teaching of arithmetic had to be exclusively deductive. Based on this conception, it was that, until very recently, the subtraction was processed as follows:
>
> Ex: $\begin{array}{r} 123 \\ -\ 98 \\ \hline 25 \end{array}$
>
> This operation was carried out as follows: of 3 you cannot take 8, 3 lends 1 from 2, which is worth 10, and thus 3 becomes worth 13; of 13 you take 8 = 5. 2 becomes worth 1. You cannot take 9, so you lend 1 of the next house, which is worth 10, and thus 1 becomes worth 11. Of 11 you deduct 9 = 2. 1 is worth zero, and the calculation is completed (deduction). (ROCHA, 1958, p. 4–5).

Within, Rocha (1958) points out that this method was being replaced by the inductive method. According to Maciel (2019), the *inductive* method consisted in memorizing through repetition. It was widely used until the end of the nineteenth century, because teachers had difficulty in replacing this method by the "new method," that is, the intuitive method. That is, Rocha

(1958) criticizes this method of repetition, which seems to have remained for a long time in the teaching of arithmetic.

In Rocha's notebook (1958), the normalist mentions a work of Thorndike, The *methodology of arithmetic*, and states, based on this work, that the child needs to "count things, such as balls, pebbles, etc". (1958, p. 5), this would be an appeal from this author to "observe reality".

Still based on Thorndike's precepts, the author points out that "the old methods presented a great flaw. They taught arithmetic by arithmetic, without regard to the realities of life" (1958, p. 5). This is because the previous methods presented "kilometric accounts and useless subtleties" *(ibid).*

The arithmetic problems appear throughout this notebook. The normalist records that the inductive method recommended the use of "real problems, calculations that constantly appeared in daily life" (ROCHA, 1958, p. 5), because in the previous method, the deductive, the problems considered satisfactory were the following:

> 1) Alice had $^{3}/_{8}$ of a thousand réis, Berta $^{14}/_{16}$, Maria $^{3}/_{25}$ and Nena $^{3}/_{4}$. How much did they have together?
> "Only in a madhouse, a problem with these characteristics could appear" (Thorndike).
> 2) A man is 1.80m tall and weighs 83 kgs. What will be the height of his wife, knowing that her weight is 62 kgs and her stature is proportional to that of his husband?
> The author says: "This problem is futile and extravagante". (ROCK, 1958, p. 6).

The author, to whom the normalist refers, is Thorndike. About the problems, Rocha (1958) states that, even in the 1950s, there were still these types of real problems: "I spent 2/3 of the money I had and 1/5 more of the rest. How much did I have, knowing that I still came home with *x*?" (1958, p. 6). The normalist points out that, although it is a type of real problem, it is unlikely to occur in real life. The author explains that Thorndike says that the new methods ask for a teaching "in absolute harmony with the real situation of life" *(ibid).*

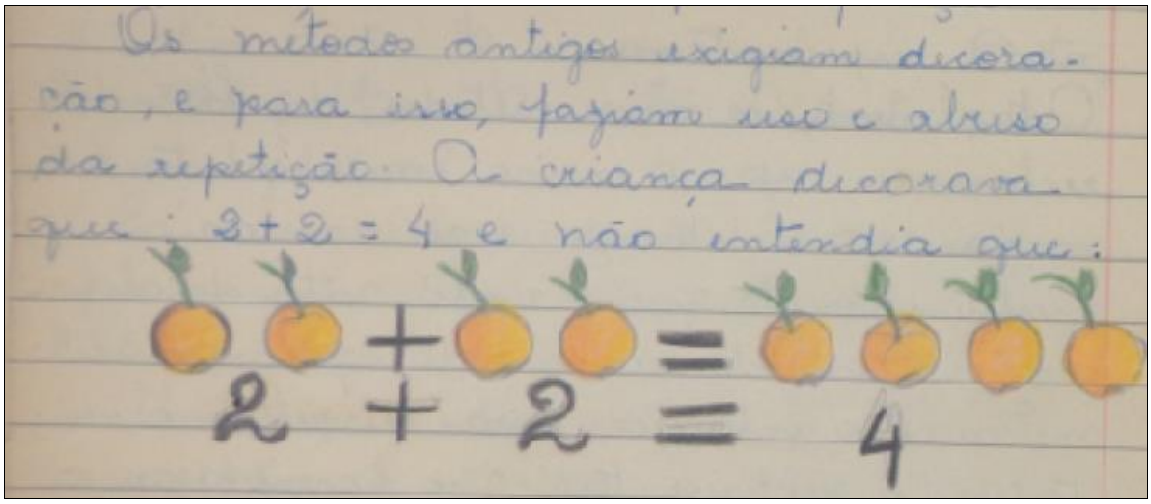
Therefore, according to Rocha (1958), Thorndike indicates that an important item in the methodology of arithmetic is the *interest*. The teacher should awaken it in the children, so it was not recommended the use of very extensive problems to copy, because in this way they could lose interest when calculating them.

The suggestion would be to use "graduated notebooks", or mimeographed paper. These resources would serve to save the children's energy, so that they could use it to solve the calculations. Rocha (1958, p. 7) reiterates that it is necessary "understanding, not simple repetition". The normalist seems to emphasize that the child must understand the problem and, for this, should use their interest to involve them in teaching arithmetic.

The author speaks, from Thorndike, that the arithmetic initiation must be done in three phases: "The first consists in the presentation of reality, the second in the presentation of drawings corresponding to reality, and finally the presentation of symbols or graphic signs corresponding to reality" (ROCHA, 1958, p. 7).

That is, first it would be to show children the real objects, for example, an orange. Then, the drawing of an orange. And only then would they present the numbers, so that they could relate them to the objects studied, because only by presenting the abstract form (the numbers) would the children not understand the meaning of the reason 2 + 2 be equal to 4.

FIGURE 3: Concretely explained addition



Source: Rocha (1958, p. 7)

Through Figure 3, it is possible to see that the child will be able to understand by the visual form that two oranges plus two oranges will be four oranges. Apparently, the normalist is in favor of observation with objects, to then teach the numbers, as Thorndike recommended.

For this normalist, teaching should start from the concrete to the abstract. An example of this would be to show orange illustrations and add them up. Thus, the child would be able to visualize the quantity and add up to the oranges at the end. A more abstract problem would be to ask the child how many siblings she/he has, so she/he would have to imagine the quantity and speak the number.

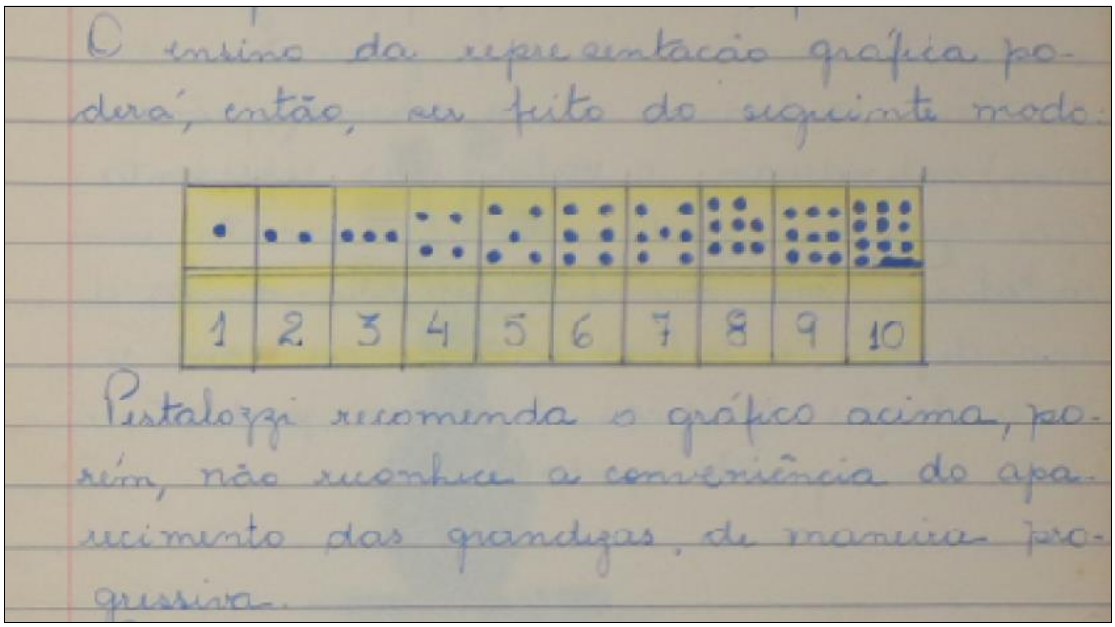
The author indicates that these kinds of problems could be done orally, or use drawings made on the blackboard or posters, as a didactic resource. Then, initially the students would see the picture and would have to say the word "three". But over time, they would learn to write and relate this word to the graphic form "3". The normalist emphasizes that the child needs to understand what the problem is asking for and not just solve it by mere repetition.

In this same notebook, however, the author also defends the use of Pestalozzi's conceptions. For mathematical initiation, this author points out three distinct phases: first, to verify what the child knows as soon as she/he

enters school; then, to give her/him the notion of number; and finally, to make her/him recognize group of things and objects (ROCHA, 1958, p. 9). It is understood by this orientation that only after these three elementary notions should one begin to teach the four fundamental operations.

For the teaching of graphic representation, the author states that Pestalozzi recommends the *associated* form, that is, "the graphic representation is associated with the corresponding drawing" (ROCHA, 1958, p. 10). Initially, the normalist suggests the use of ball drawings, "because it is easier for the child to copy" (1958, p. 10), to then make drawings of more "animated" objects for children, such as butterflies, birds, fish, etc.

FIGURE 4: Example of Pestalozzi's chart in Rocha's notebook (1958)



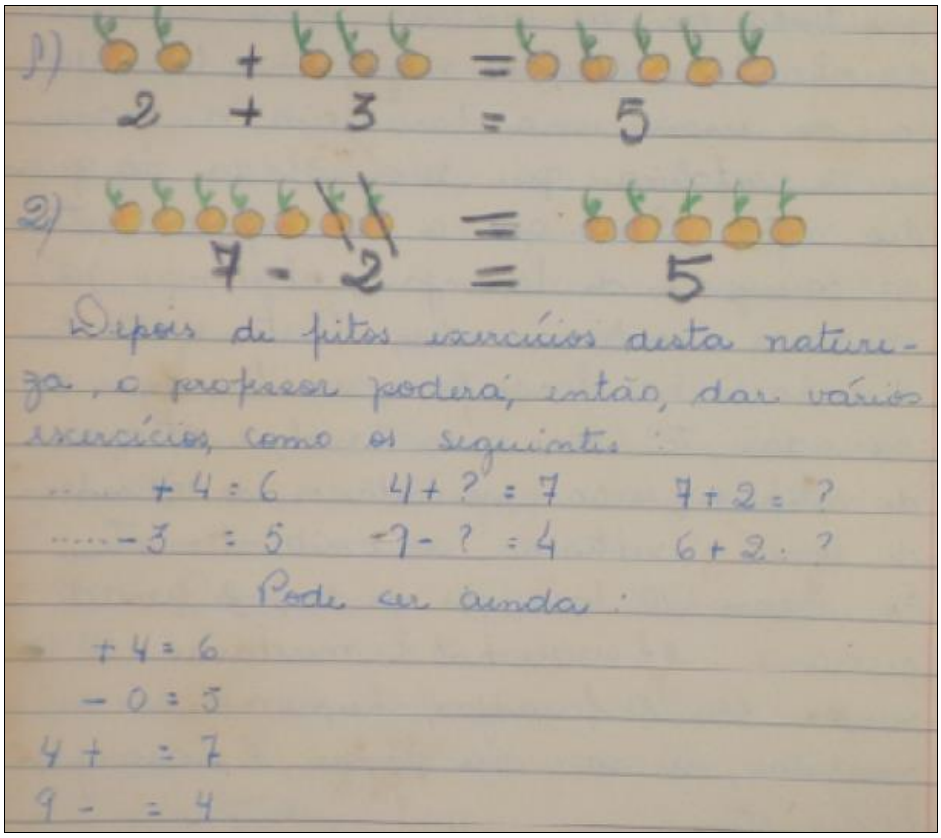
Source: Rocha (1958, p. 10)

In order for children to master the graphical representation of numbers, Pestalozzi recommends other steps, as the author records: "1. careful observation of the graph that represents at the same time, the quantity and its respective graphical representation" (ROCHA, 1958, p. 10), that is, the graph presented in Figure 3, which contains the quantities and numbers. "2. lightly delete the signs representing the quantities so that the corresponding graphic signs better

impress the sense of view" (1958, p. 10) and "3. completely delete the quantities and require the repetition of the reading and copying of the graphic representations already studied" (1958, p. 11). It is understood that this would be to gradually erase the drawings until leaving only the figures.

The normalist Rocha (1958) indicates in the notebook that the recommendation given at the time was that the teaching of the four operations appear concomitantly. Everything indicates that this would be the first phase, from concrete objects. After that, it would be possible to proceed to the drawings, where the child sees a relationship between the object and a drawing of this object, as can be seen in Figure 5.

FIGURE 5: Processes for teaching addition and subtraction from concrete



Source: Rocha (1958, p. 14)

Figure 5 shows that the normalist understands that teaching numbers from drawings would be a way for the child to understand them. After this phase, the teacher could give exercises only with

numbers, adding or subtracting, either to find the final result or one of the missing numbers in the account.

In the notebook of Rocha (1958), it is pointed out that one of the contents of primary education would be the problems. To solve or exemplify some of these, the normalist uses objects or illustrations. To introduce them to children, one would first have to pass the stage of arithmetic initiation, which consisted of showing the object, then the drawing of this object and only then relate the quantity to the numbers.

According to the normalist, "the notions of division must be presented before the notions of multiplication. The child understands division more easily than multiplication" (1958, p. 16). She proposes an exercise to introduce the notion of division as a first step. The activity consists of the teacher holding eight pencils in her hands and asking the children "what is the half of 8?", and after "various tricks", the children could answer that the half of eight is equal to four.

Another activity that the normalist suggests for the teacher to teach the notion of multiplication would be to draw three trees, each with three fruits. And, based on this drawing, she/he could ask the questions. Perhaps the children would count the amount of fruit, but this would make them understand the meaning of multiplication and then the multiplication table.

In this notebook, after presenting all the arithmetic content that should be given to children, the normalist makes a comparison of the first and last exercise that a textbook, entitled *Raciocine com a criança*, by Claedmar French, proposes for each year of school, presented in Chart 1.

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CHART 1: Suggested exercises and serial problems

	First exercise	Last exercise
1st grade	Draw little balls for the children to identify the corresponding numbers (up to number 5).	Problem: "One piece of lace measures 27 meters. How tall are 3 equal pieces?"
2nd grade	Teaching the unit, ten, hundred and thousand, by means of a picture.	Problem: "From a basket with 2 hundred oranges, I took the 5th part to distribute among 8 children. How many oranges did each child receive?"
3rd grade	Problem: "How much are 43 pears at Cr$1.60 each?"	Problem: "The $\frac{2}{8}$ Cr$298.50 is the price of one meter of silk. How much will I pay for 3,40m?"
4th grade	Problem: "The price difference between 2 objects is Cr$149.00. Knowing that the most expensive cost Cr$324.00, what is the price of the other?"	Problem: "I bought a wallet for Cr$180.00 and I sold it for Cr$130.00. What is the percentage of loss?"

Source: produced by the authors from Rocha (1958, p. 21–22)

For the 1st year, the normalist says that she prefers to work with oranges instead of balls with children. However, for the other exercises and problems she makes no comment on the indication that the textbook *Raciocine com a criança* proposed, although she criticizes problems that are not real and defends that they have to be the interest of the child. These problems, presented in Chart 1, from this textbook, seem to approach real life, but it is not known if they are in the interest of the children.

These mathematical contents, pointed out by Rocha (1958), emerge as a teaching object, which, according to Valente *et al.* (2017) can be called a Mathematics *to be taught*.

Rocha (1958) also suggests that the teacher use other documents to prepare their primary classes, as *Começando a calcular*, by Lília N. P. Visani; *Nossa vendinha*; o *Programa Escolar*; o *Manual do ensino primário*, de Miguel Milano; and others.

From Pestalozzi to Thorndike, knowledge to teach arithmetic problems

It is noted that in the Rocha's notebook (1958), the authors Pestalozzi and Thorndike transit, subsidizing professional knowledge. Pestalozzi systematized a teaching work tool, the intuitive method, based on the senses and the most sensitive look with those who are learning. Thorndike, in the same way, elaborated a tool, from the researches related to the conexionist psychology, a *Mathematics for teaching*, taking into account aspects such as child interest. And such tools are present in the records of Rocha (1958) and denote the construction of a professional knowledge of the teacher who teaches Mathematics in the early years.

The use of illustrations in classes demonstrates the concern with the child's understanding, since numbers and quantities are abstract questions for the child at the beginning of learning, as well as the definitions of numbers that were present in arithmetic books in times of traditional pedagogy.

The idea was to respect the graduation of the student's development and not the development of the curricular component. Mathematics was presented in traditional teaching, with definition, rules, examples and exercises. Therefore, to think of concrete objects to walk to the construction of the concept of number would be to move from the easy to the difficult.

In this scenario, using resources such as illustrations, concrete objects, Parker's maps, etc., would be an easy way to move from the easy to the difficult.[13], to have a sequence to teach the fundamental operations, to graduate the teaching, were seen as innovations in the period. In Rocha (1958), you can see that the normalist indicates the use of some teaching resources, among them are the maps of Parker.

This graduation, as shown in the notebook of Rocha (1958), can be seen when it has a model of arithmetic problem for the beginning and end of the school year, as illustrated in the Chart 1.

[13] Sequenced posters with the purpose of teaching the numbers and operations. See Valente (2008).

Thorndike (1936) brought detailed analysis of fundamental operations, but argued that even students who knew how to perform calculations might not be able to solve problems. Throughout his book, this author defended interest and motivation as a means of point the curiosity of children to learn. To do so, the problems would be excellent proposals, as long as they followed the recommendations bellow:

> Every problem should preferably (1) deal with situations that are likely to occur many times in real life; (2) treat them as they would in practical life; (3) present them in a way that is neither much more difficult nor very easy. (THORNDIKE, 1936, p. 153).

Rocha (1958) also has strong influence of this author, because she is concerned about the interest of the child. With this triad, Thorndike (1936) proposed possible situations both to happen and to be solved by each child.

She also notes that the teacher would need to observe three elements in the solution of a problem: "(1) the exact understanding of the issue, (2) the knowledge of the facts that should be used to solve it, (3) the use of these facts in correct arithmetic relations" (THORNDIKE, 1936, p. 154).

Another highlight given by the author is that not every problem, just by the fact of carrying in its enunciation words of everyday childish life, would arouse their interest. It was necessary to mobilize other forms. In the normalist's notebook, the same criticism can be noticed regarding the everyday problems, since some of them would not make much sense to the children. Although she agrees with this, some problems of this kind still appear in her notebook: real, but difficult to happen.

Thorndike, having a basis in connectionist psychology, argued in his book that the problems were excellent tests of intelligence and would serve to "inform the teacher of the absolute capacity of each student" and "to stimulate the teacher to assist the class and improve the quality of the work" (THORNDIKE, 1936, p. 279). The problems were means to test the students.

Finally, as Giusti (2020) pointed out, it is possible to see that this miscellany of orientations brought by the pedagogical manuals gives these manuals the *status* of a *Mathematics for teaching* (VALENTE et al., 2017). This Mathematics *for teaching* can be seen as a tool for the primary school teacher, as Hofstetter and Schnewly (2017) stated.

Conclusion

As can be seen, Rocha's notebook (1958) has numerous references both to Pestalozzi, who advocated the intuitive method at the end of the 19th century, and to Thorndike, a representative of the psychology of the connectionist, of the early 20th century. Although the normalist always refers to previous methods as "outdated", she relies on methods constructed and thought out by authors of the intuitive method and the new school. It is worth noting that the 1950s were marked by changes in school teaching, according to Giusti (2020), and in the mid–1960s the signs of a new method for teaching Mathematics began: the Modern Mathematics Movement.

Thus, what is noticeable is that although the normalist criticizes the previous methods, in fact the modern methods have only improved those that were disseminated in a previous time. Pestalozzi used intuition for observation, the senses as a way of learning. Thorndike stated that with observation the child would acquire abstraction and that the classes had to take into account the interest of the child and her/his practical life.

However, when Rocha (1958) suggests some problems that were in a textbook without questioning them, it is noted that not always the teaching of problems would be related to the interest of the child or that it would be possible to start from the concrete to the abstract.

It should be noted that Rocha (1958) was starting her practices in primary school, from tools acquired in normal school. It is possible that, at that time, the course she took did not provide more depth on theories about child

development and the ideas that Pestalozzi and Thorndike advocated, which would contribute to a homogeneous view on the teaching of arithmetic.

It is understood that the relationship established by Mathematics *to be taught* and the Mathematics *for teaching*, highlighted in the article, shows itself as a professional teaching knowledge.

These indications lead to the hypothesis that in this course of improvement, and possibly in others of teacher training, it was not taken into account whether one author was contrary to the ideas of the other, or whether they were part of opposing currents of thought. It seems that for the formation of teachers in the 1950s, each author selected what proved to be the most interesting and possible to be applied in daily school life. In short, it is possible to perceive signs of a professional knowledge of the teacher who will teach mathematics in primary school.

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