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The collection *Mathematics, methodology and complements for primary school teachers*, by Ruy Madsen Barbosa: an interpretative analysis

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RESUMO

Este artigo tem como objetivo analisar a coleção *Matemática, metodologia e complementos para professores primários*, utilizando o referencial teórico metodológico da Hermenêutica de Profundidade, proposto por Thompson (1995), em conjunto com a ideia de Paratextos Editoriais, de Genette (2009). A Coleção, cujo autor é Ruy Madsen Barbosa e cuja primeira edição é de 1966, é composta por três volumes. A análise dessa obra nos permite apontar tanto elementos que, nela, nos pareceram inovadores do ponto de vista didático e pedagógico, quanto o modo como, nela, aparentemente, seu autor defende valores próprios à prática científica da Matemática em detrimento do que se considera ser adequado como guia à formação de professores.

PALAVRAS-CHAVE: Análise de Livros. Movimento Matemática Moderna. História da Educação Matemática Brasileira. Ensino Primário. Formação de Professores.

The collection Mathematics, methodology and complements for primary school teachers, by Ruy Madsen Barbosa: an interpretative analysis

ABSTRACT

The main goal of this text is to perform a hermeneutic examination of a set of books entitled *Mathematics, Methodology and Complements for*

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primary school teachers. Such analysis was developed according to Thompson´s Depth Hermeneutics and the idea of paratexts as proposed by Genette. The book set is comprised of three volumes published in 1960s by Ruy Madsen Barbosa. From a didactical and pedagogical point of view, such hermeneutic exam shows some points that can be considered innovations. However, elements of a conservative nature could also be detected, such as those in which the author seems to defend some values related to the scientific practice of mathematics instead of values considered more suitable to guide teachers in their teaching practices.
KEYWORDS: Textbook analysis. New Math Movement. History of mathematics education in Brazil. Elementary school. Teacher education.

La colección Matemáticas, metodología y complementos para maestros de primaria, de Ruy Madsen Barbosa: un análisis

RESUMEN

Este artículo tiene como objetivo analizar la colección Matemáticas, metodología y complementos para maestros de primaria, utilizando el marco metodológico teórico de la Hermenéutica de Profundidad, propuesto por Thompson, junto con la idea de Genette de Paratextos Editoriales. La Colección, cuyo autor es Ruy Madsen Barbosa y cuya primera edición es de 1966, consta de tres volúmenes. El análisis de este trabajo nos permite señalar elementos que, en él, parecían innovadores desde el punto de vista didáctico y pedagógico, y la forma en que, aparentemente, su autor defiende valores específicos de la práctica científica de las matemáticas en detrimento de lo que se considera adecuado como guía para la formación del profesorado.
PALABRAS CLAVE: Análisis de libros. Movimiento Matemática Moderna. Historia de la educación matemática brasileña. Escuela primaria. Formación de profesores.

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Introduction

Professor Ruy Madsen Barbosa's family[3] donated part of his library to the collection of the Oral History and Mathematical Education Research Group (*Grupo de Pesquisa História Oral e Educação Matemática* - GHOEM). Professor Madsen Barbosa died in July 2017, and by 2018, the books were already available for consultation, at the GHOEM archives, located in Bauru (SP), however still in the early organization stages. Among such materials there were two volumes of a collection of three books entitled Mathematics, methodology and supplements for primary school teachers (*Matemática, metodologia e complementos para professores primários)*. The first edition of Volume I of the collection was published in 1966. What peaked our interest in the collection was the target audience, primary school teachers, as, until then, we were unaware of that author's work focused on early schooling years, which then comprised the so-called grade school. Prior to that, we had only had access to two printed books published by the same author, both single volumes: A modern elementary course in combinatorial analysis (*Um curso moderno elementar de análise combinatória*), in 1963, and Combinatorics and Probability (*Combinatória e probabilidades*), in 1964. Regarding his production of school textbooks, Professor Madsen Barbosa published a total of 16 titles[4], 13 of which single volumes, 2 double volumes (published in 1975 and 1984) and the collection Mathematics, methodology and supplements for primary school teachers (Coleção *Matemática, metodologia e complementos para professores primários*), containing three volumes.

Our research group has a significant history of textbook analyses, all developed according to the same theoretical methodological framework known as Depth Hermeneutics (DH). Therefore, it seemed natural to use

[3] We would like to thank Professor Antonio José Lopes, Bigode, for making the first contacts for the assignment and transference of such work.
[4] Total declared by Professor Ruy Madsen Barbosa himself in the Lattes platform and verified by us.

that same framework to study that collection. DH was proposed by British sociologist John B. Thompson in his 1995 book *Ideology and Modern Culture*. It deals with the interpretation of what the author calls symbolic forms[5], which, can be synthetically characterized, as human constructions intentionally taken as the focus of those who, when in contact with such forms, are willing to interpret them. In short, Depth Hermeneutics can be presented as a theoretical-methodological framework proposed by John Thompson rooted in two realms, sociology and philosophy, whose aim is to interpret symbolic forms. Whereas the role of philosophy is to nourish the referential with considerations regarding the limits and potential of interpretations (or readings), sociology must steer hermeneutics towards understanding the ideology that surrounds/constitutes symbolic forms. This emphasis on the need to address the ideology that surrounds symbolic forms may be the difference between this framework and others currently used.

According to Thompson, every symbolic form operates to create (or maintain) certain power relations of one group over another. We live in a world of powers and counterpowers, in a universe that is, forcibly and hopelessly, ideological, where institutions are characterized by the maintenance or imposition of a particular power relation. According to the discussion proposed by Thompson, studying ideology encompasses studying the ways through which meaning serves to establish and sustain domination. The author lists some of the ways through which ideology operates. It operates by *legitimation* (as relations of domination are always seen as legitimate, that is, as fair and worthy of support);

[5] According to Thompson (1995, p. 182), symbolic forms are characterized by five aspects: (i) intentional, as they are created with a purpose (in the case of books, they result from the author's intention); (ii) conventional, as each form follows some sort of convention, thus this aspect enables communication between symbolic form and hermeneut (in the case of books, there are patterns that can be distinguished and interpreted, such as the sequence of assumptions, clarity and correction of language, the symbology of the area the book was written about, etc.); (iii) structural that, in the case of books, is shown in the organization of the symbolic form, in the way the internal contents of the volume(s) are structured and are related, (iv) referential: a symbolic form always alludes to something, addresses something or someone: the books we study are focused on teaching, the discussion of content, aimed at a certain reader, school, discipline, etc.; (v) contextual aspect, as every symbolic form is produced and circulates within a given context. In the study of textbooks, it is important, for example, to understand the school culture and educational policies of the time when it was produced and published.

universalization (as global institutional agreements result from the interests of some groups or individuals, though considered the best for all); *concealment* (as power relations are always hidden, obscured, (*mis*)represented, in order to divert attention, aiming to lead individuals to disregard elements considered undesirable); *fragmentation* (segmenting groups and peoples in order to avoid ideas that can become troublesome for dominant groups); and *reification* (which leads to the invention of artificial traditions, so that a transitory and historical situation is believed to be permanent and natural).

Thus, the claim that we intended to mobilize Depth Hermeneutics to analyze a certain symbolic form (a book, painting, practice, habit, oral presentation, legislation, choreography, poem, etc.) gave rise to the essential generating question of this referential, which is: "What ideology does this symbolic form create or help maintain?", which further leads to the question "How does this symbolic form establish and sustain certain relations of domination and what relations are those?"

Regarding theoretical and methodological frameworks

Although a more detailed discussion of the theoretical methodological framework of DH cannot be made here, it is important to state that Thompson proposed three analytical movements to understand the ideology surrounding a symbolic form in order to effect interpretation. Such movements have been called (i) formal or discursive analysis, (ii) socio-historical analysis, and (iii) interpretation-reinterpretation. Running the natural risk of oversimplifying it, we could say that formal or discursive analyses deal with the study of the symbolic form "*itself*", its most concrete manifestation (in this case, the books that make up the collection), while socio-historical analyses deal with the environment of production, circulation and appropriation of the form and, finally, interpretation-reinterpretation permeates, and at the same time

synthesizes, the entire interpretative process, weaving understandings obtained through the other movements, and producing what could be called a "result of"[6] of the interpretation.

According to Thompson's determinations, the formal or discursive analysis phase, not only serves to describe the symbolic form "itself", but aims to understand its "ways of being", its "place" and its "context", that is, the manner through which it is shown to the reader, who tries to analyze it in order to create an initial discourse about it, including guiding the notes that will compose the socio-historical analysis; the study regarding the "*outskirts*" of the collection, when the surroundings of the form are more thoroughly investigated. This socio-historical analysis encompasses the period the books were written, as well as materials that can help understand the author's conceptions and intentions when elaborating them. Through discursive analysis, we study the way chapters are distributed within the books; how contents are organized and presented in each chapter, also trying to understand the intentions that led the author to create the work in a specific way and not another, whereas we might say perhaps that the aim of socio-historical analysis is understanding how time and space, and the circumstances in which the work was produced led the author to make certain choices, in response to such conditions at that time.

By studying the circulation of the collection, we noticed that it was distributed and used in several Brazilian states, enjoying good acceptance, as at least four editions were published in only two years. The target audience, as the author himself indicates, were serving primary teachers, future primary teachers, primary teachers in advanced education, mathematics teachers, professors lecturing

[6] It should be noted that the characterization of the interpretation-reinterpretation movement relies strongly on the nonlinearity that marks DH, since this movement occurs throughout the process, gradually synthesizing previously described movements, while it is also responsible for producing what could be referred to as the "result" of the hermeneutics developed. Even though the formal presentation – the record of the hermeneutics performed – is linear, due to the impositions of written elaboration, it is the effort of the hermeneut to convey a process that by no means occurred in an orderly sequence of steps.

Teaching Practice or Methodology and future pedagogy teachers, that is, it was meant to encompass teachers with or without specific training in mathematics. Indeed, at this phase of the analysis the need to know the author is apparent. Professor Madsen Barbosa died before the beginning of the present research; therefore, we could not interview him. The information about him was obtained from his publications as well as the numerous interviews given to other researchers. As we know, any biographical effort that seeks to reveal who someone "really" was, how they lived etc., is as naïve as it is impossible. What can be done, in an attempt to get to know the person biographed, is seek remnants, indications, clues of what they did, how they presented themselves, with whom they lived, the work to which they dedicated themselves, etc., by talking to them in person, or with people who lived with them, and examining texts related to that individual. Such efforts enable the creation of a plausible version of someone (an identity) as well as assign meaning to what the person has done and/or been, under a variety of perspectives, as no one is a monolithic unit throughout their lifetime, but everyone is comprised of a multitude of desires, practices, achievements, dreams, and circumstances.

Professor Madsen Barbosa was one of the founders of the mathematics course created at the College of Philosophy, Sciences and Arts of Araraquara (*Faculdade de Filosofia, Ciências e Letras* - UNESP) in 1966, the same year the books studied herein were published. Also, in the 1960s, he was one of the founders and member of the first board of directors of the Study Group on Mathematics Teaching (*Grupo de Estudo do Ensino da Matemática*- GEEM) and the Regional Center for the Development of Mathematics Teaching (*Centro Regional de Aperfeiçoamento do Ensino de Matemática* - CRAEM), both collectives had a significant influence in the dissemination of the Modern Mathematical Movement (MMM) in Brazil. Those groups were directly linked to the MMM, and we observed that the books were handbooks of sorts for the

movement, thus, we also came to know and understand more about that movement. We also know that Professor Madsen Barbosa ministered courses to teachers and was himself a professor of pedagogy at the College of Philosophy, Sciences and Arts of Araraquara. These are important elements for our understanding of the reasons why Professor Madsen Barbosa, at that point in his career, devoted efforts to producing a textbook, more specifically, a manual focused on primary education.

Moreover, becoming acquainted with the author and understanding the phases of production of a specific symbolic form is as important as knowing aspects of its circulation, acceptance, target audience (in this case, the readers), the people involved in the MMM, the specific places and situations (historical and social contexts) associated with the production and appropriation process, the various reasons for its creation and the power relations that marked the symbolic form.

DH is flexible, it can be operated and articulated with other research techniques. In the case of the present work, we also chose to operate with the idea of Editorial Paratexts presented by Gérard Genette (2009), going through the peritexts (internal paratexts) and epitexts (external paratexts) of the collection.

Genette (2009, p.9) defined paratexts as all the elements that make a book a book, perceived as a book by readers and any audience in general. Evidently, Genette deals specifically with written texts. It is worth reiterating that Thompson endeavored to conduct a broader, more focused interpretation of symbolic forms. By understanding that a written text is a symbolic form (which the development of DH will show, even more clearly, during the analyses, increasingly revealing the symbolic form, even though it has already been taken as such), while attempting to analyze the collection *Mathematics, methodology and supplements for primary school teachers*, the approximation of Genette and Thompson became possible.

In an analysis whose objective is to deal with paratexts, the focus turns both to "internal" content of the work and other strategies and elements "surrounding" it, that is, the "external" content (GENETTE, 2009). Those are paratexts, thus, are on the horizon of our attention, for example, the body of the volume, the way themes are organized and treated, as well as the colors, shapes, headings, subheadings, foreword, dedications, illustrations, appendices, etc., that is, we seek to make a detailed and stringent description of each of the three volumes analyzed. Materials related to the work, such as publisher's lists that refer to the book, brochures, newspapers and magazines articles about the work, folders, advertisements, critiques, references used and recorded by the author, should also be considered. According to Genette (2009), such elements are also paratexts, as they make the work what it is, aimed at (and reaching) a specific audience, defending specific concepts, being presented, and discussed publicly in specific ways by the author, editor, reader, unlike any another work.

The interpretation/reinterpretation movement, in turn, is dedicated to weaving a link which intertwines the two "previous" analyses, discerning approximations and divergences between the "results", interpreting and creatively constructing a discourse about the work and the ideology that surrounds it, attributing possible meanings; "an interpretative explanation of what is represented or is said" (THOMPSON, 1995, p.375).

While readers analyze a book, a text, or even this dissertation, they weave interpretations and reinterpretations about it, as the meaning is neither in the text nor is it static: it is flexible, as it is assigned by the reader[7].

[7] We believe it is important to highlight this idea, as, usually the meaning is seen as being IN THE TEXT, and the job of the interpreter is to EXTRACT such meaning (the record of the author's intentions) from the text. We believe this position is the expression of a significant misunderstanding, which represents a whole theoretical field now outdated. As the highlighted in the sentence meaning is ASSIGNED to the text by the reader/interpreter, according to a series of predispositions, circumstances, and comprehensions of this reader/interpreter.

Therefore, the theoretical-methodological approach attributed to Thompson is taken here as a tool to interpret the information organized during the analysis process. "Hermeneutics presupposes a text[8] or expression which has something to convey and that can be interpreted or re-stated in another way" (GARNICA, 1993, p.46). Thus, it is a movement of thought through which the hermeneut must transcend the information presented to them as a static product, in the case of textbooks, in an attempt to highlight their own point of view, the intentions expressed by the author and, the way such intentions reach a certain audience, according to the interpreter's understanding. "Symbolic forms represent something; they convey something about something, and it is this transcendent character that must be understood in the interpretation process." (THOMPSON, 1995, p. 376).

Thus, we strive to understand the relations between production, circulation, and the interference of the sociopolitical context in this hermeneutic movement. From a reporting standpoint (a dissertation or thesis, book or article, oral presentation, for example), the interpretation/reinterpretation phase, which disseminates the hermeneutics conducted, can be viewed as the culmination of the interpretative process[9].

Starting the analysis

To construct a discursive analysis with a heedful gaze, it is necessary to review and try to attribute meaning to the three volumes that comprise the collection *Mathematics, methodology and complements*

[8] At this stage it should be clear to the reader that the word "text" here does not refer solely to written text, but also to the symbolic configurations that, within the world, are given to interpretation. This is a concept akin to that of Paul Ricoeur.

[9] Several studies have been conducted using DH and including DH as the object of analysis. Aiming at consistency, we tried to outline a panorama of such referential, so that the attentive reader can take it apart, if so desires, based on the references listed at the end. However, it is worth highlighting that the analyses (phases or moments) which constitute DH, in its elaboration processes, are neither sequential nor linear (contrary to the presentation process of these analyses), generally happening simultaneously.

for primary school teachers. Our analysis was based on three editions of the first volume: the first (1966), the second (1967), and the fourth edition (1968), which may be a sign of how well the books were accepted by the target audience at the time. This, in turn, may have occurred because the collection was part of dissemination efforts of the Modern Mathematics Movement (MMM) in Brazil, during that period, effected mainly by the mass-production of textbooks. The analyses of volumes II and III were conducted with the first edition (1966).

The way the themes are organized and treated enabled us to detect the interference of the MMM in the three volumes, as everything was presented according to an approach rooted in set theory (further explored, from a theoretical-formal point of view, in the first volume). The books included certain topics that nowadays will only be developed in higher education; in modern algebra courses, such as the discussion on isomorphisms. While the three volumes have similar characteristics, such as being guides for teachers, they also display striking differences. According to the foreword to Volume I, the collection comprises three parts: the "First part: Theoretical-practical arithmetic", which deals with mathematical content (from a formal point of view), the "Second Part: Methodology" which presents a discussion about the methodology that teachers should use in the classroom, and the "Third part: Complements", which, as the title indicates, is a supplement to the previous parts. By studying the three volumes, we noticed that each part corresponds to one volume of the collection.

Later in this article we will present an overview of the analyses of each of the volumes of the collection, followed by a conclusion regarding the analysis movement in its entirety, which considered the three volumes of the collection at concomitantly. The reader will certainly understand that it is not possible to present a detailed account of all the peculiarities and particularities of the hermeneutics performed. To readers interested in such

a level of detail, we suggest reading the research report (MILANEZ, 2020) through which the interpretation was initially made public[10].

Theoretical-practical arithmetic

According to the author, Volume I, reserved to theoretical and practical treatment of Arithmetic, was organized in a linear sequence of contents. There were not many pictures, however, those that have been included display quite simple editing art which, while exacted no significant visual appeal, could be easily reproduced by teachers on the blackboard.

Geometry is never used as support, and the contents are treated in an essentially algebraic form. Exemplar are the properties of sets, operations on sets, and the thoroughly covered basic arithmetic operations. Throughout the book, the idea of sets is very strongly used, and provides support, either implicitly or explicitly, to all discussions, and modern algebra underlies all concepts developed in the volume.

Mathematical rigor is also an indisputable characteristic. Whereas due diligence towards mathematical language is perceptible, there is a blatant disregard for the mother tongue, diagramming, graphic and textual reviews. Even though, according to the author, the justification of results presented should be given in the form of formal demonstrations, which are always suggested, though relegated to “second study”; there are no in-depth discussions/debates regarding the topics; most often present are rules and "*ways of doing*", derived from examples.

Briefness is another element that characterizes the volume: all chapters and respective topics are quite short, which led us to assume that contrary to what one might imagine based on the foreword, the author envisioned his ideal reader as a teacher who was already familiar with the concepts and operationalizations presented. There are no pedagogical

[10] More specifically, we suggest reading that work as it presents extracts of the collection that justify each of the assertions made about it herein.

discussions accompanying mathematical treatment, which, it must be pointed out, is quite formal.

In the sections intended for exercises, some were supposed to be tackled following the presentation of content, while others should be left for a second study[11]. It is also noted that this "second study", which the author often mentions (when demonstrations and the most "sensitive" exercises would be done), is composed of a series of demonstrations, of which only a few steps are presented, with little detail, probably aimed at teachers with specific mathematics training, who are part of the author's target audience. Few chapters include what the author calls problems, and the chapters that do include problems, include mostly directive questions such as: calculate, determine, verify, represent. Some answers are presented, but none of them with any deeper elaboration.

From proposed methodology to primary school teachers

Volume II of the collection, as announced in the foreword to the first volume, focuses on the study of methodology for teaching arithmetic, aiming to instruct the teacher. It is marred by an authoritative and authoritarian tone, regarding what they "should" do in their daily practice to treat the contents presented in the first volume. The text is quite descriptive, with many examples of didactic resources, and, in this sense, distant from that theoretical text focused on mathematical content that characterizes Volume I.

Although the idea of sets is also very marked, the author's efforts to present practical questions and procedures are more noticeable in this second volume, abandoning some of emphasis on formal language that characterized the first, clearly anchored in the Modern Mathematics

[11] It should be pointed out that "second study" is not a specific topic or volume of the collection, but a simple indication left in some passages, meaning that the content (exercise, demonstration, or explanation of concepts) should be revisited by the teacher at another time, not necessarily in the order it appears within the body of the text.

Movement. Even though it would be a mistake to reduce the MMM solely to the emphasis on formal use of language, whether as a guideline given, or as the way that guideline was understood and effected, concern for mathematical language is one of the elements that have become a hallmark of those beliefs. Regarding the MMM, there is also the author's own record of having tested his suggestions in real classrooms. However, it is unclear in the text, whether such tests were systematic and supported by any specific protocol, which followed ideas of the MMM, or if they were only based on the author's personal experiences during his long career as a teacher.

The text seems more like a conversation between the author and teachers, always seeking to bring students and their difficulties into focus, as well as the procedures considered most appropriate for teaching. A significative feature of this volume is the attention to teachers' individuality, their circumstances, and the variety and peculiarities of students' profiles at the beginning of schooling, as well as the emphasis on strategies for working with teaching aids, trivia, games, and problem solving. Thus, a certain asynchrony between discourses could be detected, when comparing the author's approaches. At times he relies on arid and stringent formalism without concessions, other times approaches teachers with a style that seems to be an attempt to create an atmosphere of complicity. Sometimes, the experienced mathematician speaks, making no concessions regarding the requirements for rigor, language, and formalization. Then, it is the teacher who speaks, seemingly driven by the lacking and precarious teacher education, at the time, dealing with this *new* mathematics being introduced. However, in both cases it is a discourse imbued with a good deal of authoritarianism, sometimes based on extensive knowledge of the field in which he works, other times guided by the excellence of his teaching practice. Nonetheless, both discourses are disjunct; one identity crumbling in face of the other.

Several passages contained the author's explicit criticism towards other authors and their work, some of whom he even blamed for misusing the ideas of the MMM in Brazil. Even though it is strongly suggested that both memorization and rote treatment of operations should be discouraged, the rules, many of which only enunciated, but insufficiently justified (if ever), prevail throughout this and other volumes of the collection. This impression was reinforced by the prevalent idea of methodology as an articulated sequence of steps; step-by-step descriptions, characterizations of concepts into items and sub-items, use of (and emphasis on) specific nomenclature, and exhaustive treatment of cases.

The graphic treatment of the volume is quite elementary. This can be seen in the absence of colors, the simplicity of schemes and drawings, as well as printing problems, such as many blurs and flaws. The language used by the author, although familiar to teachers, was not the most correct from a grammatical standpoint. The use of certain words and expressions brings methodological discussions closer to the experience, apparently lived by the author, and common sense, rather than specific authors and work regarding the subject, which would have finer, well-worked approaches from a theoretical-educational standpoint. In some cases, the justification for the author's suggestions to teachers seems to be rooted in his knowledge about what mathematics is, his *expertise* on mathematical content and methods, instead of specifically on well-founded didactic and pedagogical knowledge. In any case, the final format of the volume, regarding its materiality, is careless, both from the point of view of graphic design and textual review.

The frequent and emphatic suggestion that the teacher should use several teaching resources (such as games, narratives, drawings, diagrams, etc.) brings a different tone to the text, marking it remarkably. Moreover, even when not making concessions, which we often deemed necessary, the author claims to be aware that his text will be used by

teachers whose training is deficient and who keep a certain "distance" from the mathematics that they should teach.

Complements

Having reached Volume III, it is interesting to reiterate the organization of collection to account for its title. The first volume deals more specifically with what could be called theory of mathematics and, as the author himself claimed, aims to lead the teacher to learn, as they must “know more” than they teach. Therefore, the first volume, according to the author, contains much more information than teachers will use to teach primary school. It is permeated with formal mathematical language, definitions, and specific nomenclature, as this is a book written by a teacher for teachers, not for a lay audience.

The objective of Volume II, on the other hand, is to deal with methodology (the practical part) of teaching, and recommendations for teaching each specific content. It is a volume of “pedagogical guidelines”, comprised of texts dedicated to teachers, encouraging them to guide students. According to Oliveira, Silva e Valente (2011, p. 134), during the MMM in Brazil, primary school teachers were expected “to take the role of guiding discoveries, primarily intuitive, that would gradually be systematized, formalized and treated without great concern for symbology”. From such recommendations, it can be understood that the collection strictly follows only those regarding fostering discoveries; a suggestion thoroughly emphasized by the author throughout. However, although the author suggests teachers rely on intuition, it is abundantly clear that the collection does not renounce formal symbology, notation, and nomenclature. Considering Oliveira, Silva, and Valente (2011), it can be said that whereas the emphasis on the often precocious and barely discussed systematization and formalization leads the book to fiercely defend the Modern Mathematics Movement, it also subverts it, perhaps

due to the author's plentifully stated confidence in the excellence of his mathematics education, which thus serves as a guide for the elaboration of the collection, rather than the MMM.

Volume III, which contains the complements mentioned in the title, revisits each of the contents already presented in previous volumes, incorporating some new items into them. According to the author, the objective of the third volume is support teachers who aim at more varied and, in some cases, even more amusing classes.

The collection is not a series of three books to actually be adopted in the classroom, as students are not their target audience; it is a collection that can help teachers in their school routine, something that, in more modern jargon could be called "*paradidactic*", that is, a series of supplemental didactic-pedagogical support texts. A statement by the author, on page 99 of Volume II, clearly explains the intention of his collection: "/.../ assist the training of primary school teachers, or /.../ that this book serves as a reference". It is important to remember that, also according to the foreword to the first edition of Volume I, this third volume should serve only as a reference book for primary teachers and pedagogy professors, as a supplemental resource for mathematics teachers, and as a reference book or textbook for professors of the course: "Teaching Practice and Methodology".

Themes related to geometry were quite neglected within the collection, occurring only in this third volume, as a "complement". Thus, it should be noted that, in that sequence, the treatment of *volume* precedes that of *area*. In an interview to Lima (2006), Professor Madsen Barbosa reported that the treatment once given to volume of bodies, was basically reduced to spatial geometry theorems, "which /.../ [were] a bit heavy for the level of our students at that time", and that the idea of working with the Cavalieri principle arose from the Study Group on Mathematics Teaching (GEEM). It is worth pointing out that, within the collection, the treatment of "volumes of bodies" is succinct, like that regarding area. Since the former

is not a prerequisite to teaching the latter, this might indicate that the sequence chosen to present them in the book was fortuitous, or, that the third volume of the collection was elaborated as a set of independent topics, to be used according to readers' needs.

Let us, once again, consider the assertion of Oliveira, Silva e Valente (2011, p. 163):

> Far from being abandoned by the author, the teaching of geometry is presented as a new proposal, in which two significant trends are identified. One that incorporates geometric transformations, derived from the approach advocated by Félix Klein; the other, hegemonic, reinforcing Euclidean geometry with a different approach, whether through the incorporation of new axioms, or inclusion of experimental geometry (OLIVEIRA, SILVA, VALENTE, 2011, p. 163).

Regarding geometry, the collection examined herein does not display any such characteristics.

In volumes II and III it is usual to find texts akin to conversations between the author and teachers/ readers, and between the readers / teachers and students. The author suggests courses of action, possibilities and even anticipates students' doubts, to which teachers will have to respond, and the comments that they should consider or make. However, his tone is authoritative, prescriptive; and phrases beginning with "The teacher must..." are constant. However, at times the author tries to weave, say, a lighter discourse, whose aim seems to be creating an atmosphere of intimacy and complicity with readers.

Throughout the collection it is easy to see that most of the content covered in the previous chapters is reused, continued, complemented, and exemplified, aiming to build a bridge between themes that have been, are being and will be discussed. The elaboration sequence and logic of the volumes of the collection is dictated by the author, who certainly had to follow the legislation and regulations for academic content in force at the time. Nonetheless, the author's discourse is charged with an authority that he

gained, and invokes, through his previous performance as a mathematician and a mathematics professor.

The idea of sets permeates and supports all work regarding mathematical content; however, this explicit emphasis is diluted from volume to volume. Especially in Volume I, in which mathematical concepts are discussed and some demonstrations given, this central idea underlying the Modern Mathematical Movement, as well as ideas of formalization and structuring are evident.

In all volumes mental or approximate calculations are recommended, as well as working with games and an educational approach that uses real and everyday situations as relevant motivations. At the same time, it is clear that although there is criticism regarding memorization and rules "given" by the teacher, the three volumes of the collection often use these very recourses which the author criticizes. This is quite clear in all volumes, but specifically conspicuous in the third, in which the formulas for volume and area abound, without any detailed justification. Also, in the third volume, an approach we know the author holds near and dear is most noticeable, the so-called *Recreative Mathematics*. He would subsequently explore it in other books, and in dissertations and theses he oversaw when acting as an advisor in graduate courses.

Throughout the three volumes, simple black and white pictures, without any graphic enhancements prevail. While addressing elements of geometry, the meager number of illustrations is even more noticeable in the third volume, which is also the only one in the collection that contains a list of references.

Summarizing to close

The expression "hermeneutic analysis" must be viewed as synonymous to "reading", thus a hermeneutic analysis such as the one we

conducted cannot be definitive. It is not a question of any reading, but a specific reading whose aim is to attribute plausible meanings, anchored in the object analyzed and the history of such object and its context, and the history of how such a symbolic form, circulates in the world. Therefore, such an analysis can always repeated, complemented, increased, or reduced, depending on who conducts it, and the materials available regarding what is analyzed.

In the case of the present work, we wanted to present a reading of the work Mathematics, methodology and complements for primary teachers, by Professor Madsen Barbosa, a collection comprised of three volumes, produced in the 1960s. We contemplated the relations and approximations among the volumes, which were evident to us, and the ideas of the MMM, which led us to seek elements to understand this movement, while trying to attribute meaning to the collection, as the analyses of the object always give rise to searches for complementary materials that help understand it. The understanding deriving from such materials, in turn, feeds possible readings of the object being analyzed.

This conclusion is close to what has been called the "hermeneutic circle": a continuous movement through which one interprets to understand and, while understanding, interprets. Thus, every reading takes place within this circle that can always be restarted, redone, revised, resumed, and complemented.

Our reading was conducted in addition to what we believe is a detailed and careful description and interpretation of the three volumes that comprise the collection, elements of the author's life and academic production, as well as educational aspects of the period when the work was produced. Interweaving the analysis of the text "itself", that is, the books, and the analysis of the context (the time of production, circulation area, the author, links to certain ideas, etc.), as proposed by Thompson, and driven by the work's paratexts, as proposed by Genette, we believe we have presented a reading which highlights aspects that can be seen as

didactic and pedagogical innovations, based on the author's history, as well as the historical scenario in which they were produced, and more conservative aspects, such as advocating values and concepts germane to the practice of mathematics, such as, the precedence of content linked to the need for formal treatment, at times quite premature and disconnected from the needs of schools and the training of teachers of his time. It seems that the shift from scientific practice of mathematics into pedagogical practice is one of the fundamental characteristics of the ideology surrounding the collection. The option for focusing on problem solving, through various heuristics, recreational mathematics, the care for the children's psychological aspects, the use of specific instructional materials, advocating the need to refuse rote-learning approaches, memorization, and repetition etc., are among aspects that, at the time, responded to a massive movement of renewal of mathematics teaching. Nonetheless, due to that renewal movement, it is impossible to assess if such option was merely declaratory or legitimate, as within the collection there were shifts in practices, as previously mentioned.

For our analysis, we used interviews given by the author to various researchers, as well as several editions of each volume and, of course, complementary materials; basically books, articles, academic papers and clippings from journals and newspapers. The present work resulted from all such materials, focusing on the collection Mathematics, methodology and complements for primary teachers, by Professor Madsen Barbosa. There are still aspects to be further explored, such as the "transit" of Professor Madsen Barbosa's collection within the GEEM, a group in which the approach of GRUEMA (Cf., p.e., Vilela, 2009), regarding primary education, seems to have been more influential and significant. Further apart from the values advocated by Professor Madsen Barbosa, which we believe were rooted in the practices and values promoted within pure mathematics and opting for an elaboration driven towards the education of primary school teachers, regarding the history of Brazilian mathematics

education, the works published by GRUEMA enjoyed more widespread and significant circulation and permanence. Considering the number of editions and the editorial and graphic characteristics of Professor Madsen Barbosa's collection, for example, we can see that they are disparate from those of the books prepared by GRUEMA. Such observations, although inceptive, may constitute openings for a complement to the present hermeneutic. Likewise, we believe that a more detailed study could be made on the references used by Professor Madsen Barbosa to support the elaboration of his collection. The list of authors and works cited by him regarding books and researchers considered germane for the MMM is lacking, and noteworthy the absence of references to foreign authors (even those who had attended events in which Professor Madsen Barbosa participated in Brazil) who, during interviews, he claimed were important for his immersion in the movement. At first glance, in the author's list of references, the echoes of the New School are more distinctive... These are two of the various themes that could still be explored to complement our reading regarding the collection Mathematics, methodology and complements for primary teachers.

At the time of the elaboration and publishing of the collection, Prof. Madsen Barbosa was involved in several activities. In addition to being a member of GEEM, he served on the Group's board, taught courses, gave lectures, worked in a specialization course, participated in meetings, and produced teaching materials. Therefore, it was in this context of varied and intense production that the collection was designed, prepared, and published. One of the author's concerns, underlying all such activities, was teacher education, which he considered insufficient and inefficient, resulting in inadequate training, which led to unprepared teachers going into classrooms. It is worth pointing out that the adjective "unprepared" (used by the professor himself) apparently not only referred to the insufficiency of mathematical training, but also qualifies those teachers who did not adhere to the MMM. Whereas this can be perplexing, as

Professor Madsen Barbosa was aware of the teacher-training scenario and its problems in relation to mathematics, which would certainly bring difficulties for teachers using the collection, it can also justify the emphasis on form that pervaded the whole collection. Discussing mathematics with teachers was not enough; it was also necessary to urge them to approach and accept a specific ideal, that of the MMM, under the terms proposed by the author.

Finally, we must point out that we conducted this study considering the possibility of divulging it not only within the academic research environment, but also in mathematics undergraduate courses, hoping to foster new readings, of this and other works, ultimately contributing to the education of teachers who teach mathematics and the history of Brazilian mathematics education.

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