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Manifestations of future teachers who teach mathematics in actions involving quantities and measures

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SUMMARY

This article aims to discuss future teachers' manifestations in the face of a Learning Trigger Situation about quantities and measures, specifically about volume. In the space of the Mathematics Club of the Federal University of Santa Maria (UFSM), academics participated in studies and actions related to the teaching and learning of mathematics, based on the Historical-Cultural Theory, especially the Activity Theory and the Teaching Guiding Activity, originating the empirical data of this work. The analysis presented indications that future teachers, when faced with Learning Trigger Situations, do not always overcome the appeal to sensory material as the only way to find a solution, which characterizes empirical thinking. As a consequence of the results, in the initial training, the relevance of access to spaces that allow reflections to go beyond the knowledge considered as a deepening of the future teaching area is pointed out.

KEY WORDS: Initial Teacher Training. Quantities and Measures. Math Club.

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Manifestaciones de futuros profesores que enseñan matemáticas en acciones que involucran grandezas y medidas

RESUMEN

Teniendo como objeto de reflexiones la formación inicial, este artículo se propone discutir sobre manifestaciones de futuros profesores delante de em situación desencadenadora de aprendizaje relativa a grandezas y medidas, específicamente sobre volumen. Em el espacio del Club de Matemáticas, de la Universidad Federal de Santa Maria (UFSM), académicos participaron de estudios y acciones relacionadas a la enseñanza y el aprendizaje de matemáticas, pautados em la Teoría Histórico-Cultural, especialmente em la Teoría de la Actividad y em la Actividad Orientadora de Enseñanza, originando los datos empíricos de este trabajo. El análisis presentó indicativos de que futuros profesores, cuando puestos delante de situaciones desencadenadoras de aprendizaje, ni siempre superan el llamado al material sensorial como único modo de búsqueda de solución, lo que caracteriza el pensamiento empírico. Em virtud de los resultados, se señala, em la formación inicial, la relevancia del acceso a espacios que posibiliten discusiones y reflexiones que permitan ir más allá de los conocimientos considerados como profundizaciones de la futura área de enseñanza.

PALABRAS CLAVE: Formación Inicial de Profesores. Grandezas y Medidas. Club de Matemáticas

Manifestações de futuros professores que ensinam matemática em ações que envolvem grandezas e medidas

RESUMO

Tendo como objeto de reflexões a formação inicial, este artigo propõe-se a discutir sobre manifestações de futuros professores diante de uma situação desencadeadora de aprendizagem relativa a grandezas e medidas, especificamente sobre volume. No espaço do Clube de Matemática da Universidade Federal de Santa Maria (UFSM), acadêmicos participaram de estudos e ações relacionadas ao ensino e aprendizagem de matemática, pautados na Teoria Histórico-Cultural, especialmente na Teoria da Atividade e na Atividade Orientadora de

Ensino, originando os dados empíricos deste trabalho. A análise apresentou indicativos de que futuros professores, quando colocados diante de situações desencadeadoras de aprendizagem, nem sempre superam o apelo ao material sensorial como única forma de busca de solução, o que caracteriza o pensamento empírico. Como decorrência dos resultados, aponta-se, na formação inicial, a relevância do acesso a espaços que possibilitem reflexões que permitam ir para além dos conhecimentos considerados como aprofundamentos da futura área de ensino.

PALAVRAS-CHAVE: Formação Inicial de Professores. Grandezas e Medidas. Clube de Matemática.

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Introduction

Considering that mathematics is a historical and cultural product and that, from it, it is possible to create new interactions in the social field, it allows to understand it beyond a curricular component, reduced to numbers, formulas and memorization. It acquiesces in the possibility of seeing its meaning as an instrument to be used in solving socially established problems. And it is the necessity to solve these problems, derived from situations experienced by humanity, that generates reasons for the human being to transform the objective world, leading to new knowledge. This movement ensures that humanity develops, because it is from the appropriation of knowledge that subjects develop their maximum intellectual capacities, establishing themselves as human beings. In this way, we have mathematics as a fundamental instrument in the process of humanization, since it, like so many others, provides new conditions for subject to humanize themselves through their activity.

Leontiev (1988), in his Theory of Activity, helps us to understand human activity as a psychological process that, starting from a need, leads the subject to appropriate the products historically produced. Thus, it is not any action that will make them develop new psychic aptitudes, these need to

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come from a particular need that will allow them to set themselves in motion, guided by a reason, tracing out actions that require knowledge.

Conducting this discussion to the educational field, Moura (1996) proposes the Teaching Guiding Activity (AOE), understood as a theoretical basis referenced in the Historical-Cultural Theory and, more specifically, in Leontiev's Theory of Activity, and methodological, bringing elements that make it possible to organize the teaching of mathematics, understood as a cultural element, derived from human needs and, as such, the right of acquisition of all.

To think about the process of appropriation of knowledge aiming at development, requires objectifying theoretical knowledge, which is the one that promotes the interrelations between the internal and external aspects of the object, the totality and the appearance, the original and the derivative (DAVIDOV, 1982). This implies that the subjects - both students of Basic Education and teachers in initial training - are placed in situations that promote the need to mobilize knowledge that, when used as a tool for solving problems, can lead to the acquisition of new intellectual capacities.

From this perspective, we understand that the future teacher, as the subject whose main activity will be teaching, is being constituted from the appropriation of knowledge that favors him to occupy this new social role (no longer of student, but of teacher). Hence the relevance that, in the graduation course, there are opportunities of appropriation of the object of knowledge with which he will deal - such as mathematics - in order to allow him to unveil its essence through its particularities and its internal details, as well as through the very process of constitution of the object. And this comes from a learning based on theoretical thought (DAVIDOV, 1982).

This article is inserted in the context of learning to be a teacher who will teach mathematics, with the purpose of discussing future teachers' manifestations in the face of a Learning Trigger Situation regarding quantities and measures, more specifically about volume. Thus, we discuss actions developed in the extension project of the Mathematics Club (CluMat) of the Federal University of Santa Maria (UFSM), the core of this research, with scholars4 from the undergraduate courses in Special Education, Mathematics and Pedagogy.

The research that we present here refers specifically to the study developed among the members of the project which was triggered by a situation focused on the concept of volume, which was intended to foster discussions regarding its teaching and learning, since it is a content worked in the initial and final years of elementary school.

Therefore, we will briefly present some theoretical notes, as well as the paths crossed by the research. We will then discuss the manifestations of future teachers in the face of the proposed situation. Finally, we will bring some considerations about the study carried out.

Some theoretical notes

Mathematics, as a curricular component, is mostly understood as a list of rules and formulas to be memorized for the moment of evaluation and then forgotten. However, by understanding systematized knowledge - like mathematics - as a cultural and historical product, it supplants this reductionist vision and starts to assume its social meaning as an instrument of interaction, action and transformation of the space in which the subject lives.

Knowing that different interactions take place in school and that it is there that systematized knowledge is presented to the student, we can recognize it as an established place to enable the appropriation of knowledge produced socially and historically, such as mathematics. Thus,

> School education, understood as the satisfaction of a collective need to include newcomers to the community, should lead to the

[4] The author uses the term "scholars" in the female gender in the original language, since the group was composed only of women.

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> appropriation of knowledge that will allow them to be identified as part of that community. (MOURA; SFORNI; LOPES, 2017, p.89).

From the perspective of what the authors bring us, the school is (or should be) one of the places intentionally organized for the subject to appropriate the historically elaborated culture that will enable them to act and insert themselves in the social life of the group to which they belong. Knowledge, as well as its organization in the form of curricular components, is the result of a process that is part of human history and, therefore, its appropriation should serve as an instrument for the development of the maximum intellectual capacities of the subject.

It is in the human process of appropriation and transformation of knowledge that new concepts are constituted, with new qualities, as a cultural product that favors social interaction.

> Thus, knowledge, as a cultural product to be learned, becomes an instrument of intervention of the subject, by appropriating, in its own way, a form of intervention in its cultural environment. It is a matter of giving meaning to what is learned, not only for the subject, but also for the entire school institution. Giving meaning to the learning of mathematics is also giving meaning to education. (MOURA, 2011, p.54, Translated from Spanish)[5].

Among so many types of knowledge that help in the subject's intervention in the social environment, we have the mathematical knowledge that, perceived as the output of a historical process, is an important agent to solve collectively established problems, resulting in the improvement of human life. This justifies the importance of learning this discipline at school.

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[5] "Así, el conocimiento, puesto como produto cultural a ser aprendido, se torna um instrumento de intervención del sujeito, al apropiar-se , a su modo,de una fomra de intervención em su médio cultural. Se trata de dar significado al qué aprender; no sólo para el sujeto, sino también para toda la institución escolar. Dar significado al aprendizaje de la matemática es dar sentido también a la educación."

> Learning a mathematical language is more than just learning codes and rules. It is learning a method of knowing and transmitting what is known. It is also knowing how to apply what is known in the solution of problems that are proper to them in the coexistence with others. It is making oneself human. (MOURA, 2011, p.54, Translated from Spanish)[6]

Mathematics is much more than a knowledge materialized as a static curricular component, it is a language produced in response to human needs and facilitates the subject's insertion into community life, whose interactions will reflect in new qualities in their social actions.

But the appropriation of social experience is not an innate process. It happens on two levels, as Vigotsky (2007, p. 23) teaches us.

> In cultural development, all functions enter the scene twice, on two different levels: first, on the Social, then on the psychological level; first as a form of cooperation between people, as a collective and interpsychological category, then as means of individual behaviour, as an intrapsychological category.

The movement of higher psychic functions exists first on the social plane, and then it passes to the individual plane - of each subject - which leads us to understand the importance of teaching in this process. For Vigostki (1982, p. 237, translated from Spanish), "instruction is fundamentally ahead of development"7. In other words, it is the social learning - especially that acquired through an intentional process of the one who teaches - that will subsidize the development of each subject.

In this direction, we have that teaching is a promoter of development, and the teacher, in organizing it, needs to take into account the learning

[6] "Aprender un languaje matemático es más que aprender códigos y reglas. Es aprender un método de conocer y transmitir lo que se conoce. Es también saber aplicar lo que se conoció em la solución de problemas que les son proprios em la convivencia com otros. Es hacer-se humano."

[7] "*la instrucción se adelanta em lo fundamental al desarrollo*"

that will subsidize the acquisition of new skills by the students, which is not provided by any activity. Thus, the activity is not understood as a simple action, but that which provides psychic development,

> by activity, we designate the processes psychologically characterized by what the whole process is directed to (its object), always coinciding with the objective that stimulates the subject to perform this activity, i. e. the reason. (LEONTIEV, 1988, p. 68)

In the case of school education, for the student to be active in such a way as to promote the development of higher psychological functions through a certain knowledge, there must be a need to drive the motive, for them to acquire socially established knowledge, proposed by the teacher.

> The first condition of every activity is a necessity. However, necessity in itself cannot determine the concrete orientation of an activity, for it is only in the object of the activity that it finds its determination: it must, so to speak, be found in it. Since necessity finds its determination in the object (if it is "objective" in it), the said object becomes the motive of the activity, that which stimulates it. (LEONTIEV, 1978, p. 115)

The student's activity - perceived from this point of view - is part of a need that is embodied in the motive, responsible for directing their study actions, and involves motives, desires, needs and emotions. It is the activity that moves him towards the appropriation of knowledge.

However, two types of knowledge must be considered, the empirical and the theoretical, which "correspond to two types of thinking, equally empirical and theoretical, so that the individual can approach reality in two very different ways" (RUBTSOV, 1996, p.120). The empirical knowledge is achieved by comparing and observing objects, creating only generalizations from the attribution of judgment, and is the result of the direct relationship with reality. The author points out that any empirical knowledge is based on observation and reflects only the external properties of objects, relying entirely on concrete representations.

Theoretical, on the other hand, originates from a transformation of objects and reflects the relationships between their properties and their internal links. And "as soon as thought reproduces an object in the form of theoretical knowledge, it surpasses sensory representations" (RUBTSOV, 1996, p.129). Thus, theoretical knowledge becomes a mental action, in which

> The first moment allows the man to become aware in the thought process that independently of it the object exists, given as a premise of the activity. This premise communicates to the concept the moment of passivity, contemplative character and independence from the objective content. And nevertheless, to have notion of the given object supposes to report it mentally, to construct it. This operation of constructing and transforming the mental object is equivalent to the act of understanding it, explaining it and revealing its essence. (DAVIDOV, 1982, p.301, translated from Spanish)[8]

Bringing this discussion to school education, Freitas (2016, p.403-404) explains that

> For Hedegaard and Chaiklin (2005), theoretical knowledge can be considered as a tool to associate the core concepts of a subject with daily, local and personal knowledge, and it is this character that should be emphasized in the students' learning. Studying and learning an object in this way makes the student able to rely on concepts as a general perspective to act considering the

[8] "El primer momento le permite al hombre tomar conciencia en el proceso del pensamiento que independientemente del mismo existe el objeto, dado como premisa de la actividad. Esta premisa comunica al concepto el momento de pasividad, caráter contemplativo y de independencia respecto al contenido objetivo. Y sin embargo, tener noción del objeto dado supone reporducirlo mentalmente, construirlo. Esa operación de construir y transformar el objeto mental equivale al acto de comprenderlo, explicarlo y revelar su esencia."

> perspective of the particular, local context, and understanding the particular and local is like a concretization of the general.

But, as the author herself warns, this is not a process that occurs mechanically and automatically. It is pertinent to adequately organize the teaching in order to enable students to appropriate the objects of knowledge and their respective theoretical concepts, aiming at the development of their maximum intellectual capacities. In this direction, Rubtsov (1996, p.131) indicates that

> The adoption of a theoretical attitude in face of reality and the use of appropriate means to acquire theoretical knowledge are considered to be specific needs and sufficient reasons for learning.

Hence the relevance of proposing to students learning problems confronting them with a situation "whose solution in all concrete variants calls for an application of the generalist theoretical method" (RUBTSOV, 1996, p.131) and allowing them to create general forms of action to solve them.

In view of the appropriation of theoretical knowledge, understood as the synthesis of the production of culture from human needs, we highlight the Teaching Guiding Activity (AOE) as a theoretical and methodological basis, which helps us to organize the teaching of mathematics from some guiding principles, in such a way that it contemplates the movement passed by humanity when producing knowledge. The AOE considers the activity as a precursor of the subject's development because it seeks to bring the student closer to the human need to create the concept through what it calls the Learning Trigger Situation.

> In teaching and learning actions the triggering problem appears under certain unique conditions, i.e. materialized in the form of a learning trigger situation. This situation seeks to create the

> means or instruments so that the subjects engage in the process of solving that problem, which synthesizes the concept one wants to teach. (NASCIMENTO, ARAUJO, 2019, p.681, italics in the original)

It is through the Learning Trigger Situation that the teacher will enable students to appropriate the social product summarized in the knowledge. In the search for general ways to solve the problems collectively the subject can learn a new knowledge.

It should be noted here that we believe that in the initial training of teachers, this process can also contribute to the acquisition of new knowledge needed for teaching. In the initial training process, future teachers attribute new meanings both to the exercise of pedagogical work and to the object of their teaching, because, even if they already have experiences related to this, they have been frequenters of the school environment for a long time and need to go beyond the pupil's eyes.

Authors like Pimenta (1999, p.120) have been pointing out to us for some time that, when students arrive at the graduation course, they bring with them knowledge about what it is like to be a teacher, through their experiences of interacting with different teachers in their school life. Such experiences allow them to say, for example, "which were the good teachers, which were good in content, but not in didactics, that is, they did not know how to teach". Then it can be said

> […] the challenge, then, posed to the initial training courses is to collaborate in the process of passage of the students from their seeing the teacher as a student to their seeing themselves as teachers. That is, to build his identity as a teacher. For what the knowledge of the experience is not enough. (PIMENTA, 1999, p. 20)

It is in this challenge presented by the author over two decades ago, but still to be faced, based on the assumptions listed in this item, that our research is placed, the paths of which will be presented below.

Research Paths

Initial training is a space in which students need to take on a new social role: that of teacher. This knowledge will result in possibilities to contribute with their future pedagogical practice, with their need to organize the teaching when they are before their students.

From this premise comes the relevance that in undergraduate courses subjects have opportunities to appropriate different knowledge, which leads us to the space where the research was developed: the Mathematics Club. UFSM's CluMat began its actions in 2009 and, since then, it has focused on the interaction between the university and the school of Basic Education, continuously providing the movement of study, organization, development and evaluation of teaching activities triggered in public schools in Santa Maria.

The participants in the research were 11 undergraduate students, 2 from Special Education, 4 from Mathematics and 5 from Pedagogy. During the year in which the research was developed, these academics participated in the development of actions involving different mathematical contents. However, in this article we will present a cutout of the study carried out, directing our gaze to those that involved the knowledge of quantities and measures, more specifically about volume. The actions were the following: individual reading; collective discussion of texts; and development and collective discussion of Learning Trigger Situations.

By providing these actions focused on mathematical knowledge, the intention was to pay attention to the teaching movement, understanding that discussing knowledge considered basic - which should already be appropriated in Basic Education - allows the attribution of new meanings to

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them, since their gaze will no longer be of a student, but of a subject constituting a teacher. Although later actions have been developed with students in Basic Education, these will not be addressed here, since our focus at this time is on future teachers.

All moments were recorded in audio and video with the proper authorization of the participants and following the guidelines of the Ethics and Research Committee of our institution (CEP / UFSM). Later, because we understood that "written or spoken phrases, gestures and actions that constitute scenes may reveal interdependence between the elements of a formative action" (MOURA, 2004, p. 276), we transcribed the speeches and these, originated the scenes, selected for analysis.

In the next sub-item, we will discuss two scenes about situations developed, in view of our objective in this article, which is to discuss the manifestations of future teachers in the face of a Learning Trigger Situation about quantities and measures, more specifically about volume.

The Learning Trigger Situation as a Discussion Point

We understand knowledge, as previously explained, as a premise for the interaction of the subject with the social environment in order to appropriate human culture. Thus, from a very early age, we have acquired diverse spontaneous knowledge that allows us to interact in the social groups in which we live, such as family, friends and school. However, to create new forms of interaction and even transform the established ones, it is necessary that the knowledge is expanded and goes beyond empiricism. This requires that the subject be offered possibilities and experiences in which to use the knowledge woven by humanity, advancing from the empirical level to reach the theoretical, since it is not inserted directly into the social environment.

It is with this idea that we have supported ourselves in the proposal of Moura (1996) of the Teaching Guiding Activity, by means of Learning

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Trigger Situations, aiming at the appropriation of concepts. As the author proposes, these should be problematized based on the movement of the concept passed by humanity, aiming that the subject puts themselves in the need of its use as a tool to solve the proposed situation. Therefore, when we intend to discuss the concept of volume with future teachers, we understand that we could not start from their syntheses materialized in mathematical formulas, but from a Learning Trigger Situation proposed by an everyday situation.

The 11 participants were divided into two groups and were presented with a problem concerning the transport of Montessori Golden Beads. The criterion for their formation was the participation of at least one component of each course in each group, to eliminate possible distinctions derived from the initial formation. To develop the CluMat actions in the schools, the academics often use some pieces of the sets available from the University's School Mathematics Laboratory (LEME), but they do not take the complete box[9] (due to the difficulty of transportation) and need to carry them in another container (cardboard box, pot,...). Several times the construction of cardboard boxes specifically for this transport had already been considered, which led to the following learning problem: "How can we find out which is the smallest possible box to carry the material in order to use the smallest amount of cardboard?". Each group received the material that was normally taken to school: one cube (thousand), ten plates (hundreds), ten bars (dozens) and ten little cubes (units) of the Montessori Golden Beads. Being able to handle this material, they would have to discuss how to find the smallest cardboard box that could be assembled to contain this material.

In order to discuss the initial demonstrations, we bring in Scenes 01 and 02 the first resolution sent by the groups. We emphasize that, after these discussions, there was a presentation and general discussion between

[9] The Montessori Golden Beads are based on one of the materials designed by the physician and educator Maria Montessori to work with arithmetic, especially the Decimal Numeral System. The sets (organized in wooden boxes) that make up LEME's collection are made up of pieces of wood or EVA foam: 10cm edge cube, representing the thousand; 10cmx10cmx1cm plates, representing the hundreds; 10cmx1cm bars, representing the dozens; and 1cmx1cm little cubes, representing the units.

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the two groups, seeking a collective synthesis, but because of the space in this article and because it is not the objective at this time, it will not be presented here.

Chart 01: Scene 1- Group 1's Resolution of the Learning Trigger Situation

Description of Scene 1: The following dialogue refers to the discussion of the solution by Group 01.
1. Rose: *We actually started by looking at the sizes and thinking a little bit, but we didn't stop at the calculations. Here are all the possibilities we tried... [showing the various ways they found of "piling up" the material]* **2. Adviser:** So, [...] you were making your assumptions from the material? **3. Rose:** *Yes, and we all saw, in fact, the same thing [showing the way they all found the best]. We thought about how the children would do it in their process. Then we arrived in a box that would fit two thousand and two hundred, in this case there was room for nine bars [the total volume of the bars was 2110].* **4. Adviser:** *So, you made a little box that fits two thousand and two hundred [units of cubes]?* 5. **Rose:** *That's right.* **6. Adviser:** *Did you get to two thousand and two hundred later, or did you first start from it to build the box?* **7. Rose:** *Later. First we assembled the measurements as we thought it needed to be.* **8. Adviser:** *What measure did you find by putting the little cubes together?* **9. Rose:** *10, 11 and 20* [parallelepiped's edges]. [...]

Source: Math Club Collection

With this scene, which presents the resolution of the problem by Group 1, we want to draw attention to a demonstration: the call for sensory experience. The dialogue indicates that the solution was found only by handling the material in order to arrive at the hypotheses to represent the size of the container that would be built. It should be noted that they arrived at the answer much earlier than the other group.

The handling of sensory material as the only way to arrive at a response characterizes empirical thinking, since it begins by examining the object's external relations and attributes, allowing its apprehension in an immediate way. The resolution of a problem based only on the process of handling and visualizing the sensory material allows the subjects to appropriate knowledge, however, they are situated in the empirical scope. Davidov (1988) recalls that the principle of visual and concrete character (object) supports empirical knowledge and that, although mental actions

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promoted by this type of knowledge may be important, they should only be the beginning of the process towards theoretical knowledge. The objective of the organization of teaching as well as the insertion of subjects into the school environment is to develop theoretical thinking.

> The content of theoretical thought is mediated, reflected, essential existence. The theoretical thought is the process of idealization of one of the aspects of the object-practical activity, the reproduction, in it, of the universal forms of things. Such a reproduction takes place in the labor activity of people as a peculiar object-sensorial experiment. Then this experiment acquires more and more a cognitive character, allowing people to pass, with time, to the experiments carried out mentally. (DAVIDOV, 1988, p. 125, translated from Spanish)[10]

It is based on theoretical knowledge that man appropriates the most elaborate human culture that, used mentally, dispenses the appeal to the sensory object and begins to develop new generalizations.

Let's see, below, the resolution of Group 02.

Chart 02: Scene 2- Group 2's Resolution of the Learning Trigger Situation

Scene 02 Description: This scene happened at the same time as Scene 1, however it presents the conclusion that Group 2 came to the proposed Learning Trigger Situation.
When organizing the material, the academics started by placing the plates, the bars and the little cubes on the big cube and noticed some difficulties. 1. **Luna:** *The problem will be the little cubes.* *2.* **Lara**: *There are ten cubes. Now what?* 3. **Manuella:** *And if we put [bars] a little there [base length] and a little here [bars on the side] and these [little cubes] here [edge of the cube].* 4. **Márcia**: *Like a row here [cube edge]?* [...] 5. **Sophia**: *Must it necessarily be [bases and faces] a square?* 6. **Luna:** *I don't think so, it could be a rectangle too. It can be a rectangular shape that occupies less space, so you can analyze it standing or lying down, but it needs to occupy the smallest space.*

[10] "El contenido del pensamiento teórico es la existencia mediatizada, reflejada, essencial. El pensamineto teórico es el processo de idealización de uno de los aspectos de la actividad objetal-prática, la reprodúcion, en ella, de las formas universales de las cosas. Tal reprodúcion tiene lugar em la actividad laboral de las personas como peculiar experimento objetal-sensorial. Luego este experimento adquire cada vez más um carácter cognoscitivo, permitiendo a las personas passar, com el tiempo, a los experimentos realizados mentalmente."

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7. **Lara:** *I think we can continue with the size of the cube.*
8. **Luna:** *But there's either one missing or one leftover. I think we'll move it to one side [assemble the rest of the materials next to the cube], but it doesn't necessarily have to... let's think about calculations: we need to have a base, one needs to be even and the other needs to be odd.*

[...]

9. **Manuella:** *It needs to be in cube shape, because here [base length] it will yield a lot if we multiply the lengths.*

[...]

10. **Luna:** *So the shape we occupy has nothing to do with whether or not we lack space.*
11. **Sophia:** Do we need to complete it, make it "right"?
12. **Researcher:** *Not necessarily. You need to have the smallest space.*

[...]

13. **Luna:** *So it's measuring the volume. Because it won't just take up the flat area [base], it will take up all the space.*
14. **Sophia:** *So we have to calculate the volume he occupies??*

[...]

15. **Luna**: *Since all the units complete the little cube, we can use the little cube as a unit of measurement, like a unit, and calculate the volume. The bar that is a rectangular figure is the length, width and height.*
16. **Alice:** The order does not change the product. Regardless of the space it occupies, the volume will be the same because of the multiplication of the three dimensions.

[...]

17. **Luna:** *What is the volume of the pieces again?*
18. **Sophia:** *Two thousand one hundred and ten.*
19. **Luna:** *If we make a shape that's cubic, the volume of the cube is cubic. So I need to find out if there's a cubic root for that value, to see if it's accurate. If it is, it needs to be a number that ends in zero.*
20. **Manuella:** *We have to see if it's right. But I don't think it's going to be accurate because that time the whole cube didn't work, so the measurements are not accurate..*
21. **Luna:** *So it's not going to be a cube shape, it's going to be a rectangular shape, because when we decided to find the root of the cube we didn't get an exact root, so that means the measurements wouldn't be exact.*

[...]

22. **Luna:** *We thought the following: we could have tried for hypotheses, but thinking a little that man created a formula to calculate the volume, and knowing that we already knew the volume of the space, we tried to use the formula of volume. Since we have rectangular figures, we thought about the volume of the cube to see if we could assemble a cube. When we calculated it, we saw that it was not exact.*

[...]

23. **Adviser:** *How did you get to the two thousand one hundred and ten?*
24. **Luna:** *By calculating the volume of each piece and adding them up..*

[...]

25. **Luna:** *Starting from the value two thousand one hundred and ten, we did the basic: since it ends in zero, it is divisible by ten. From that we found two measures and tried to take the other measure with hypotheses, trying to divide. Then we started from the assumption of trying to decrease the volume to see if there would be any pieces left, but it didn't work. But when we increased the volume, we saw that, as it increased ten units, there was a little bar left. From that we made the calculation and increased that volume to ninety and we saw that there were nine bars left. In another case, we increased it by fifty and there were five bars left over. But then we saw that we could have tried with little cubes. So, in the end we came to the conclusion that the maximum that was left was five bars, finding the dimensions ten, twelve and eighteen, reaching a volume of two thousand one hundred and sixty.*

Source: Math Club Collection

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This scene presents the resolution of the Learning Trigger Situation by Group 2, from the exploration of the material to the final conclusion, bringing the way it was performed and the solution found for the box, which was built later. Following the lines, we would like to discuss three manifestations related to the actions developed: the need to use the calculations - the formula as a synthesis that surpasses the sensorial empirical experience; the sharing as a propeller of knowledge; the possibility of approaching the theoretical concept.

The mathematical concept can be considered as an object of the human mind with a set of meanings, which goes beyond the different social experiences and favors understanding and appropriation of knowledge. As Moura puts it (2011, p. 55), it is

> [...] a concrete object produced to be useful to a subject who wants to understand a certain phenomenon, be it physical or social. The mathematical concept is na object of the human mind, produced by producing objects and reflecting on natural forms that may have some meaning for life. The mathematical ontente, as a social object, contains a set of meanings that should be shared. (Translated from Spanish)[11]

In the case of our participants, the dialogue runs through various concepts that support the hypotheses created on how they could solve what was being proposed, which were not explicitly listed in the Learning Trigger Situation, but were used, even if empirically.

In lines 1, 2 and 3 of Scene 2, the academics reflect on the material used (Montessori Golden Beads), which was already known to them, and thus, starting from their measurements, they could not find an exact value,

[11] "un objeto concreto producido para ser útil a un sujeto que quiere comprender un certo fenómeno, sea él físico o social. El concepto matemático es un objeto de la mente humana, producido al producir objetos y al reflexionar sobre formas naturales que puedan tener algún significado para la vida. El contenido matemático, como objeto social, encierra un conjunto de significados que deberán ser compartidos."

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because, as Luna declares (line 1) the little cubes represented "the problem". So, when Sophia questions whether the base and the edges of the container need to be square (line 9), two hypotheses are raised, one by Luna (line 10), stating that it could be a rectangular shape, and another by Lara, (line 11), who is not very convinced by the rectangular shape, preferring to continue with the idea of the cube (based on a square shape). This situation shows that although our goal was to work with the concept of volume, the Learning Trigger Situation required other knowledge (square, rectangle, base, edge). This reminds us of Vigotsky (2003, p. 72), in explaining that

> The formation of concepts is followed by their transference to other objects: the subject is induced to use the new terms when talking about objects other than experimental blocks, and to define their meaning in a generalized way.

When the Learning Trigger Situation was presented, the academics were confronted with a certain problem, in which, at each new hypothesis, new terms appeared as a necessary part to solve that situation, determining a movement beyond the initial object. However, the Learning Trigger Situation presented started from a daily situation, and the subjects were familiar with the problem and also with the material. Based on Davidov (1982, p. 297), we can say that, while purely sensory experience, the solution to the problem may not have surpassed the empirical knowledge derived from reflections and direct use with that material, and thus may not have reached theoretical generalization.

Later the discussions of Group 2 on these hypotheses advanced to the need to use calculations to verify which would be the smallest possible size of the container they would build, as Luna states (line 12). And this is the first manifestation to which we draw attention, since, in the development of actions, we visualize the need to overcome the sensorial experience through the use of calculation to find a more precise, more exact result. In line 25,

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the participant Luna explains the way they found out whether or not the container could be a cube, stating, after performing the calculation, that the container is not shaped like a cube, because the cubic root of 2,110 is not exact. The reference to the cubic root came from the formula for the volume of the cube ($V=a^3$).

Having to build the box to fill it with the material had already led students to relate it to the "space" the material occupied. But, in recurrence to the calculations, it is possible to identify the correlation they make with the volume. We observed a movement in which, initially, they were able to find a way to verify the possibilities of solving the problem with the material, but this was no longer enough. Thus, the use of calculus as a way to solve the proposed problem, taking into account that the sensory hypotheses (of assembling the material) were not satisfying their needs to do it in the most exact way, point to a path similar to the human movement in order to create faster and more effective ways to solve social problems, bringing indications of the appropriation of mathematical formulas as syntheses that surpass the sensorial empirical experience.

We return again to Moura (2011), when he claims that in a first moment we have the imitation as a conductor of direct appropriation of knowledge, in which, after making use of the word that makes justice to what he has learned, the subject will be developing new ways of making use of it, being able, therefore, to appropriate the empirical knowledge in a direct way with the social environment and, thus, evolve into theoretical knowledge. Knowing that empirical knowledge is different from theoretical knowledge and that it is only by the latter that the person has the possibilities to develop his maximum intellectual capacities, it must be considered that the organization of teaching must allow the learning subject to go beyond what the sensory experience provides, to go beyond empirical knowledge. In our case, the subjects already had knowledge, but this was empirically related to the material.

Vigotsky(2007) explains that learning occurs from the social level to the individual, with the intrapsychic relations (of the subject) constituting the interpsychic relations (between subjects). Thus, it is through the relationships that the subject appropriates the socially established knowledge, which is, in a first moment, of social nature and then passes to the individual.

By thinking about the development of the subjects, interacting among themselves, creating hypotheses and suppositions, we show possibilities of appropriation of universally systematized knowledge through the logical and historical human movement. In Scene 2, we perceive signs of manifestation of a shared process, with dialogues that make the other subjects rethink their hypotheses, creating new conditions to arrive at the answer, as in lines 9, 10, 11 and 12, in which the scholars reflect about the geometric shape of the container.

We also highlight a third manifestation, taking up the reference of the scholars to the word "space", especially in lines 7, 10 and 14. In these lines we can identify that, even before explicitly referring to the volume that the material occupies, there are already elements referring to this greatness which, as Bendick (1965, p.22) states, is "the space occupied by anything". We understand that this cannot be seen as a unique element of appropriation of theoretical knowledge, since the concept of volume goes beyond what the author points out. However, we consider - as a result of the movement of previous discussions of the group - an indication that the actions developed were in the direction of approaching the concept of volume.

Conclusion

We understand from what Davidov (1982, 1988) and Rubtsov (1996) tell us that theoretical knowledge is the content of the activity of the subject who learns and promotes the development of theoretical thinking, since it is

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predominant in what concerns learning and allows the knowledge of the object in its totality, involving the universal and singular aspects (DAVIDOV, 1982). But the development of theoretical thought does not happen spontaneously, it requires the intentionality of the organization of teaching which, in Rubtsov's understanding (1996, p.131), involves placing the subject in the face of a learning problem that allows him to "acquire the general forms of action and, with them, the acquisition of theoretical knowledge". In this way, it is expected that the subject in a learning situation puts themselves into activity, in Leontiev's perspective (1978, 1988).

From these basic assumptions we have brought to light manifestations of the resolution of a Learning Trigger Situation solved by future teachers who will teach mathematics, divided into two groups that presented distinct approaches.

The first group found a solution to the proposed problem based exclusively on the manipulation of the material it had at its disposal. Although the answer found enabled the appropriation of knowledge (which would allow, in this case, the construction of the box), at that moment it did not surpass the empirical one, since this is based entirely on sensory representations, reflecting only the external properties of the objects, as sustains Davidov (1988,1982).

We can understand that the proposed problem is configured for these future teachers - at least initially - as a practical problem (RUBTSOV, 1996), since it allowed them to reach only the immediate solution. The group's need was to find a solution for the size of the box and not a general way of building boxes, which clearly shows that it did not materialize as an activity in Leontiev's perspective (1978,1988). This requires us to reflect on what possibilities have limited the learning of these subjects, and some indications can be found in the analysis of the second group's resolution.

The second group tried a different path, guided by a collective discussion among its components, leading them to a possible general way of solving the problem, materialized in the formula of obtaining the volume of the box, understood as sufficient for the "space" to be occupied by the material. This may be indicative that for them it has been configured as a learning problem, since the formula can be understood as a model, transformed into a basis "which constitutes the common orientation to complete the concrete actions concerning a class of problems" (RUBTSOV, 1996, p.133).

In other words, the formula they found is not only for the problem of the box they needed to produce, but also for calculating the volume of any parallelepiped, approaching generalization. Possibly the members of the other group also had knowledge of the parallelepiped's formula, but the unique appeal to the sensory aspect did not make it possible for them to overcome the empirical knowledge, possibly rooted in the model they experienced in their school experience.

We are not saying that Group 2 has reached theoretical generalizations, but it is possible to identify the differences between the solutions of the two groups. The first is very close to what can normally be observed in Basic Education, derived from the emphasis of the empirical daily experience of schoolchildren, which, although we consider of extreme relevance, especially in the first years of schooling, needs to be surpassed.

> For Davydov, empirical thinking supports a didactic and organizational conception of teaching in which, mistakenly, "empirical" means sensory, palpable, concrete and theoretical means abstract, verbal, general. In this didactic conception, teaching is configured as the transmission of knowledge and not as a process of investigation of its origin and development in the form of a concept (FREITAS, 2016, p.397, emphasis added by the author).

What we present here is a cutout of a broader process and was succeeded by a subsequent moment of seeking a collective synthesis (as

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forwarded by the Teaching Guiding Activity), in order to give all participants the opportunity to approach the theoretical knowledge on quantities and measures, more specifically volume. This moment was not presented here, because our intention was to discuss the initial manifestations of future teachers in front of a Learning Trigger Situation.

As outcome of the results, we pointed out the relevance of the insertion of future teachers in spaces where they have the possibility to discuss and reflect, allowing them to go beyond the knowledge taken as a further development of their future teaching area, since it was possible to realize that, even being in a college course, some of them rely solely on empirical experiences, without theoretical mastery of knowledge that we could consider as basic, since they are part of the curricular organization of Basic Education. This is because, in case this does not happen, the consequence will be the perpetuation of the current model, since, when teachers, the organization of teaching will be based on their knowledge, which means that it will be difficult to achieve the development of theoretical thinking.

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