

Comparative Analysis of Mathematics Programs: Industrial Technical Courses versus School (1942-1965)¹

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ABSTRACT

From historical studies on the subject of Mathematics in secondary education, a gap was found concerning the discipline in industrial education. The purpose of the article is to compare mathematics in the technical courses of the National Technical School with the discipline of the high school at the time. To carry out the investigation, the comparative history and history of school subjects were used. The research sources were Mathematics programs, legislation and textbooks. We can conclude that, among the similarities, we have the way of teaching and the nomenclature of some of the contents, and among the differences, the distribution of the discipline, the workload, the purposes and the textbooks.

KEYWORDS: Industrial Education. ETN. Comparative Study.

Análise comparativa dos programas de Matemática: cursos técnicos industriais versus colégio (1942-1965)

RESUMO

A partir de estudos históricos sobre a disciplina de Matemática no ensino secundário, verificou-se uma lacuna com relação à disciplina no ensino industrial. O objetivo do artigo é comparar matemática nos cursos técnicos da Escola Técnica Nacional com a disciplina do 2º ciclo do ensino secundário da época. Para realização da investigação, utilizou-se a história comparada e história das disciplinas escolares. As fontes de pesquisa foram os programas de Matemática, legislação e manuais didáticos. Podemos

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concluir que, entre as similaridades, temos a forma de ensinar e a nomenclatura de alguns dos conteúdos, e entre as diferenças, a distribuição da disciplina, a carga horária, as finalidades e os manuais.

PALAVRAS-CHAVE: Ensino Industrial. ETN. Estudo Comparativo.

Análisis comparativo de programas matemáticos: cursos técnicos industriales versus escuela secundaria (1942-1965)

RESUMEN

A partir de estudios históricos sobre el tema de las matemáticas en la educación secundaria, se encontró una brecha en relación con la disciplina en la educación industrial. El propósito del artículo es comparar la matemática en los cursos técnicos de la Escuela Técnica Nacional con la disciplina del 2º ciclo de educación secundaria en ese momento. Para llevar a cabo la investigación, se utilizó la historia comparativa y la historia de las asignaturas escolares. Las fuentes de investigación fueron programas de matemáticas, legislación y libros de texto. Podemos concluir que, entre las similitudes, tenemos la forma de enseñar y la nomenclatura de algunos de los contenidos, y entre las diferencias, la distribución de la disciplina, la carga de trabajo, los propósitos y los manuales.

PALABRAS CLAVE: Educación industrial. ETN. Estudio comparativo.

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Introdução

Valente (2007) performed a historical investigation on how mathematics was educationalized in this country from 1730 to 1930. According to the author, classic school mathematics was developed during this period. The discipline experienced a consolidation process of the technical knowledge, which was previously linked to engineering, to become a school proficiency associated to general culture. At the time, mathematics was divided into Arithmetic, Algebra, and Geometry. Valente (2007) notes that a new educational mathematics was founded on progressive educational ideas after the 1930s.

Along with the first reform of education in the beginning of the Vargas Era, the three fields of mathematics were unified into one single discipline in secondary education. The very process that institutionalized the country's secondary education began with the reform. It consisted of two cycles: the first was an Elementary cycle, with duration of five years; and the second was a complimentary course which lasted two years and corresponded to the student's particular field of interest in higher education. In 1942, the Capanema reform changed many aspects of secondary education, including the names of the two cycles and their durations. The first one had its name changed to gymnasium and its duration reduced to four years. The second cycle course was divided into classic and scientific, with duration of three years. Additionally, the second cycle course lost its preparatory characteristics.

According to Valente (2011), the historical process of establishing Mathematics in Brazilian secondary education was made up of different processes, which resulted in two disciplines: gymnasium Mathematics and school Mathematics. The former “[...] originates from the appropriation of the 1st International Movement of Modernization of Mathematics Education, made possible because of the heritage of more than a century of arithmetic, algebra, and geometry divided exams” (VALENTE, 2004, p.16); the latter, however, was constituted based on the so-called Complementary Courses of the Francisco Campos reform.

The Capanema reform instituted the country's industrial education through the Organic Law of Industrial Education and aimed at training professionals to work in the industry, promoting professional qualification to young people and adults without diplomas, improving or specializing the skills of workers already with diplomas, and disseminating knowledge on technical modernity (Brasil, 1942).

The industrial education considered the same cycling division as secondary education. The 1st cycle offered basic industrial courses, with four years of duration, and was associated with the gymnasium; the 2nd cycle was

equivalent to school (scientific and classic courses) and offered technical courses, with three years of duration.

In this research, we are interested in the Mathematics discipline of schools and technical courses. Given that these two educational systems were offered in different modalities, but with this single discipline in common, we raised the question of whether or not it can be considered the same. Thus, we have researched academic materials that could assist us in this initial survey. The studies of Alvarez (2004), Dassie (2001), Marques (2005), Otone (2011) and Oliveira Filho (2013), Pires (2004), Ribeiro (2006; 2011), and Valente (2004; 2007; 2011) helped us in the process of characterization of the Mathematics disciplines in secondary education. Moreover, other works assisted us in this process. On the other hand, Maciel (2018) found that studies on mathematics for industrial education were limited to six academic works in the period in which he carried out his investigation. These studies presented the first efforts towards identifying Mathematics in professionalizing schools linked to industrial education.

We have found that there is a gap on the history of Mathematics in the industrial courses, more specifically the technical ones. Therefore, identifying which contents were taught and which textbooks were used can contribute to the understanding of how the discipline was conceived in a historical perspective. Unlike secondary education, industrial school programs were not released by the government, and therefore it is important to understand how these curriculums were created for these school systems. We have identified programs in a school that is currently known as the Celso Suckow da Fonseca Center for Technological Education (CEFET/RJ). This institution underwent several transformations before reaching the current name. At the time of the Capanema reform, it was known as the National Technical School (ETN) and the programs that will be analyzed in this work are from that time.

We have adopted the historical research, inserted within the field of Mathematics Education History. In this article, we intend to analyze and compare the contents of the school programs (secondary education) with those

of the technical course (industrial education). No historical investigation should dismiss the strategy of confronting information, since “[...] the introduction of new elements in any field of knowledge without comparing them with the already known” (CARDOSO; BRIGNOLI, 1981, p. 410).

To carry out the comparative methodology, one must know the subject of comparison well before confronting common or distinct characteristics of a given social structure with others, since it is important to appreciate individuality and specific characteristics (CARDOSO; BRIGNOLI, 1981).

Chervel (1990) considers academic disciplines as spontaneous creations of the school, and not a simple vulgarization of scientific knowledge. And educational knowledge is the focus of the research field he developed. Therefore, the History of Educational Disciplines (HDE) is inserted in cultural history, which “[...] aims at identifying how a given social reality is constructed, thought of, and given to read in different places and time” (CHARTIER, 2002, p.6).

Chervel’s (1990, p.184) contribution was to show that the HDE study is not neutral and that “[...] it is dedicated to finding the beginning of an investigation and a specific historical description, in the very own school”. Accordingly, we believe that their social perceptions are by no means neutral: they produce strategies and practices (social, educational, political) that tend to impose an authority at the expense of others, whom they undervalue, in order to legitimize a reform project or justify, for the very individuals, their choices and conducts (CHARTIER, 2002, p. 16-17).

The educational concept of discipline initially brings us to the contents or subjects. Chervel (1990, p. 180) characterizes these words as “[...] a form of disciplining the spirit and providing the methods and rules to address the different domains of thought, knowledge, and art”.

In a historical analysis, educational discipline occurs through a development process that contains steps that may trigger the creation of a new educational discipline, because this process takes place under the analysis of changes in content, teaching methodology, and activities until

these practices become more uniform and in accordance with the objectives, either by legislation or by a pedagogical action.

The history of teaching is also confronted by the problematic of the school purpose because it is complex and subtle (CHERVEL, 1990), and the study of this problem partially depends on the so-called history of educational disciplines. According to the author, at a given period, schools receive different demands from society, and this reflects on the purposes of that institution. School education occurs by putting these imposed objectives into practice, and thus causing convenient acculturation (CHERVEL, 1990).

Contents are present in teaching programs and in the didactic materials, as well as other resources, used by the teacher. For the aforementioned author, these contents are like “sui generis entities, typical of the educational class, independent, to a certain extent, of all the cultural reality outside the school” (CHERVEL, 1990, p. 180).

Goodson (1997) states that it is necessary to research educational curriculum in a comparative perspective. Additionally, according to him, we have:

[...] That which is needed to comprehend the stability and curricular changes are forms of analysis that seek to examine internal affairs together with external relations, as way of developing different views on organizational change and on changes in broader institutional categories (GOODSON, 1997, p. 30).

In other words, when conducting curriculum studies, one needs to understand the internal issues along with the external ones. As an example of external interference in the programs defined for the schools, we can point out the government issue and the legislations that were created for the school environment, as for the internal issue we can highlight the teachers, instituted programs, direction, and teaching materials.

The History of ETN

The decree-Law number 7566, of September 23rd of 1909, from the President Nilo Peçanha, established the creation of the Schools for Apprentices and Craftsmen, associated with the Ministry of Agriculture, Commerce, and Industry.

The Normal School of Arts and Crafts Venceslau Brás was created in 1917 by decree of the city of Rio de Janeiro, which was the Federal District territory at that time. The purpose of the school was to train teachers and foremen to work with professional education and teacher training for primary schools that taught manual work to their students (MACIEL, 2018). In 1937, the Normal School had declared the institution closed to demolish the old building and make room for a lyceum. However, due to the delay in construction, a new guideline was instituted that culminated in the promulgation of the Organic Law of Industrial Education, in 1942. Thus, the National Technical School (ETN) was created from the decree-Law number 4127, of 25th of February of 1942, which defined the organization bases of the industrial education institutions of the federal network. The motivation for the technical schools was to train workers for the industry.

In 1965, the national capital was transferred from Rio de Janeiro to Brasilia, and as a result there was a change in the name of the school to Federal Technical School of Guanabara (ETFGB). In 1967, with the passing of its Director, the institution paid tribute by changing the name to Federal Technical School Celso Suckow da Fonseca (ETFCSF).

In the year of 1978, new guidelines for technical education transformed the ETFCSF into Federal Technological Education Center Celso Suckow da Fonseca. This law was intended to verticalize and promote the continuity of industrial training from basic to higher education.

School Programs

The Francisco Campos reform created the Complementary Course, the second cycle of secondary education. These courses were intended to have a duration of two years, with intensive studies, exercises, and individual practical activities (BRASIL, 1931). The second cycle courses were supposed to prepare the individual to enter Law School, Dentistry, Pharmacy and Medicine School, Engineering, and Architecture. Therefore, we consider that there were three types of Complementary Courses: Pre-medical, Pre-Legal, and Pre-polytechnical. Each of these courses had their own list of disciplines since their objectives were to prepare for the entry in specific higher education courses. The institutions that were able to offer were those that had secondary education, under a government inspection regime. Valente (2011) states that based on the analysis performed on the programs of the Complementary Courses, it was possible to observe that Mathematical contents were of the following fields: Algebra, Higher Algebra, Vector Analysis, Analytic Geometry, and Trigonometry.

The material taught in these courses gave way to the didactic texts used. The textbooks produced in these times were written in two modalities: the first considered the program as points or lessons, therefore the subjects were condensed in one book; and the second modality emphasized the independence of the subjects. Thus, books would cover only one single subject.

The Capanema reform was responsible for a new format of secondary education. The Organic Law for Secondary Education had the following purposes: develop the integral character of the adolescent, patriotic and humanistic awareness, and preparation for higher education. The Complementary Courses became later schools, which were divided at the time in classic and scientific courses, with the duration of three years each. The classic course emphasized in

intellectual training of Philosophy and Ancient Letters. The scientific course, on the other hand, emphasized in scientific education.

When analyzing the structures of the classic course, we realize that the disciplines of human science were present in every grade, while the disciplines of exact science were present from the 2nd grade forth. Mathematics was present in every grade. However, the workload consisted of three hours for the 1st and 2nd grades and two hours for the 3rd grade.

On the other hand, the scientific course focused more on Science and Mathematics, leaving lower workloads for the disciplines of human science. In the scientific course, Mathematics also appeared in the three grades, and the weekly workload was of four hours each.

According to Romanelli (2013), the generalist and humanistic culture characteristic given to both courses in their curriculums is undisguisable. Although Mathematics is considered to be a discipline of general culture, it can be noted that there were two different programs for each course, in order to meet specific demands. These differences consisted of including new material in the scientific course in comparison with the classic one.

The school's Mathematics programs were issued by Ordinance number 177, of 16th of March of 1943. We will now present the Mathematics programs of the classic course by each grade and, because the two courses have symmetries in their contents, at the end we will indicate which extra subjects are included in the scientific course. The programs of the three grades are presented in Tables 1, 2, and 3.

Table 1 – Mathematics program for the 1st grade of the classic course

Theoretical Arithmetic
Unit I – Numerical divisibility: 1. General theorems on divisibility. 2. Divisibility characters. 3. Theories of L.C.M and G.C.D. 4. Prime number theory; applications.
Algebra
Unit II – Polynomials: 1. Algebraic operations on polynomials. 2. Long division theory for polynomials in x by $x \pm a$; Briot Ruffini rule and practical device. Unit III – The 2 nd degree trinomial: 1. Decomposition into 1 st degree factors; trinomial signs; 2 nd degree inequalities. 2. Notions of variable and function; variation of the 2 nd degree trinomial; graphic representation.
Geometry
Unit IV – The plane and line in space: Definition of plane. 2. Intersection of lines and planes. 3. Parallel lines and planes. 4. Perpendicular lines and planes. 5. Perpendiculars and obliques from a point to a plane. 6. Dihedra; planes perpendicular to one another. 7. Understanding polyhedral angles. Unit V – Polyhedra: 1. General notions. 2. Study of prisms and pyramids and their trunks; area and volume of these solids.

Source: Brasil (1943)

The program for the 1st grade of the classic course was divided into: Theoretical Arithmetic, Algebra, and Geometry. The scientific course program had the same contents as the ones presented in Table 1, differing in Theoretical Arithmetic where the following subjects were included: Fundamental Arithmetic Operations and Fractional Numbers. The workload for both courses was the same.

Table 2 – Mathematics program for the 2nd grade of the classic course

Algebra
Unit I – Progressions and logarithms: 1. Study of arithmetic and geometric progressions. 2. Theory of logarithms; use of application boards. 3. Solving simple exponential equations. Unit II – Newton's binomial: 1. Understanding combinatorial analysis. 2. Newton's binomial.
Geometry
Unit III – Round bodies: 1. Understanding the generation and classification of surfaces. 2. Study of cylinder and cone: areas and volumes. 3. Study of the sphere; sphere, zone and spherical area; volume of the sphere.
Trigonometry
Unit IV – Vector: 1. Scalar and vector quantities. 2. Notion of vector; equipollence. 3. Resultant or geometric sum of vectors. 4. Sliding vectors over an axis; algebraic measure; Chasles' theorem. Unit V – Projections: 1. Orthogonal projection of a vector over an axis. 2. Carnot's theorems. 3. Vector projection value. Unit VI – Circular Functions 1. Generalization of notions of arc and angle; congruent arcs; arcs of the same origin and associated ends. 2. Circular or trigonometric functions: definitions, variations, reducing to the first quadrant. 3. Relationships between the circular functions of the same arc. 4. Calculation of the circular functions for the same arc. 5. Calculation of circular functions for 30°, 45° and 60° arcs. Unit VII – Resolution of triangles: 1. Relationships between the elements of a triangle. 2. Use of trigonometric tables. 3. Resolution of right triangles.

Source: Brasil (1943).

The program for the 2nd grade of the classic course had subjects of: Algebra, Geometry, and Trigonometry. The scientific course program emphasized mathematics subjects. Some of them were not present in the classic course, such as exponential functions, determinants, and trigonometric transformations. The workload for both courses set by law was also the same in this grade.

Table 3 – Mathematics program for the 3rd grade of the classic course

Algebra
Unit I – Functions: 1. Notion of a real variable function. 2. Cartesian representation. 3. Notion of limit and continuity. Unit II – Derivatives; 2. Definition; geometric and cinematic interpretation. 2. Calculation of derivatives. 3. Differentiation of elementary functions. 4. Determining maximum and minimum, and study of variation of simple functions.
Geometry
Unit III – Plane curves: 1. Definition and fundamental properties of the ellipse, hyperbola, and parabola. 2. The conic sections. 3. Definition and fundamental properties of cylindrical helix.
Analytic Geometry
Unit IV – Fundamental notions: 1. Concepts of Descartes. 2. Coordinates; axis over a line; rectilinear coordinates in the plane. 3. Distance between two points; point dividing a segment in a given ration. 4. Defining direction; angle between two directions. Unit V – Locus: 1. Natural equation for locus; its interpretation. 2. Moving from the natural equation to the rectilinear rectangular equation. 3. Line equation. 3. Circle equation. 4. Reduced equations for ellipse, hyperbola, and parabola.

Source: Brasil (1943).

The 3rd grade program of the classic course was structured by: Algebra, Geometry, and Analytic Geometry. Among the contents that were studied, the following subjects stood out because of the strong relationship between algebra and geometry: functions, derivatives, conic sections, and line and conic section equations. In this grade the most detailed study of functions was to be carried out. Additionally, Analytic Geometry was present, as well as line and circle equations.

The 3rd grade program of the scientific course used the same structure as the classic one. However, it contained five extra units with the following contents: Series, Complex Numbers, Algebraic Equations, Transformation of

Equations and Metric Relations. The workload for this scientific course grade was twice as much as the classic course.

When analyzing the Mathematics programs of each course, it can be noted that their structures are very similar, despite the insertions of contents in the scientific course. In fact, at that time, the textbooks were the same for both courses.

In 1951, with the creation of the Minimum Programs by Minister Simões Filho, the classic and scientific course contents were reorganized, just like in the gymnasium, as mentioned before.

The Ordinance number 966, of 2nd of October of 1951, determined the contents for each course, indicated methodological instructions, and redefined the workload of the Mathematics discipline, which became of 3 hours per week for all grades, in both courses. Few differences between corresponding disciplines were kept, and thus the programs became quite similar.

The courses were structured as follows: 1st grade: Arithmetic, Algebra, and Geometry; 2nd grade: Algebra and Trigonometry; and 3rd grade: algebra (Otone, 2011). In each grade, the contents of both courses appeared in a single program, different from what had happened during the Capanema reform. Therefore, there was one single distribution. According to Otone (2011), it could be observed that the contents were arranged to be taught in a single grade, making Mathematics more homogeneous in terms of curricular proposal for both courses. This resembled the practices that were being carried out in the late 1940s. With the Minimum Program, textbooks were reissued to meet these new demands.

Mathematics Programs in Technical Courses

The technical courses were made up of the academic disciplines of general and technical culture, as well as educational practices. From 1942 to 1959, the curriculum distribution followed guidelines established by the Organic Law for Industrial Education and the Ordinances number

162, of 1st of March of 1943, and 163, of 13th of March of 1943. In 1959, with the change in legislation, industrial education institutions became more autonomous.

In the technical course, Mathematics had two different culture disciplines. The first one, called Mathematics, was offered only in the 1st grade, with a five-hour workload per week. The other one was called Complements of Mathematics, and it was only offered in few technical courses. This allows us to open a discussion on how Mathematics, as an educational knowledge, was linked to general culture as well as technical culture for the Industrial Education. This also brings us back to the thesis, which proposes to investigate whether Mathematics can be considered the same discipline that was being taught in secondary education. This discussion allows us to question the role of Mathematics in the Industrial courses and whether it followed a more practical approach better adapted to workshops. Hence, we question ourselves how the technical courses were conceived, given that we would have two situations: a course that only had the one discipline of Mathematics and another one that had Mathematics and the Complements of Mathematics.

We found programs for such courses in two documents. The first one was a 1946 ETN report, and the other one was the 1964 program of the Technical Industrial Course for Machines and Engines.

In the 1946 report, we have found three programs: a Mathematics discipline program, a Mathematics complements program, and a Mathematics and Complements of Mathematics for the Aeronautics Technical Course. According to the document, because no proposal had been sent from the Industrial Education Division (DEI), the institution's own teachers organized the courses. A letter sent from DEI to ETN by Francisco Montojos mentions that the program sent by the school would be present to other technical schools on an experimental basis. It was not possible to locate the curriculum proposal sent from the DEI to the other technical schools.

Electrotechnics courses, Technical Design courses, and Buildings and Interior Decoration courses had the same Mathematics program as the ETN. Next, we present the program of this discipline.

Table 4 – Mathematics Programs of Technical Courses

Algebra
1. 2nd degree Trinomial; determining its roots; variations of signs and the trinomial; graphic representation. 2. Square trinomial. 3. Irrational Equations. 4. Systems of quadratic equations. 5. Arithmetic and geometric progressions. 6. Theory of logarithms; compound interests and annuities.
Trigonometry
1. Oriented circumference; gradian, degree, and radian. 2. Trigonometric functions: definitions, variations of signs, and values; graphic representation. 3. Determining simple arc function values. 4. Reducing to the first quadrants. 5. Fundamental formulas. 6. Determining the functions of any arc. 7. Arc operations. 8. Logarithmic transformations. 9. Resolution of right triangles. 10. Resolution of oblique triangles.
Algebraic Analysis
1. Combinatorial analysis; Newton's binomial. 2. Notions on limits; continuous functions; derivative theory; differentiation of elementary functions. 3. Increasing and decreasing functions; maximum and minimum. 4. Notions on Integrals, immediate primitives; integration by decomposition, substitution, and by parts. 5. Defined integrals; calculation of area.

Source: Report (1946, p. 73-74).

We identified the structured program containing three fields: Algebra, Trigonometry, and Algebraic Analysis. The contents of these fields were common to all technical courses offered by the institution. Furthermore, it is easy to observe that, for these courses, there were contents of Differential and Integral Calculus.

Complements of Mathematics was offered by the following ETN courses: electrotechnics and building and construction of machines and engines. It is noteworthy that this discipline was part of a list of disciplines of specific culture. The program is presented as follows:

Table 5 – Complements of Mathematics Program of 1946

<p>Vector Analysis</p> <ol style="list-style-type: none"> 1. Scalars and vectors; vector; the nature of vectors. 2. Point and vector sum; adding and subtracting vectors. 3. Multiplying a real number by a vector; unit vectors; vector projection. Representing a vector using two others; representing a vector using three others; reference trihedrals. 6. Scalar product; analytical properties and expression; angle between two vectors. 7. Vector product between two vectors; construction of the vector product; analytical properties and expression; angle between two vectors.
<p>Analytic Geometry Notions</p> <ol style="list-style-type: none"> 1. Axis, determining a point and a direction. 2. Distance between two points; dividing a segment in a given ration. 3. Straight line: representation; various forms of equation; intersection of lines; the distance from a point to a line; triangle center of gravity; equation of lines that pass through two point. 4. Circumference: equation; intersection of a circle with a line and another circle. 5. Ellipse: equation; intersection of an ellipse with a line and an axis; tangent equation. 6. Hyperbole: equation; intersection of a hyperbole with a line and an axis; tangent equation. 7. Parabola: equation; intersection of a parabola with a line and an axis; tangent equation.

Source: Report (1946, p. 74).

This program has been divided in two groups: Vector Analysis and Analytic Geometry. When analyzing their contents, we find that they are part of a Mathematics general culture. We do not see information that leads us to specific issues of the technical course professions, at least not apparently. Both programs were organized by the ETN professor João Dias dos Santos Júnior.

The Mathematics and the Complements of Mathematics programs for the Aeronautical Construction course were presented as a single program for both disciplines. Therefore, it was possible to verify the differences between the structure of this program and other courses. Moreover, the faculty staff of this course was not a part of the institutions, initially. Next, we present the unified program.

Table 6 – The Mathematics and the Complements of Mathematics Program for the Aeronautics Construction technical course (1946)

Arithmetic and Algebra Review
a) Fundamental operations. Powers and roots. Divisibility. Multiples and submultiples. Prime numbers. Theories of L.C.M and G.C.D. b) Fractions. Fraction operations; c) Complex numbers. Metric system and English system of weights and measures. Ratio and proportions. Proportional quantities. d) Algebra, Concepts, definitions. Algebraic expressions. Algebraic operations: polynomial addition, subtraction, and multiplication. Notable products: monomial and polynomial powers. Monomial roots. e) Dividing by monomials and by $x \pm a$, Factorization. Greatest Common Divider and Least Common multiple. Algebraic fractions. f) first degree equations with unknown inequalities. Linear equations with two unknown variables. Systems of Equations.
Algebra
a) Linear equations with three or more unknown variables. b) Graphic presentation of equations. Quadratic equations. Complex numbers. c) Arithmetic progressions. Geometric and harmonic progressions. d) Partial-fraction decomposition (first case). e) Newton's Binomials – combinatorial analysis. Newton's binomial general term. f) Limits. Understanding series. Series: convergent, divergent, and alternate-convergence criteria. g) Logarithms: introduction, definition, operations, and properties. Different logarithm systems; change of base. Common logarithm. The use of boards. Notions of exponential equations.
Plane Geometry
a) Introduction; definitions. Line and angles. b) Metric relations of triangles (right and obliques). c) Metric relations of circles. d) Regular polygons. Rectifying the circumferences. Areas.
Geometry of Space
a) Intersection of lines and planes. Line and perpendicular planes. Dihedral angles. Perpendicular planes. b) Projecting a line over a plane. Projecting a plane area. c) Polyhedral angles. Polyhedra: general notions, definitions. Prism: lateral area, volume. Parallelepiped. Pyramid: volume and lateral area of right pyramids. Pyramid trunk; prism trunk; volumes. d) Round bodies: general notions, definitions. Cylindrical surface, conical surface, and surfaces of revolution. Cylinder. Circular based cylinder: lateral surface, volume. Cone of revolution: lateral surface; volume. Trunk of the cone of revolution; lateral surface; volume. Sphere: generalities; surface and volume. e) Symmetry, homothetic preferences, and similarity.
Slide Rule
a) Theory. b) Common scales. Multiplication, division, power (square, cube), roots, logarithms. c) Special scales and particularities of types of Slide rules. Scales LL1-LL2-LL3-LL0-LL00. Combined operations.
Trigonometry
a) Introduction, arcs and angles, Angle measurements; measurements in radians. Conversions. b) Trigonometric functions. Important functions of angles. Angles in general. Reducing to the 1 st quadrant. c) Tables: in radians, of trigonometric functions, logarithmic (layout and application). d) Relationship between functions. Fundamental relationships. Adding and subtracting arcs. Sum and subtraction functions of angles. Multiplication and division of arcs. e) transforming sums into products. Functions of angles expressed in terms of functions of half or double of the angle. f) Pythagorean relations. Identities. Trigonometric equations. g) Resolution of right triangles. Introduction. Formulas. Classic cases. Solving triangles using the slide rule. h) Use of the slide rule to determine small angles in vector problems. i) Resolution of any triangle. Formulas. Laws of sines and cosines. General formula for calculating the area of a triangle. Classic cases of resolution of oblique triangles. j) Understanding geometric and trigonometric representations of the imaginary and of Moivre's formula.

Source: Report (1946, pp. 110-111).

The program presented in Table 6 is divided into six groups: Arithmetic and Algebra Review, Algebra, Plane Geometry, Geometry of Space, Slide Rule, and Trigonometry. It can be noted that this is not a combination of other technical course programs at ETN (Table 4 and Table 5). It has a greater amount of content and detail. The course for which it was intended held a greater demand.

This course was initially offered in agreement with the Aeronautics Technical Office, directed by engineer Luiz Felipe Marques (ETN Reprot, 1946). The contract that was signed by the office and MEC guaranteed that an amount would be paid for each hour of class, with the determination of 840 hours of technical culture for the 1st grade, 1000 hours for the 2nd grade, and 1040 hours for the 3rd grade. This was 2880 hours altogether. We also identified that Mathematics was taught by members of the office, and the other subjects of general culture was taught by the ETN faculty.

The curriculum for this course was structured according to trimesters. Additionally, in order to allow comparisons with the North American courses, the number of classes of each discipline also followed the North American standard. The school program based itself on the Aeronautical Engineering Bachelor of Science course, offered in the United States of America (USA) after secondary school (ETN REPORT, 1946; ABRANCHES, 1958). However, the disciplines taught at ETN did not have the same efficiency because of lack of materials and laboratories.

The course had a low enrolment rate, training only 25 technicians in the ten years (1946-1956) in which it was offered. Abranches (1959) gives two possible explanations for this result: its high-demanding characteristic which led to several withdrawals; and that the aviation industry was not able to absorb this number of trained technicians. The National Engine Factory (FNM) opened in 1942. However, it did not succeed in its initial proposal to build aircraft engines, nor did it succeed in its subsequent one to perform aircraft maintenance, which according to Abranches (1958), were projected and tested outside of the country. Soon after, the factory began to operate in other fields.

In 1959, ETN became more autonomous because of the Decree number 47038, of 1959. Therefore, the discipline gained more freedom and part of the decisions were analyzed by the Council of Representatives and the Council of Teachers. However, we identified in the CEFET/RJ files a program proposal suggesting that the discipline of Mathematics be offered in two grades (ATA, 1963).

After 1964, it is noted that there was no longer a distinction in the Mathematics discipline between ETN's technical courses, and that there was only one single program. The program for this new format was found in the document titled 'Programs of the Industrial Technical Course for Machines and Engines'.

Next, we present the Mathematics the 1st grade program for the technical courses of 1964.

Table 7 – 1st grade Mathematics Programs for Technical Courses (1964)

Algebra
1.1 Arithmetic Progressions. 1.2 Geometric Progressions. 1.3 Exponential Function. 1.4 Logarithmic Function. 1.5 Theory of Logarithms. Use of Mathematics boards. 1.6 Exponential Equations. 1.7 Compound Interest, Capitalization, and Amortization. 1.8 Slide Rule.
Geometry of space
2.1 Prism. 2.2 Pyramid. 2.3. Cylinder. 2.4 Cone. 2.5 Sphere and its parts.
Trigonometry
3.1 Basic notions of vector analysis. 3.2 Oriented circumference, degree, gradian, and radian. 3.3 Arcs with connected ends. 3.4 Trigonometric functions; definitions, variation in sign and value, and graphical representation. Arcs of the same function. 3.5 Determining the values of single arc functions. 3.6 Reducing to the first quadrant. 3.7 Fundamental formulas, resulting formulas. 3.8 Determining the functions of arbitrary arcs. 3.9 Arc operations. 3.10 Logarithmic transformations. 3.11 Resolution of right triangles. 3.12 Resolution of arbitrary triangles. 3.13 Simple trigonometric equations.

Source: Programs (1964, p. 13).

This program was structured in three groups: Algebra, Geometry of space, and Trigonometry. It presents a novelty in school curriculum, which is the inclusion of Geometry of Space. In Algebra, Exponential Function and Exponential Equation were inserted in the discipline's contents.

Table 8 presents the 2nd grade program for the ETN technical courses.

Table 8 – 2nd grade Mathematics Programs for Technical Courses (1964)

Algebraic Analysis
1.1 Combinatory Analysis. Newton's binomial. 1.2 Elementary theory of determinants. 1.3 Complex numbers. 1.4 Limits. Continuous functions. 1.5 Derivatives. Differentiation of elementary functions. 1.6 Increasing and Decreasing functions. Maximum and Minimum. Graphic study of functions. 1.7 Integral; integral concept; immediate primitives; integration by decomposition, substitution, and parts.
Analytic Geometry
2.1 Concepts of Descartes. 2.2. General notions of coordinate systems. 2.3 Relationship between Cartesian, rectangular, and polar coordinates. 2.4 Distance between two points. 2.5 Coordinates of a point that divides a line in a given ratio. Triangle barycenter. 2.6 Defining direction 2.7 Angle between two directions. 2.8 Area of the triangle. 2.9 Line. 2.10 Coordinate transformation. 2.11 Circumference. 2.12 Ellipse. 2.13 Hyperbole. 2.14 Parabola. 2.15 Intersections.

Source: Programs (1964, p. 13).

The 2nd grade program has the same contents as the Mathematics and Complements of Mathematics program (Tables 4 and 5). Apparently, almost all the content was assimilated by the new discipline of Mathematics, except for Vector Analysis.

The disciplines of the ETN technical courses were taught using materials produced by the institution. In the CEFET/RJ files, we were only able to find in the trigonometry material. Later, the school published a collection of Arlindo Clemente's books, called Mathematics for the Industrial Technical Courses (Maciel, 2018). This collection was a compilation of the material produced by the teacher at ETN, since the trigonometry content of the book was identical to the material found in the CEFET/RJ files. Additionally, the author produced other Mathematics books, which we were unable to locate, but were referenced by the other content present in the collection and may have been identical, as in the case of the aforementioned material.

As for the contents' presentation, there was an introduction using definitions, explanations, or theorems, followed by examples, and then by direct exercises in the same format as the examples and explanations (MACIEL & FACHADA, 2019).

Comparing Programs.

To compare the two programs, we resort to the considerations of Cardoso and Brignoli (1981), who affirm that it is necessary to understand the subjects matters of comparison. Therefore, we consider it important, in addition to the analysis of the programs, to understand the structures that were established by legislation for each course, their contents, and the books and pedagogical practices.

We began our analysis by looking at the structures of each course. According to the Organic Law of Secondary Education, the purposes of this modality were: to form the integral personality of the adolescents; to elevate spirituality, and patriotic and humanistic consciousness; and to provide intellectual preparation to continue to higher educational studies (BRASIL, 1942b). On the other hand, the Organic Law of Industrial Education had the following purposes: train professionals to exercise industrial activities, provide training for the industry's young and adult workers, improve worker's abilities, and disseminate knowledge of technical updates (BRASIL 1942a). We have found that the training provided by one of them was geared towards a more humanistic and preparatory training for higher education. The other one was aimed at professionalization for the industry.

As for the school curriculum, the discipline was distributed among three grades, with a three-hours-a-week workload. The distribution of the discipline in the curriculum of the technical course was initially concentrated in the 1st grade. There were two subjects, one of general culture and another of technical culture.

As for the programs, the secondary educational program was determined by Ordinance number 177, of 16th of March of 1943; by law, the technical course program should have followed the same guidelines, however, when consulting the ETN documentation, we discovered that the program had not been sent. Thus, the teachers from that institution had to follow the program that was found in the institution's 1946 report, which may possibly have been used as model for other institutions.

When comparing the Mathematics program for technical courses (Table 4) with the one presented for the scientific and classic courses, both from Capanema reform and the Minimum Program of 1951, we have noticed great differences. Mainly because of the large amount of content that was not included in the ETN program. The fields that did not even have contents were Theoretical Arithmetic, Plane Geometry, Geometry of Space, and Analytic Geometry. Other explicit contents such as Polynomials and Conics were not a part of the curriculum.

The contents of the Complements of Mathematics program (Table 5) were part of the school programs of 1943 and 1951. However, as mentioned before, this discipline was not offered for all technical courses, and thus it can be considered that there was no uniformity amongst the mathematical content that was taught in the technical courses. The discipline was technical culture. Even if Mathematics and the Complements of Mathematics contents were to be added, we would still not have the same explicit material as schools. We noticed that the discipline's emphasis was on Vector Analysis and Analytic Geometry.

The contents of the Aeronautics Construction technical course program (Table 6) were more similar to those of the scientific and classic courses than any other technical course program. However, we noticed that the program did not have all the secondary education contents. Subjects such as Conical Sections did not appear in the School program. In reality, it was distributed among other disciplines such as Review of Arithmetic and Algebra, Algebra, Plane Geometry, Geometry of Space, Slide Rule, and Trigonometry. We did not find any other information about this discipline in the ETN archives, mainly because this discipline was taught by a hired Aeronautics Technical Office.

As for the programs of the technical course of 1964 (Tables 7 and 8), these had their contents inserted in the school program, however they did not represent the classic and scientific courses, entirely. There were

subjects that were not included in this program, like Plane Geometry and Geometry of Space (e.g., lines and planes).

The proposals presented in the programs of 1964 (Tables 6 and 7) is somewhat in consonance with Cunha (1990), who states that secondary education and technical courses became close in the constitution of a single secondary education. However, in the ETN program, we noticed that Mathematics was not yet available for the three grades, and workload also wound up being different.

When analyzing the contents presented by the ETN programs, we have found that they are not equivalent to those of secondary education courses which they should have articulated with, according to the legislation of that time. We found several differences, such as disciplines of different natures, courses that had two Mathematics disciplines, and others that had only one. Not to mention different programs for one single course. There was no uniformity of contents within the school, and there were evidence indicating that their Mathematics disciplines were different from school Mathematics.

According to Maciel (2018), the way contents are presented is basically the same: introduction, examples, and exercises. Additionally, the number of pages in the technical course books was significantly inferior to those of the secondary education course. Moreover, the exercises of similar topics were also different. Generally, in secondary education textbooks, there were a variety of exercises with straightforward activities and problems. In the industrial education, however, there was always an emphasis on a specific type of exercise.

Final Considerations

When comparing technical course programs with school programs, we realized the need to compare legislations and textbooks, to better understand the discipline. Therefore, we found that there were similarities and differences.

Among the similarities, we find it noteworthy that common subjects had the same denomination and content presentation.

Regarding the differences, we can mention: the way a subject is distributed among each grade, the amount of content is not the same, the purposes foreseen by law identify different propositions, and the use of specific materials for each modality.

We have found that the discipline of Mathematics at ETN was conceived by appropriation of the secondary education discipline, however, we have noticed the construction of a representation of its own for the modality to which it was associated with.

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