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TESTE DE VIDA SEQÜENCIAL APLICADO A UM TESTE DE VIDA ACELERADO: UTILIZAÇÃO DO MÉTODO DA MÁXIMA VEROSSIMILHANÇA PARA ESTIMAR OS TRÊS PARÂMETROS DE UM MODELO DE AMOSTRAGEM WEIBULL

A SEQUENTIAL LIFE-TESTING APPLIED TO AN ACCELERATED LIFE-TESTING: USING A MAXIMUM LIKELIHOOD APPROACH TO ESTIMATE THE THREE PARAMETERS OF AN UNDERLYING WEIBULL MODEL

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ABSTRACT

The sequential life-testing approach is an attractive alternative to that of predetermined, fixed sample size hypothesis testing because of the fewer observations required for its use, especially when the underlying sampling distribution is the three-parameter Weibull model. It happens that sometimes the amount of time available for testing could be considerably less than the expected lifetime of the component. To overcome such a problem, there is the accelerated life-testing alternative aimed at forcing components to fail by testing them at much higher-than-intended application conditions. One possible way to translate test results obtained under accelerated conditions to normal using conditions could be through the application of the "Maxwell Distribution Law." In this work we will be life-testing a new industrial product. To estimate the three parameters of the Weibull model we will use a maximum likelihood approach for censored failure data. We will be assuming a linear acceleration condition. To evaluate the accuracy (significance) of the parameter values obtained under normal conditions for the underlying Weibull model we will apply to the expected normal failure times a sequential life testing using a truncation mechanism developed by De Souza [1]. An example will illustrate the application of this procedure.

Keywords: Accelerated Models, Sequential Test, Acceleration, Maxwell Distribution Law, Maximum Likelihood.

RESUMO

O mecanismo de teste de vida seqüencial é uma alternativa plausível ao de um teste com tamanho de amostra pré-fixado, devido utilizar um número pequeno de observações, especialmente quando a distribuição de amostragem é o modelo Weibull de três parâmetros. Acontece que mesmo com o uso desse mecanismo seqüencial, algumas vezes o tempo disponível para a realização do teste poderá ser consideravelmente menor do que a vida esperada do componente. Para a solução desse problema, existe a alternativa de um teste de vida acelerado direcionado a forçar os componentes a falharem, submetendo-os a condições de teste muito mais severas do que as encontradas em condições normais de uso. Uma maneira de traduzirmos os resultados obtidos sob uma condição de aceleração para uma condição normal poderá ser através da aplicação da Lei de Distribuição de Maxwell. Neste trabalho, testaremos um novo produto industrial. Para estimarmos os três parâmetros do modelo Weibull, utilizaremos o estimador de Máxima Verossimilhança para uma condição de teste de vida truncado por falhas. Assumiremos uma condição de aceleração linear. Para avaliarmos a precisão dos valores dos parâmetros do modelo Weibull, obtidos em condições normais de uso, aplicaremos aos tempos esperados de falhas em condições normais um teste de vida seqüencial utilizando um mecanismo de truncagem desenvolvido por De Souza [1]. Um exemplo ilustrará a aplicação desse procedimento.

Palavras-Chave: Modelos Acelerados, Teste Seqüencial, Aceleração, Lei de Distribuição de Maxwell, Estimador de Máxima Verossimilhança.

1 – INTRODUCTION

The sequential life testing approach is an attractive alternative to that of predetermined, fixed sample size hypothesis testing because of the fewer observations required for its use, especially when the underlying sampling distribution is the three-parameter Weibull model. It happens that even with the use of a sequential life testing mechanism, sometimes the number of items necessary to reach a decision about accepting or rejecting a null hypothesis is quite large; see De Souza [2]. Then, a truncation mechanism for this life-testing situation was developed by De Souza [1] and an application of this mechanism was presented by De Souza [3]. But it happens that sometimes the amount of time available for testing could be considerably less than the expected lifetime of the component. To overcome such a problem, there is the accelerated life-testing alternative aimed at forcing components to fail by testing them at much higher-than-intended application conditions. To go from the failure rate obtained at high stress to what a product or service is likely to experience at much lower stress, under use conditions, we will need additional modeling. These models are known as acceleration models.

One possible way to translate test results obtained under accelerated conditions to normal using conditions could be through the application of the "Maxwell Distribution Law." In this work we will be life-testing a new industrial product. To estimate the three parameters of the Weibull model we will use a maximum likelihood approach for censored failure data, since the life-testing will be terminated at the moment the truncation point is reached. We will be assuming a linear acceleration condition. To evaluate the accuracy (significance) of the parameter values obtained under normal conditions for the underlying Weibull model we will apply to the expected normal failure times a sequential life testing using a truncation mechanism developed by De Souza [1]. An example will illustrate the application of this procedure.

2 – THE ACCELERATING CONDITION

When only thermal stresses are significant, an empirical model, known as the Arrhenius model, has been used with relative success as an accelerated model. The Arrhenius model is given by Equation 1 below:

$$R_{rate} = e^{-E/KT_n + C} \qquad (1)$$

Here, R_{rate} is the rate of reaction, E represents the energy of activation of the reaction, K the gas constant (1.986 cal/ (mol.Kelvin), T_n the temperature in degrees Kelvin (273.16 plus the degrees Centigrade) at normal condition of use, and C a constant.

The acceleration factor $AF_{2/1}$ (or the ratio of the specific rates of reaction R_2/R_1), at two different stress temperatures, T_2 and T_1, will be given by:

$$AF_{2/1} = \frac{R_2}{R_1} = \frac{e^{-E/KT_2 + C}}{e^{-E/KT_1 + C}} = \exp\left[\frac{E}{K}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right] \qquad (2)$$

Applying natural logarithm to both sides of this equation and after some arithmetical manipulation, we will obtain:

$$\ln\left(AF_{2/1}\right) = \ln\left(\frac{R_2}{R_1}\right) = \frac{E}{K}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \qquad (3)$$

How does such a relationship come about? Maybe it can be related to the "Maxwell Distribution Law." This law, which expresses the distribution of kinetic energies of molecules, is given by the following equation:

$$M_{TE} = M_{tot}\ e^{-E/KT} \qquad (4)$$

Here, M_{TE} represents the number of molecules at a particular absolute Kelvin temperature T with kinetic energy greater than the energy of activation of the reaction E. The term M_{tot} represents the total number of molecules present. Equation 4 expresses the probability of a molecule having energy in excess of E.

The ratio of the number of molecules having energy E at two different temperatures will be given by

$\frac{M_{TE}(2)}{M_{TE}(1)} = \frac{e^{-E/KT_2}}{e^{-E/KT_1}}$. Applying natural logarithm to both sides of this equation and after some algebraic manipulation, we will obtain:

$\ln\left(\frac{M_{TE}(2)}{M_{TE}(1)}\right) = \frac{E}{K}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$, which is very similar to Equation 3.

From Equation 3 we can estimate the term E/K by testing at two different stress temperatures and computing the acceleration factor on the basis of the fitted distributions. Then;

$$\frac{E}{K} = \frac{\ln\left(AF_{2/1}\right)}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} \qquad (5)$$

The acceleration factor $AF_{2/1}$ will be given by the relationship θ_1/θ_2, with θ_i representing a scale parameter or a percentile at a stress level corresponding to T_i. Once the term E/K is determined, the acceleration factor $AF_{2/n}$ to be applied at the normal stress temperature is obtained from Equation 2 by replacing the stress temperature T_1 with the temperature at normal condition of use Tn. Then:

$$AF_{2/n} = \exp\left[\frac{E}{K}\left(\frac{1}{T_n} - \frac{1}{T_2}\right)\right] \qquad (6)$$

De Souza [4] has shown that under a linear acceleration assumption, if a three-parameter Inverse Weibull model represents the life distribution at one stress level, a three-parameter Inverse Weibull model also represents the life distribution at any other stress level. The same reasoning applies to the three-parameter Weibull model. We will be assuming a linear acceleration condition.

In general, the scale parameter and the minimum life can be estimated by using two different stress levels (temperature or cycles or miles, etc.), and their ratios will provide the desired value for the acceleration factors AF_θ and AF_φ. So, we will have:

$$AF_\theta = \frac{\theta_n}{\theta_a} \qquad (7)$$

$$AF_\varphi = \frac{\varphi_n}{\varphi_a} \qquad (8)$$

Here, φ_n represents the minimum life or location parameter at normal using condition of the three-parameter Weibull sampling distribution, and φ_n is the minimum life at accelerated using conditions. θ_n is the scale parameter at normal using condition and θ_a is the parameter at accelerated using conditions. Again, based on the paper by De Souza [4], for the Weibull model the cumulative distribution function at normal testing condition $F_n(t_n-\varphi_n)$ for a certain testing time $t = t_n$, will be given by:

$$F_n(t_n-\varphi_n) = F_a\left(\frac{t_n - \frac{\varphi_n}{AF}}{AF}\right) =$$

$$= 1 - \exp\left[-\left(\frac{t_n - \frac{\varphi_n}{AF}}{\theta_a\, AF}\right)^{\beta_a}\right] \qquad (9)$$

In this paper, the terms β_a and β_n represent, respectively, the shape parameter of the three-parameter Weibull model at accelerated and normal using conditions. Equation 9 tells us that, under a linear acceleration assumption, if a three-parameter Weibull model represents the life distribution at one stress level, a three-parameter Weibull model also represents the life distribution at any other stress level. The shape parameter remains the same while the accelerated scale parameter and the accelerated minimum life parameter are multiplied by the acceleration factor. The equal shape parameter is a necessary mathematical consequence of the other two assumptions; that is, assuming a linear acceleration model and a three-parameter Weibull sampling distribution. If different stress levels yield data with very different shape parameters, then either the three-parameter Weibull sampling distribution is the wrong model for the data or we do not have a linear acceleration condition.

Again, since $R_n(t_n-\varphi_n) = 1 - F_n(t_n-\varphi_n)$, we will have:

$$t_n = \theta_n\left[\ln\left(\frac{1}{R_n(t_n-\varphi_n)}\right)\right]^{1/\beta_n} + \frac{\varphi_n}{AF} \qquad (10)$$

3 – MAXIMUM LIKELIHOOD ESTIMATION FOR THE WEIBULL MODEL FOR CENSORED TYPE II DATA (FAILURE CENSORED)

The standard maximum likelihood method for estimating the parameters of the three-parameter Weibull model can have problems since the regularity conditions are not met; see Murthy, et al. [5]; Blischke [6]; Zanakis and Kyparisis [7]. To overcome the resulting "no regularity" problem above mentioned, we will apply a modification proposed by Cohen et al. [8]. De Souza [4] presents a complete discussion of this matter for use with the three-parameter Inverse Weibull model. The same reasoning applies to the three-parameter Weibull model.

The likelihood function for the shape, scale and minimum life parameters of a Weibull sampling distribution for censored Type II data (failure censored) will be given by:

$$L(\beta;\theta;\varphi) = k!\left[\prod_{i=1}^{r} f(t_i)\right]\left[1 - F(t_r)\right]^{n-r}$$

$$L(\beta;\theta;\varphi) = k!\left[\prod_{i=1}^{r} f(t_i)\right]\left[R(t_r)\right]^{n-r}\ ;\ t > 0 \qquad (11)$$

With $f(t_i) = \frac{\beta}{\theta^\beta}(t_i-\varphi)^{\beta-1}\ e^{-(t_i-\varphi/\theta)^\beta}$ and with $R(t_r) = e^{-(t_r-\varphi/\theta)^\beta}$, we will have:

$$L(\beta;\theta;\varphi) = k!\ \frac{\beta^r}{\theta^{\beta r}}\left[\prod_{i=1}^{r}(t_i-\varphi)\right]^{\beta-1} \times$$

$$\times\ e^{-\sum_{i=1}^{r}(t_i-\varphi/\theta)^\beta}\left[e^{-(t_r-\varphi/\theta)^\beta}\right]^{n-r}$$

The log likelihood function $L = \ln[L(\beta;\theta;\varphi)]$ will be given by:

$$L = \ln(k) + r\ln(\beta) - r\beta\ln(\theta) + (\beta-1)\sum_{i=1}^{r}\ln(t_i-\varphi) -$$

$$-\sum_{i=1}^{r}\left(\frac{t_i-\varphi}{\theta}\right)^\beta - (n-r)\left(\frac{t_r-\varphi}{\theta}\right)^\beta$$

To find the value of θ, β and φ that maximizes the log likelihood function, we take the θ, β and φ derivatives and make them equal to zero. Then, applying some algebra, we will have:

$$\frac{dL}{d\theta} = -\frac{r\beta}{\theta} + \frac{\beta\times\sum_{i=1}^{r}(t_i-\varphi)^\beta}{\theta^{\beta+1}} +$$

$$+\ \frac{\beta(n-r)(t_r-\varphi)^\beta}{\theta^{\beta+1}} = 0 \qquad (12)$$

$$\frac{dL}{d\beta} = \frac{r}{\beta} - r\ln(\theta) + \sum_{i=1}^{r}\ln(t_i-\varphi) - \sum_{i=1}^{r}\left(\frac{t_i-\varphi}{\theta}\right)^\beta \times$$

$$\times\ln\left(\frac{t_i-\varphi}{\theta}\right) - (n-r)\left(\frac{t_r-\varphi}{\theta}\right)^\beta\ln\left(\frac{t_r-\varphi}{\theta}\right) = 0 \quad (13)$$

$$\frac{dL}{d\varphi} = -(\beta - 1)\sum_{i=1}^{r}\frac{1}{(t_i - \varphi)} +$$

$$+ \frac{\beta \times \left[\sum_{i=1}^{r}(t_i - \varphi)^{\beta-1} + (n-r)(t_r - \varphi)^{\beta-1}\right]}{\theta^{\beta}} = 0 \quad (14)$$

From Equation 12 we obtain:

$$\theta = \left(\frac{\sum_{i=1}^{r}(t_i - \varphi)^{\beta} + (n-r)(t_r - \varphi)^{\beta}}{r}\right)^{1/\beta} \quad (15)$$

Notice that, when $\beta = 1$, Equation 15 reduces to the maximum likelihood estimator for the two-parameter exponential distribution. Using Equation 15 for θ in Equations 13 and 14 and applying some algebra, Equations 13 and 14 reduce to:

$$\frac{r}{\beta} + \sum_{i=1}^{r}\ln(t_i - \varphi) - \frac{r \times \left[\sum_{i=1}^{r}(t_i - \varphi)^{\beta}\ln(t_i - \varphi) + (n-r)(t_r - \varphi)^{\beta}\ln(t_r - \varphi)\right]}{\sum_{i=1}^{r}(t_i - \varphi)^{\beta} + (n-r)(t_r - \varphi)^{\beta}} = 0 \quad (16)$$

$$\left[\frac{\sum_{i=1}^{r}(t_i - \varphi)^{\beta} + (n-r)(t_r - \varphi)^{\beta}}{r}\right](\beta - 1)\sum_{i=1}^{r}\frac{1}{(t_i - \varphi)} + \beta \times$$

$$\times \left[\sum_{i=1}^{r}(t_i - \varphi)^{\beta-1} + (n-r)(t_r - \varphi)^{\beta-1}\right] = 0 \quad (17)$$

To overcome the "no-regularity problem," one of the approaches proposed by Cohen et al. [8] is to replace Equation 17 with the equation

$$E(\varphi) = \varphi_n = t_1 - \frac{\theta_n}{n^{1/\beta}}\,\Gamma\left(\frac{1}{\beta} + 1\right) \quad (18)$$

Here, t_1 is the first order statistic in a sample of size *n*. In solving the maximum likelihood equations, we will use this approach proposed by Cohen et al. [8]. Appendix 1 shows the derivation of Equation 18.

4 –THE SEQUENTIAL TESTING

The three-parameter Weibull density function is given by:

$$f(t) = \frac{\beta}{\theta}\left(\frac{t - \phi}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t - \phi}{\theta}\right)^{\beta}\right]; \quad t \geq 0$$

The hypothesis testing situations were given by Kapur and Lamberson [9], and De Souza [2]:

1. For the scale parameter θ: H_0: $\theta \geq \theta_0$; H_1: $\theta < \theta_0$

The probability of accepting H_0 will be set at $(1-\alpha)$ if $\theta = \theta_0$. Now, if $\theta = \theta_1$ where $\theta_1 < \theta_0$, then the probability of accepting H_0 will be set at a low level γ.

2. For the shape parameter β: H_0: $\beta \geq \beta_0$; H_1: $\beta < \beta_0$

The probability of accepting H_0 will be set at $(1-\alpha)$ if $\beta = \beta_0$. Now, if $\beta = \beta_1$ where $\beta_1 < \beta_0$, then the probability of accepting H_0 will be also set at a low level γ.

3. For the location parameter φ: H_0: $\varphi \geq \varphi_0$; H_1: $\varphi < \varphi_0$

Again, the probability of accepting H_0 will be set at $(1-\alpha)$ if $\varphi = \varphi_0$. Now, if $\varphi = \varphi_1$ where $\varphi < \varphi_0$, then the probability of accepting H_0 will be once more set at a low level γ. The sequential probability ratio (SPR) will be given by SPR = $L_{1,1,1,n}$ / $L_{0,0,0,n}$. According to De Souza [1], for the three-parameter Weibull model we will have:

$$n\ln\left(\frac{\beta_1}{\theta_1^{\beta_1}} \times \frac{\theta_0^{\beta_0}}{\beta_0}\right) - \ln\left[\frac{(1-\gamma)}{\alpha}\right] < X_i <$$

$$< n\ln\left(\frac{\beta_1}{\theta_1^{\beta_1}} \times \frac{\theta_0^{\beta_0}}{\beta_0}\right) + \ln\left[\frac{(1-\alpha)}{\gamma}\right] \quad (19)$$

$$X_i = \sum_{i=1}^{n}\left(\frac{(t_i - \phi_1)^{\beta_1}}{\theta_1^{\beta_1}} - \frac{(t_i - \phi_0)^{\beta_0}}{\theta_0^{\beta_0}}\right) - (\beta_1 - 1) \times$$

$$\times \sum_{i=1}^{n}\ln(t_i - \phi_1) + (\beta_0 - 1)\sum_{i=1}^{n}\ln(t_i - \phi_0) \quad (20)$$

5 – EXPECTED SAMPLE SIZE OF A SEQUENTIAL LIFE TESTING FOR TRUNCATION PURPOSE

According to Mood and Graybill [10], an approximate expression for the expected sample size E(n) of a sequential life testing for truncation purposes will be given by:

$$E(n) = \frac{P(\theta,\beta)\ln A + [1 - P(\theta,\beta)]\ln B}{E(w)} \quad (21)$$

According to De Souza [1], for the three-parameter Weibull model we will have:

$$E(w) = \ln\left(\frac{\beta_1}{\theta_1^{\beta_1}} \times \frac{\theta_0^{\beta_0}}{\beta_0}\right) + (\beta_1 - 1)\, E\left[\ln(t - \varphi_1)\right] -$$

$$(\beta_0 - 1)\, E\left[\ln(t - \varphi_0)\right] - \frac{1}{\theta_1^{\beta_1}}\, E\left[(t - \varphi_1)^{\beta_1}\right] +$$

$$+ \frac{1}{\theta_0^{\beta_0}}\, E\left[(t - \varphi_0)^{\beta_0}\right] \qquad (22)$$

$$A = \frac{\gamma}{(1-\alpha)} \quad ; \qquad B = \frac{(1-\gamma)}{\alpha}$$

The solution of each component of Equation 22 can be found in De Souza [1] and De Souza [3].

6 – EXAMPLE

We are trying to determine the values of the shape, scale and minimum life parameters of an underlying Weibull model, representing the life cycle of a new electronic component. Once a life curve for this component is determined, we will be able to verify if new units produced will have the necessary required characteristics by using sequential life testing. It happens that the amount of time available for testing is considerably less than the expected lifetime of the component. So, we will have to rely on an accelerated life testing procedure to obtain failure times used in the parameters estimation procedure. The electronic component has a normal operating temperature of 298 K (about 25 degrees Centigrade). Under stress testing at 460 K, 10 electronic components were subjected to testing, with the testing being truncated at the moment of occurrence of the eighth failure. Table 1 shows these failure time data (hours). Now, under stress testing at 520 K, 10 electronic components were again subjected to testing, with the testing being truncated at the moment of occurrence of the eighth failure. Table 2 shows these failure time data (hours).

Table 1. Failure times (hours) of electronic parts tested under accelerated temperature conditions (460 K).

673.3	836.4	561.3
688.4	625.6	720.2
702.5	746.2	–

Table 2. Failure times (hours) of electronic parts tested under accelerated temperature conditions (520 K).

591.1	546.2	641.8
562.8	526.5	583.5
669.9	606.0	–

The data shown is Tables 1 and 2 were obtained from a life-test facility. Using the maximum likelihood estimator approach for the shape parameter β for the scale parameter θ and for the minimum life φ of the Weibull model for censored Type II data (failure censored), we obtain the following values for these three parameters under accelerated conditions of testing.

At 460 K. $\beta_1 = \beta_n = \beta = 7.8$; $\theta_1 = 623.4$ hours; $\varphi_{n1} = 117.0$ hours

At 520 K. $\beta_2 = \beta_n = \beta = 7.78 \approx 7.8$ $\theta_2 = 529.6$ hours; $\varphi_2 = 82.5$ hours

The shape parameter did not change with $\beta \approx 7.8$.

The acceleration factor for the scale parameter $AF\theta_{2/1}$ will be given by:

$$AF\theta_{2/1} = \frac{\theta_1}{\theta_2} = \frac{623.4}{529.6}$$

Using Equation 5, we can estimate the term E/K. Then

$$\frac{E}{K} = \frac{\ln\left(AF_{2/1}\right)}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} = \frac{\ln(623.4/529.6)}{\left(\frac{1}{460} - \frac{1}{520}\right)} = 650.1$$

Using now Equation 6, the acceleration factor for the scale parameter to be applied at the normal stress temperature $AF\theta_{2/n}$ will be:

$$AF_{2/n} = \exp\left[\frac{E}{K}\left(\frac{1}{T_n} - \frac{1}{T_2}\right)\right] =$$

$$= \exp\left[650.1\left(\frac{1}{298} - \frac{1}{520}\right)\right] = 2.54$$

The acceleration factor for the minimum life parameter $AF\varphi_{2/1}$ will be given by:

$$AF\varphi_{2/1} = \frac{\varphi_1}{\varphi_2} = \frac{117.0}{98.8}$$

Again applying Equation 5, we can again estimate the term E/K. Then

$$\frac{E}{K} = \frac{\ln\left(AF_{2/1}\right)}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} = \frac{\ln(117.0/98.8)}{\left(\frac{1}{460} - \frac{1}{520}\right)} = 674.1$$

Using once more Equation 6, the acceleration factor for the minimum life parameter to be applied at the normal stress temperature $AF\varphi_{2/n}$ will be:

$$AF_{2/n} = \exp\left[\frac{E}{K}\left(\frac{1}{T_n} - \frac{1}{T_2}\right)\right] =$$

$$= \exp\left[674.1\left(\frac{1}{298} - \frac{1}{520}\right)\right] = 2.62$$

Then, as we expected, $AF_\theta = AF_\varphi = AF \approx 2.60$

Finally, the scale parameter and the minimum life parameter of the component at normal operating temperatures is estimated to be:

$$\theta_n = AF_{2/n} \times \theta_2 = 2.60 \times 529.6 = 1{,}377.0 \text{ hours}$$

$$\varphi_n = AF_{2/n} \times \varphi_2 = 2.60 \times 98.8 = 256.9 \text{ hours}$$

Then, the electronic component life when operating at normal use conditions could be represented by a three-parameter Weibull model having a shape parameter β of 7.8; a scale parameter θ of 1,377.0 hours and a minimum life φ of 256.9 hours.

To evaluate the accuracy (significance) of the three-parameter values obtained under normal conditions for the underlying Weibull model we will apply to the expected normal failure times a sequential life testing using a truncation mechanism developed by De Souza [1]. These expected normal failure times will be acquired by multiplying the nine failure times obtained under accelerated testing conditions at 520 K given by Table 2, by the accelerating factor AF of 2.6. It was decided that the value of α was 0.05 and γ was 0.10. In this example, the following values for the alternative and null parameters were chosen: alternative scale parameter θ_1 = 1,200 hours, alternative shape parameter β_1 = 7.0 and alternative location parameter φ_1 = 200 hours; null scale parameter θ_0 = 1,377 hours, null shape parameter β_0 = 7.8 and null minimum life parameter φ_0 = 255 hours. Now electing P(θ,β,φ) to be 0.01, we can calculate the expected sample size E(n) of this sequential life testing under analysis. Using Equation 22 and the expression for the expected sample size of the sequential life testing for truncation purpose E(n), we will have:

$$E(w) = \ln\left(\frac{\beta_1}{\theta_1^{\beta_1}} \times \frac{\theta_0^{\beta_0}}{\beta_0}\right) + (\beta_1 - 1)\, E\left[\ln(t - \varphi_1)\right] -$$

$$-(\beta_0 - 1)\, E\left[\ln(t - \varphi_0)\right] - \frac{1}{\theta_1^{\beta_1}}\, E\left[(t - \varphi_1)^{\beta_1}\right] +$$

$$+ \frac{1}{\theta_0^{\beta_0}}\, E\left[(t - \varphi_0)^{\beta_0}\right]$$

Solving the above equation we obtain:

$E(w)$ = 6.637 + 6 × 5.512128 – 6.8 × 5.532708 – 2.519 + 1.0 = 0.568

Now, with $P(\theta,\beta,\varphi)$ = 0.01, $\ln(B) = \ln\left[\frac{(1-\gamma)}{\alpha}\right] = \ln\left[\frac{(1-0.10)}{0.05}\right]$ = 2.890 and also with

$\ln(A) = \ln\left(\frac{\gamma}{1-\alpha}\right) = \ln\left(\frac{0.10}{1-0.05}\right) = -2.2513$, we have:

$$P(\theta,\beta)\ln(A) + [1 - P(\theta,\beta)]\ln(B) =$$

$$= -0.01 \times 2.2513 + 0.99 \times 2.8904 = \approx 2.839$$

Then: $E(n) = \dfrac{P(\theta,\beta)\ln A + [1 - P(\theta,\beta)]\ln B}{E(w)} = \dfrac{2.839}{0.568}$

$E(n) = 4.99 \approx 5$ items

So, we could make a decision about accepting or rejecting the null hypothesis H_0 after the analysis of observation number 5. Using Equations 9 and 10 and the nine failure times obtained under accelerated conditions at 520 K given by Table 2, multiplied by the accelerating factor AF of 2.6, we calculate the sequential life test limits. Figure 1 below shows the sequential life-testing for the three-parameter Weibull model.

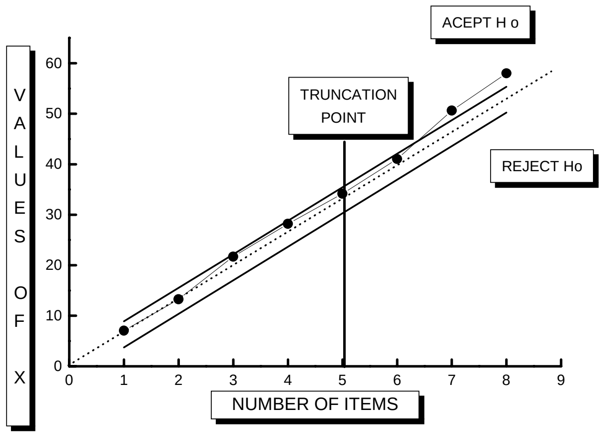

Figure 1. Sequential life-testing for the three-parameter Weibull model.

According to Kapur and Lamberson [9], when the truncation point is reached, a line partitioning the sequential graph can be drawn as shown in Figure 1. This line is drawn through the origin of the graph parallel to the accept and reject lines. The decision to accept or reject H_0 simply depends on which side of the line the final outcome lies. Obviously this procedure changes the levels of α and γ of the original test; however, the change is slight if the truncation point is not too small. As we can see in Figure 1, the null hypothesis H_0 should be accepted since the final observation (observation number 5) lies on the side of the line related to the acceptance of H_0.

7 – CONCLUSIONS

There are two key limitations to the use of the Arrhenius equation: first, at all the temperatures used, linear specific rates of change must be obtained. This requires that the rate of reaction, regardless of whether it is measured or represented, must be constant over the period of time at which the aging process is evaluated. Now, if the expected rate of reaction should vary over the time of the test, then one would not be able to identify a specific rate that is assignable to a specific temperature. If the mechanism of reaction at higher or lower temperatures should differ, this, too, would alter the slope of the curve.

Second, it is necessary that the energy activation be independent of temperature, that is, constant over the range of temperatures of interest. It happens that, according to Chornet and Roy [11], "the apparent energy of activation is not always constant, particularly when there is more than one process going on." Further comments on the limitations of the use of the Arrhenius equation could be found in Feller [12].

In this work we life-tested a new industrial product using an accelerated mechanism. We assumed a linear acceleration condition. To estimate the parameters of the three-parameter Weibull model we used a maximum likelihood approach for censored failure data, since the life-testing will be terminated at the moment the truncation point is reached. The shape parameter remained the same while the accelerated scale parameter and the accelerated minimum life parameter were multiplied by the acceleration factor. The equal shape parameter is a necessary mathematical consequence of the other two assumptions; that is, assuming a linear acceleration model and a three-parameter Weibull sampling distribution. If different stress levels yield data with very different shape parameters, then either the three-parameter Weibull sampling distribution is the wrong model for the data or we do not have a linear acceleration condition.

In order to translate test results obtained under accelerated conditions to normal using conditions we applied some reasoning given by the "Maxwell Distribution Law." To evaluate the accuracy (significance) of the three-parameter values estimated under normal conditions for the underlying Weibull model we employed, to the expected normal failure times, a sequential life testing using a truncation mechanism developed by De Souza [1]. These expected normal failure times were acquired by multiplying the nine failure times obtained under accelerated testing conditions at 520 K given by Table 2, by the accelerating factor AF of 2.6. As we saw in Figure 1, we accept the hypothesis that the electronic component life when operating at normal use conditions could be represented by a three-parameter Weibull model having a shape parameter β of 7.8; a scale parameter θ of 1,377 hours and a minimum life φ of 255 hours.

REFERENCES

[1] De Souza, Daniel I.; "Sequential Life Testing with a Truncation Mechanism for an Underlying Three-Parameter Weibull Model", ICHEAP-6, Chemical Engineering Transactions, Vol 3, pp. 557-562, Sauro Pierucci (ed), Pisa, Italy, June 2003.

[2] De Souza, Daniel I.; "Further Thoughts on a Sequential Life Testing Approach Using a Weibull Model", Foresight and Precaution, ESREL 2000 Congress, Cottam, Harvey, Pape & Tait (eds),, 14–17, Vol 2, pp. 1641-1647, Rotterdam,: Balkema, Edinburgh; Scotland. May 2000.

[3] De Souza, Daniel I.; "Application of a Sequential Life Testing with a Truncation Mechanism for an Underlying Three-Parameter Weibull Model", ESREL 2004 – PSAM 7 Conference, Spitzer, Schmoker and Dang (eds.), Vol 3; pp. 1674-1680, Springer-Verlag publishers. June 14-18, 2004. Berlin, Germany

[4] De Souza, Daniel I.; "A Maximum Likelihood Approach Applied to an Accelerated LifeTesting with an Underlying Three-Parameter Inverse Weibull Model", COMADEM 2005 Conference, Mba and Rao (eds.), , Vol 1, pp. 63-72, Cranfield University Press publisher. August 31-September 07, 2005. Cranfield, UK.

[5] Murthy, D. N. P.; Xie, M. and Hang, R., "Weibull Models." Wiley Series in Probability and Statistics, New Jersey, USA, John Wiley & Sons Inc.,. 2004.

[6] Blischke, W. R.; "On non-regular estimation II. Estimation of the Location Parameter of the Gamma and Weibull Distributions", Communications in Statistics, No 3, pp. 1109-1129. USA, 1974.

[7] Zanakis, S. H. and Kyparisis, J. "A Review of Maximum Likelihood Estimation Methods for the Three Parameter Weibull Distribution", Journal of Statistical Computation and Simulation, 25, pp. 53-73. USA, 1986.

[8] Cohen, A. C. Whitten, B. J. and Ding, Y.; "Modified Moment Estimation for the Three-Parameter Weibull Distribution", Journal of Quality Technology, No 16, pp.159-167. USA, 1984.

[9] Kapur, Kailash and Lamberson, Leonard R.; "Reliability in Engineering Design." New York: John Willey & Sons, Inc., USA, 1977.

[10] Mood, A. M. and Graybill, F. A. (1963), "Introduction to the Theory of Statistics." Second Edition, New York: McGraw-Hill., USA, 1963.

[11] Chornet and Roy; "Compensation of Temperature on Peroxide Initiated Cross linking of Polypropylene", European Polymer Journal, 20, pp. 81-84, USA, 1980.

[12] Feller, Robert L.; "Accelerated Aging, Photochemical and Thermal Aspects." Ann Harbor, Michigan, The Getty Conservation Institute. Printer: Edwards Bross., USA, 1994.

APPENDIX 1: DETERMINING AN INITIAL ESTIMATE TO THE MINIMUM LIFE φ

The *pdf* of t_1, the first failure time, will be given by:

$$f(t_1) = n\left[1 - F(t_1)\right]^{n-1} f(t_1)$$, or, since

$$F(t_1) = 1 - R(t_1)$$, we will have:

$$f(t_1) = n\left[R(t_1)\right]^{n-1} f(t_1)$$

For the three-parameter Weibull sampling distribution, we will have:

$$f(t_1) = \frac{n\beta}{\theta}\left(\frac{t-\varphi}{\theta}\right)^{\beta-1}\left\{\exp-\left[\left(\frac{t-\varphi}{\theta}\right)^{\beta}\right]\right\}^{n}$$

The expected value of t_1 is given by:

$$E(t_1) = \int_{\varphi}^{\infty}\frac{n\beta}{\theta}\, t\left(\frac{t-\varphi}{\theta}\right)^{\beta-1}\left\{\exp-\left[\left(\frac{t-\varphi}{\theta}\right)^{\beta}\right]\right\}^{n} dt$$

Making $U = \left(\frac{t-\varphi}{\theta}\right)^{\beta}$; $du = \frac{\beta}{\theta}\left(\frac{t-\varphi}{\theta}\right)^{\beta-1} dt$;

$$dt = \frac{du}{\frac{\beta}{\theta}\left(\frac{t-\varphi}{\theta}\right)^{\beta-1}}; \quad t = \theta U^{1/\beta} + \varphi$$

As $t \to \infty$; $U \to \infty$; Now, as $t \to \varphi$; $U \to 0$. Then:

$$E(t_1) = \int_0^\infty n\left(\theta U^{1/\beta} + \varphi\right) e^{-nU}\, du$$

$$E(t_1) = \theta\int_0^\infty nU^{1/\beta}\, e^{-nU}\, du + \varphi\int_0^\infty n\, e^{-nU}\, du, \text{ where}$$

$$\varphi\int_0^\infty n\, e^{-nU}\, du = -\varphi\left[e^{-nU}\right]_0^\infty = -\varphi[0-1] = \varphi$$

In solving the integral $\theta\int_0^\infty nU^{1/\beta}\, e^{-nU}\, du$ let $Z = nU$; $du = \frac{dz}{n}$; $U = \frac{Z}{n}$. As $U \to \infty$; $Z \to \infty$; Now, as $U \to 0$; $Z \to 0$. Then:

$$E(t_1) = \theta\int_0^\infty \left(\frac{Z}{n}\right)^{1/\beta} e^{-z}\, dz + \varphi$$

$$E(t_1) = \frac{\theta}{n^{1/\beta}}\int_0^\infty Z^{1/\beta}\, e^{-z}\, dz + \varphi. \text{ Finally,}$$

$$E(t_1) = t_1 = \frac{\theta}{n^{1/\beta}}\,\Gamma\left(\frac{1}{\beta}+1\right) + \varphi$$

The expected value of t_1 is given by $E(t_1) = \frac{\theta}{n^{1/\beta}}\,\Gamma\left(\frac{1}{\beta}+1\right) + \varphi$, which indicates that φ can be estimated by

$$E(\varphi) = \varphi_n = t_1 - \frac{\theta_n}{n^{1/\beta}}\,\Gamma\left(\frac{1}{\beta}+1\right) \qquad (18)$$

BIOGRAPHIC DATA

Daniel I. De Souza Jr. was born in Três Lagoas, MS in 1944. He received his B.S. in Industrial Metallurgical Engineering from Fluminense Federal University in Brazil in 1971, an M.S. in Operations Research from Florida Institute of Technology, USA in 1976 and a Ph.D. in Industrial Engineering from Wayne State University, MI, USA in 1987. He has been three times at University of Florida, Gainesville, FL, USA, as a research scholar, where he taught each time the course Industrial Quality Control and wrote several technical articles and an Industrial Quality Control workbook. He also did some research at Pennsylvania State University, USA. His research interests include life testing and Weibull and Inverse Weibull reliability estimation. His publications have appeared in IIE Transactions, ASQC Transactions, Elsevier Science Proceedings, Balkema Proceedings, Comadem Procedings (UK), ESREL Proceedings and in several Brazilian journals.