

AN APPROACH FOR THE TRANSIENT BEHAVIOR OF HORIZONTAL AXIS WIND TURBINES USING THE BLADE ELEMENT THEORY

UMA ABORDAGEM PARA O COMPORTAMENTO TRANSIENTE DE TURBINAS EÓLICAS DE EIXO HORIZONTAL (HAWT) UTILIZANDO A TEORIA DO ELEMENTO DE PÁ

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ABSTRACT

In this work, an efficient mathematical model applied to transient behavior of Horizontal-Axis Wind Turbines (HAWT) was developed. The influence of the power drive on HAWT dynamic modelling was taken into account, in order to present an improved approach for the design of wind power systems. In all simulations, a rotor with 30 m diameter operating at 20 m/s wind velocity was used. The driveline comprises the mass-moment of inertia, electromagnetic torque, and the friction torque of whole system. To solve the nonlinear dynamic equation at each time step, 4th order Runge-Kutta numerical method was considered, while a Newton-Raphson scheme was applied to the steady-state regime. In addition, to calculate the aerodynamic torque, the Blade Element Theory (BET) was implemented, since such a parameter is usually obtained through approximated mathematical functions, mainly those applied to large wind turbines as described in several works available in the literature. BET is a well-known method applied to design and aerodynamic analysis of wind turbines, which presents good agreement with experimental data. To conclude, the results show the rotational speed, output power and torque dependents on time, and depict good behavior when compared with Bao and Ye (2001). An application using BET was also carried out, which yielded consistent results.

Keywords: Wind turbine, Dynamic behavior of HAWT, Blade Element Theory.

RESUMO

Neste trabalho foi desenvolvido um modelo matemático eficiente aplicado ao comportamento transiente de Turbinas Eólicas de Eixo Horizontal (HAWT). Foi levado em consideração a influência da unidade de transmissão na modelagem dinâmica a fim de se obter uma abordagem melhorada para o projeto de sistemas de energia eólica. Em todas as simulações, um rotor de 30 m de diâmetro operando a 20 m/s de velocidade de vento foi utilizado. O sistema de transmissão compreende o momento de inércia, torque eletromagnético e o torque de atrito de todo o sistema. Para solucionar a equação dinâmica não linear em cada intervalo de tempo, o método de Runge-Kutta de 4^a ordem foi considerado, enquanto que o método de Newton-Raphson foi aplicado ao regime estacionário. Além disso, para calcular o torque aerodinâmico, a teoria do elemento de pá (BET) foi implementada, uma vez que o torque do rotor é frequentemente calculado através de funções matemáticas aproximadas, principalmente quando aplicados a grandes turbinas eólicas, tal como descrito em várias obras disponíveis na literatura. BET é um método bem conhecido aplicado ao projeto e análise aerodinâmica de turbinas eólicas, o qual apresenta boa concordância com dados experimentais. Para concluir, os resultados para a velocidade de rotação, potência de saída e torque dependentes do tempo mostram bom comportamento quando comparados com Bao; Ye (2001). Uma aplicação utilizando o método BET também foi realizado, o qual produziu resultados consistentes.

Palavras-chave: Turbinas eólicas, comportamento dinâmico de HAWT, teoria do elemento de pá.

1 – INTRODUCTION

Over the years, the study of wind turbines has been increased, and the main motivation is the power generation using clean energy with low environmental impact. This work presents a simple but efficient mathematical model, which considers a complete wind power system operating at variable rotational speed, since several model available in the literature (BAO; YE, 2001; CAMBLONG; VIDAL; PUIGGALI, 2004; MULJADI; PIERCE; MIGLIORE, 2000) take into account a fixed speed for the driveline, not reflecting practice operational condition. The variation of the wind speed always results in variation on the rotor speed, and so to consider such variation it is necessary to assess the complete driveline in order to provide the effect

of coupling between the wind rotor and electric generator. Therefore, this work shows an approach which considers the effects of inertia, friction and electromagnetic losses caused along the power drive. In addition, the methodology allows the steady-state Blade Element Theory – BET to determine the rotor power coefficient, providing a quasi-stationary scheme. BET is a well-known method applied to design and performance analysis of wind turbines, presenting good agreement with experimental data, as described by (RIO VAZ *et al.*, 2013).

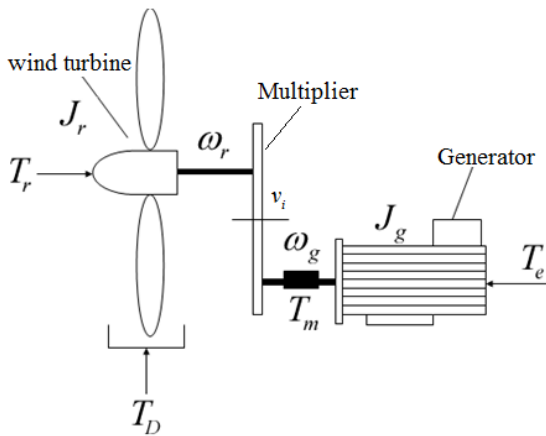
The proposed methodology brings an important contribution, leading to a generalized dynamic approach in order to consider Horizontal-Axis Wind Turbines – HAWT designed from BET. At the modeling, the wind rotor is coupled with the driveline, in which the effects of multiplier

and the electric generator are included, as well as the mass-moment of inertia necessary for the dynamic equation. To conclude, the results show the rotational speed, output power and torque dependents on time, depicting good behavior when compared with Bao; Ye (2001). An application using BET is also carried out, which yields consistent results.

2 – DYNAMIC EQUATION FOR THE POWER DRIVE

The power drive of a HAWT generally consists of a wind rotor with mass-moment of inertia J_r , connected to a generator with mass-moment of inertia J_g , through a mechanical multiplier with transmission ratio v_i , as shown in Figure 1.

Figure 1 – Illustration of the power drive (adapted from Bao; Ye (2001))



To evaluate the transient behavior of the driveline, it is necessary to take into account the dynamic equation of the power transmission system, defined by (CAMBLONG; VIDAL; PUIGGALI, 2004):

$$J_r \frac{\partial \omega_r}{\partial t} = T_r - T_D - (v_i \cdot T_m) \quad (1)$$

Where: (ω_r) is the rotor angular speed, (T_r) is the rotor torque, (T_D) is the total dissipative torque, which is related to the total friction on the driveline, and (T_m) is the reaction torque of the second shaft (Figure 1).

2.1 Rotor torque

According to Mesquita *et al.* (2014), the wind rotor torque is obtained from the aerodynamic forces acting on the rotor blades. Such interaction can be determined by the classical BET, which is the model most frequently used by scientific communities for design and aerodynamic analysis of wind rotors (VAZ; PINHO; MESQUITA, 2011). This method is essentially an integral method with semi-empirical information generated from aerodynamic forces, which act on wind blade sections. The aerodynamic forces are usually obtained from two dimensional airfoil flow model or experimental data, as given by Sheldahl; Klimas (1981), which developed a wind tunnel test series for four symmetrical airfoils. Another important reference in this

concern correspond to the classical work performed by Abbott; Doenhoff (1959), in which an important data bank on the aerodynamics of airfoil at subcritical speeds, very useful for the turbomachinery design is presented.

The forces on a blade element can be obtained through the velocity diagram shown in Figure 2, where (a) and (a') are the axial and tangential induction factors. (V) is the undisturbed freestream velocity and (W) is the relative velocity. (α) is the angle of attack, (β) is the twist angle at each blade section, (ϕ) is the flow angle, (r) is the radial position, (L) and (D) are the lift and drag forces. (F_n) and (F_t) are the normal and tangential forces, respectively. The rotor torque can be expressed as a function of the induction factors, which are given by (HANSEN, 2008):

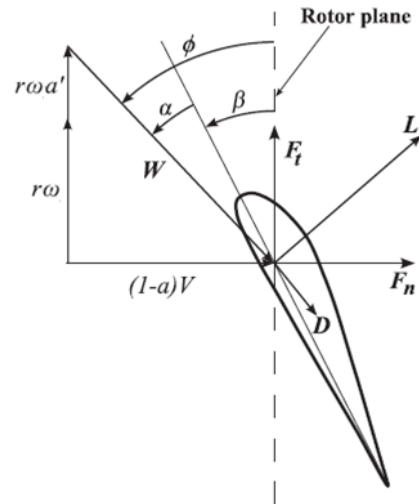
$$\frac{a}{1-a} = \frac{\sigma \cdot C_n}{4 \cdot F \cdot \sin^2 \phi} \quad (2)$$

$$\frac{a'}{1-a'} = \frac{\sigma \cdot C_t}{4 \cdot F \cdot \sin \phi \cdot \cos \phi} \quad (3)$$

Where: (σ) is the solidity of the wind rotor, and (F) is the classical Pradntl tip-loss factor, as described in Rio Vaz *et al.* (2013). (C_n) and (C_t) are the coefficients of normal and tangential forces. In this case, theoretically, the power coefficient (C_p) is defined as:

$$C_p = \frac{8}{\lambda^2} \cdot \int_0^\lambda a' \cdot F \cdot [(1 - (a \cdot F))] \cdot x^3 \partial x \quad (4)$$

Figure 2 – Velocity diagram for a rotor blade section



Where: $\lambda = \omega_r R / V_0$ is the tip-speed ratio and $x = \omega_r r / V_0$ is the local-speed ratio. The rotor torque (T_r) is given by:

$$T_r = \frac{P_r}{\omega_r} = \frac{1}{2} \cdot \frac{\rho \cdot \pi \cdot R^2 \cdot V^3}{\omega_r} \cdot C_p \quad (5)$$

Where: (P_r) is the rotor output power.

2.2 Dissipative torque

Dissipative torque is inherent to the driveline, and occurs in all constituting elements of the turbine, as rotor, bearings, gearbox, etc. In real engineering problems, the friction torque must be taken as an important contribution in this

regard, as detailed by Mesquita *et al.* (2014), which use the empirical formulation described by Palmegren (1959) in order to modeling the driveline. However, in this work, a mathematical function for the dissipative torque (T_D) is included. Such formulation has been widely used by many authors in the literature (BAO; YE, 2001; CHEN; JIANG, 2009), mainly for large wind turbine. Equation (6) was first used by Steinbuch (1989), which applied it to a 30 m diameter wind turbine, demonstrating reasonable agreement with practical results in large turbines. Thus:

$$T_D = C_1 + \frac{C_2}{\omega_r} \cdot C_3 \cdot \omega_r \quad (6)$$

Where: (C_1), (C_2) and (C_3) are appropriated constants due to the total friction of the mechanical parts imposed on the power drive, which are normally obtained by experimental fit.

2.3 Generator torque

Hong; Chen; Tu (2013) show that electricity generation systems have attracted great attention, since they can operate with constant or variable speeds using power electronic converters. Among them, variable speed generation devices are more attractive than the fixed ones due to the improvement in energy production. Variable speed power generation enables operation of the turbine at its maximum power coefficient over a wide range of speeds. Hasanien (2010) presents a control system for Permanent Magnet Synchronous Motor – PMSM, which often is used for torque ripple minimization in these sort of motors. The dynamic response of PMSM with controller is usually studied during starting process under full load torque and under load disturbance. Therefore, the permanent magnet synchronous generator is a good option for high performance energy generation in variable-speed operation. The torque of a permanent magnet synchronous generator can be given by Equation (7):

$$T_e = \frac{3}{2} \cdot p \cdot \psi i_{sq} \quad (7)$$

Where: (p) is the pole pair number, (Ψ) is the magnet flux and (i_{sq}) is the electric current one of the synchronous phases.

However, it is emphasized here, that the one of the goals is only to consider the effect of the electromagnetic torque on the dynamic behavior of HAWT. Furthermore, the proposed approach can be applied to any electrical alternator/generator, whether the electromagnetic torque is known. Detailing upon the control system, connection to the electrical grid, and the operation with constant speed or variable speed using power electronic converters were described in Hong; Chen; Tu (2013). Thus, only to reduce the complexity of the electromagnetic torque equation, the relation between synchronous generator torque and its angular speed is assumed as an approximated linear equation (MESQUITA *et al.*, 2014). In this case, a first order linear function for the electromagnetic torque dependent on the generator angular speed is used. At the

second shaft, the dynamic equation becomes (SLOOTWEG *et al.*, 2003; CAMBLONG; VIDAL; PUIGGALI, 2004):

$$J_g \frac{\partial \omega_g}{\partial t} = T_m - T_e \quad (8)$$

Where: $\omega_g = v_i \cdot \omega_r$ is the generator angular speed and (T_e) is the electromagnetic torque, which can be given by (BAO; CHEN, 1996):

$$T_e = K_e \cdot \omega_g + K_0 \quad (9)$$

Where: (K_e) and (K_0) are obtained by a linear fit with experimental data.

This methodology is reasonable, since the electromagnetic torque is always available by generator manufacturers. In this work was assumed the same values used by Bao; Ye (2001) for $K_e = 378.9$ Nms and $K_0 = -59,548$ Nm.

2.4 Solution method of the dynamic equation

Clearly, Equation (1) associated with Equation (8) present nonlinear behavior and a numerical procedure is necessary to solve the mathematical problem. Thus, 4th order Runge-Kutta method (BURDEN; FAIRES, 2010) is used, through which the transient model for the coupling rotor-generator can be solved. Therefore, combining Equations: (1), (5), (6), (8) and (9), yield:

$$J_{eq} \cdot \omega_r \cdot \frac{\partial \omega_r}{\partial t} + C_3 \cdot \omega_r^2 + [C_1 + v_i \cdot T_e(\omega_r)] \cdot \omega_r - \left[\frac{1}{2} \cdot \rho \cdot \pi \cdot R^2 \cdot V^3 \cdot (C_p - C_2) \right] = 0 \quad (10)$$

Where: $J_{eq} = J_r + v_i^2 \cdot J_g$ is the equivalent mass-moment of inertia.

Equation (10) is a first order nonlinear ordinary differential equation, where the rotation is dependent on time for any wind speed, including variable wind speed. This equation can be solved in two ways. The first one considers the unsteady behavior, becoming:

$$\frac{\partial \omega_r}{\partial t} = \frac{1}{J_{eq} \cdot \omega_r} \cdot \{(-C_3 \cdot \omega_r^2) - [C_1 + v_i \cdot T_e(\omega_r)] \cdot \omega_r + \left[\frac{1}{2} \cdot \rho \cdot \pi \cdot R^2 \cdot V^3 \cdot C_p \cdot (\lambda, \beta) - C_2 \right]\} = f(t, \omega_r) \quad (11)$$

$$K_1 = f(t, \omega_r) \quad (12)$$

$$K_2 = f\left(t + \frac{h}{2}, \omega_r + \frac{h}{2} \cdot K_1\right) \quad (13)$$

$$K_3 = f\left(t + h, \omega_r + h \cdot K_2\right) \quad (14)$$

$$K_4 = f(t + h, \omega_r + h \cdot K_3) \quad (15)$$

$$\omega_r^{i+1} = \omega_r^i + \frac{h}{6} \cdot (K_1 + 2 \cdot K_2 + 2 \cdot K_3 + K_4) \quad (16)$$

Note that Equation (11) gives the rotor speed depends on time. The power coefficient C_p is calculated by BET, Equation (4), and this is the major difference between this model and those described by Bao; Ye (2001), Camblong; Vidal; Puiggali (2004), and Muljadi; Pierce; Migliore (2000). Equation (11) can be solved in a simple procedure using any computational tool, as MATLAB, EXCEL, C, and FORTRAN. It is imperative to observe that at each time step, the BET needs to be called in order to calculate C_p through Equation (4). To solve BET is necessary to construct another routine, separately, where the input data as rotor aerodynamic characteristics and information on the rotor blade geometry are needed. A detailed procedure to calculate C_p using BET, can be found in Rueda (2012).

The second one is obtained for steady-state behavior, where Equation (10) is invariant on time, leading it to a transcendental equation:

$$C_3 \cdot \omega_r^2 + [C_1 + v_i \cdot T_e \cdot (\omega_r)] \cdot \omega_r - \left[\frac{1}{2} \cdot \rho \cdot \pi \cdot R^2 \cdot V^3 \cdot C_p - C_2 \right] = 0 \quad (17)$$

Equation (17) can be solved by Newton-Raphson method, as:

$$\Phi(\omega_r) = C_3 \cdot \omega_r^2 + [C_1 + v_i \cdot T_e \cdot (\omega_r)] \omega_r - \left[\frac{1}{2} \cdot \rho \cdot \pi \cdot R^2 \cdot V^3 \cdot C_p - C_2 \right] \quad (18)$$

$$\omega_r^{i+1} = \omega_r^i - \frac{\Phi(\omega_r^i)}{\frac{\partial \Phi(\omega_r^i)}{\partial \omega_r}} \quad (19)$$

Where:

$$\frac{\partial \Phi(\omega_r)}{\partial \omega_r} = \{ -C_2 - [C_3 + K_e \cdot v_i^2] \cdot \omega_r^2 + \frac{1}{2} \cdot \rho \cdot \pi \cdot R^2 \cdot V^3 \cdot \left[\omega_r \frac{\partial C_p}{\partial \omega_r} - C_p \right] \} \cdot \frac{1}{\omega_r^2} \quad (20)$$

Similarly, to the transient procedure, the numerical solution for Equation (19) requires also the power coefficient calculation at each time step, which can be made calling BET within the interactive scheme. Note that through Equations (11) and (19), the power drive model can be expressed as a function of the total mass-moment of inertia, the rotor aerodynamic, dissipative and generator torques. It is important to note that the aerodynamic characteristics of the blade shape were computed by BET, and it was incorporated in the numerical procedure through the power coefficient.

3 – RESULTS AND DISCUSSION

To ensure a suitable behavior of the present procedure, a numerical verification is performed using the data published by Bao; Ye (2001), in which a mathematical formulation for the power coefficient is used. Moreover, a simulation using BET coupled with the dynamic equation

of the driveline is computed, which shows better efficiency when compared with Bao; Ye (2001) as well.

3.1 Verification of the numerical procedure

The numerical procedure is verified comparing the present work with Bao; Ye (2001) approach. For that, it is necessary to consider the power coefficient as a function of the tip-speed ratio and the pitch angle, $C_p = C_p(\lambda, \theta)$, as shown in Equation (21). The input parameters for the simulation are shown in Table 1.

$$C_p(\lambda, \beta) = 0.2 \cdot \left(\frac{151}{\lambda} - 0.65\theta - 10 \right) e^{-\frac{12}{\lambda}} \quad (21)$$

Where:

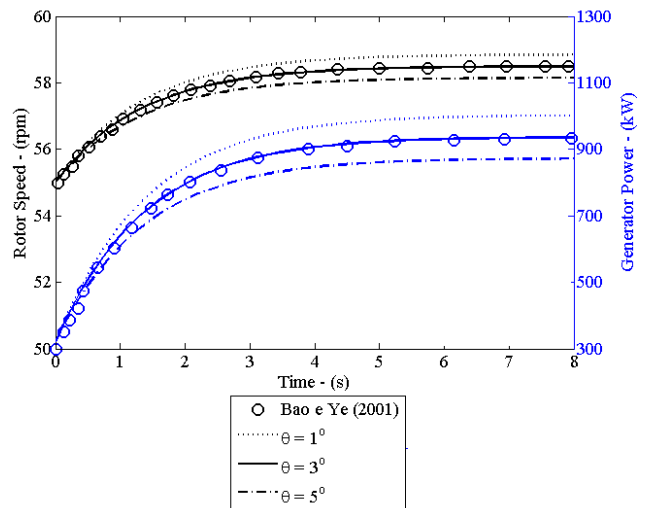
$$\tilde{\lambda} = \frac{1}{\frac{1}{\lambda - 0.001\theta} - \frac{0.0001}{\theta^3 + 1}} \quad (22)$$

Table 1 – Parameters considered in the simulation (BAO; YE, 2001)

$J_r = 350,000 \text{ kg m}^2$	$R = 15 \text{ m}$	$J_g = 32 \text{ kg m}^2$
$C_1 = 1,000 \text{ Nm}$	$C_2 = 1000 \text{ rad/s}$	$C_3 = 100 \text{ s/rad}$
$v_i = 28.32$	$r = 1.25 \text{ kg/m}^3$	$h_g = 0.90$
$V = 20 \text{ m/s}$	$w_0 = 54 \text{ rev/min}$	$h_1 = 0.97$

In Figure 3 are shown the results for the generator power and the rotor speed dependent on time, considering pitch angle $\theta = \{1^\circ, 3^\circ, 5^\circ\}$. Note, in this case, that the mathematical model described previously converges to Bao; Ye (2001) results for $\theta = 3^\circ$. It is noteworthy that Bao; Ye (2001) omitted the exact pitch angle simulated in their work.

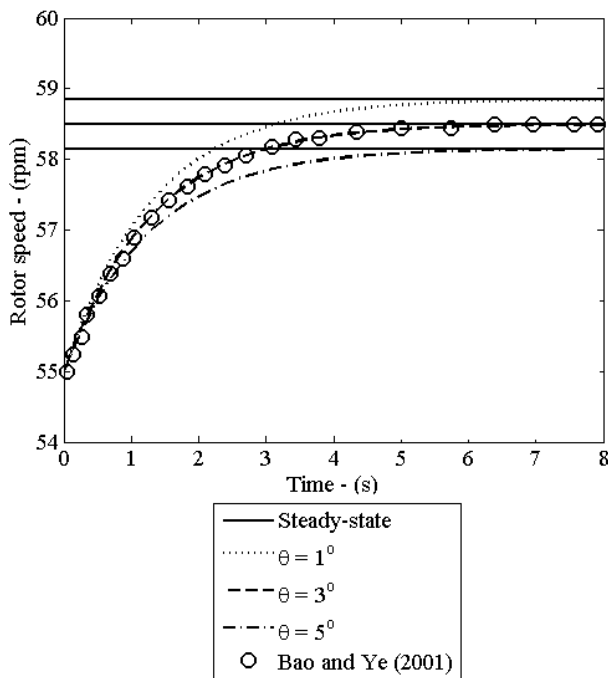
Figure 3 – Rotor speed and generator power dependent on time for some values of pitch angle



In Figure 4 is shown the good behavior of the steady-state case, converging to Bao; Ye (2001). Further, it is important to raise some relevant facts, as: (1) if the aerodynamic torque (T_r) is higher than the combined dissipative torque of the power train, the driveline accelerates ($T_r > T_D + v_i T_e$). Otherwise, (2) if

$T_r = T_D + v_i T_e$ the driveline presents a stationary behavior, and (3) if $T_r < T_D + v_i T_e$ the rotational speed decelerates. In all cases, the flow velocity is an input data, in which the aerodynamic influence needs to be taken into account at any operational condition. This aspect shows that the determination of the flow velocity for the starting rotor is strongly dependent on the dissipative torque from both generator and power train. In practice the generator torque characteristics is available from the manufacturer, and therefore, the efforts must be addressed to the friction torque model at low rotational speed. Observe that the numerical procedure presents a good behavior, allowing its use as an alternative tool for the efficient wind turbine design.

Figure 4 – Rotor speed dependent on time for some values of pitch angle

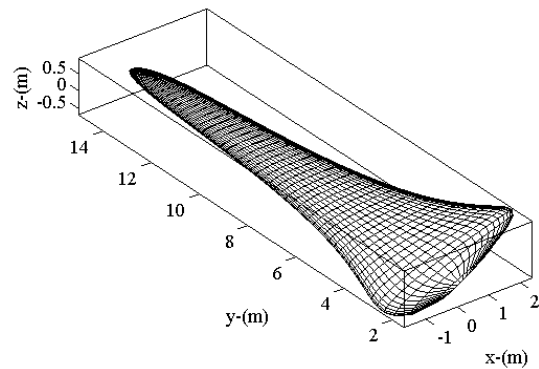


3.2 Results of the proposed methodology using BET

To evaluate the dynamic equation coupled with the classical BET as proposed in this work, a turbine with a 30 m diameter, 3 blades, NACA 0012 airfoil, hub with a 3 m diameter, constant rotor speed (59 rpm) and pitch angle of a 0° are considered. The geometry is depicted in Figure 5, which was designed using the well-known Glauert's optimization (1935), whose method is based on the general momentum theory including the rotational velocities in the near wake. According to Glauert (1935) the maximum efficiency is a function of tip speed ratio, with values ranging from zero efficiency at zero tip speed ratio to 59% at high tip speed ratios for an ideal turbine. In this case, the purpose is to extend the transient behavior of HAWT for any rotor designed by BET, extending methodologies commonly used in the literature, where the power coefficient is computed by mathematical functions with limited applicability, most appropriated to large wind

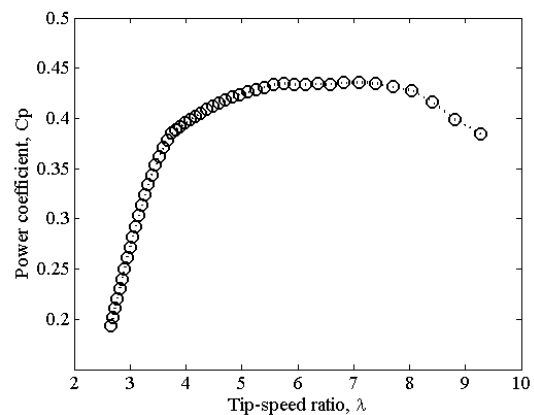
turbines. The BET method has the advantage to consider realistic information of the wind rotor, as lift, drag, number of blades, hub diameter, turbine diameter and airfoil. In this concern, BET method is better than those models based on equations as (21), since such formulations do not present enough details necessary to evaluate the effect of the turbine aerodynamic on its own transient behavior.

Figure 5 – Wind blade designed using the Glauert's optimization (1935)



The power coefficient computed with BET is shown in Figure 6. The turbine rotation evolves over time, allowing calculation of the power coefficient by BET at each time step, becoming the present approach a quasi-stationary model.

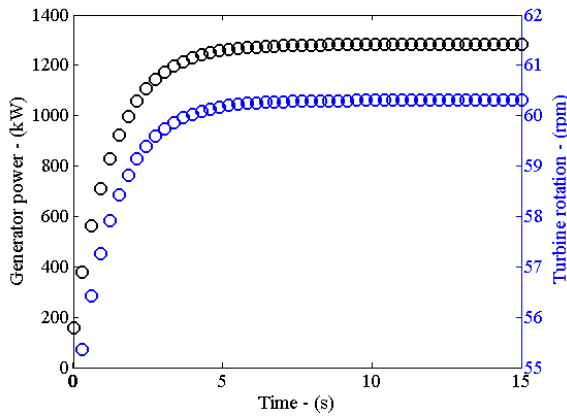
Figure 6 – Power coefficient as a function of the tip-speed ratio



In Figure 7 are shown the results obtained using constant wind velocity of a 20 m/s. The design parameters of the driveline considered in the simulation are the same described by Bao; Ye (2001) (see Table 1). The turbine rotation stabilizes at 60.3 rpm for $t > 9$ s, and the generator power stabilizes at 1,282 kW. Note that the generator power calculated by BET is rather different than the one obtained using Bao; Ye (2001). That occur because Equation (21) is not able to capture the detailed aerodynamic information of the wind turbine, admitting possible dubious response. Although experimental comparisons are necessary to confirm this proposal, it still suggests considerably modification on available methodologies, yielding an

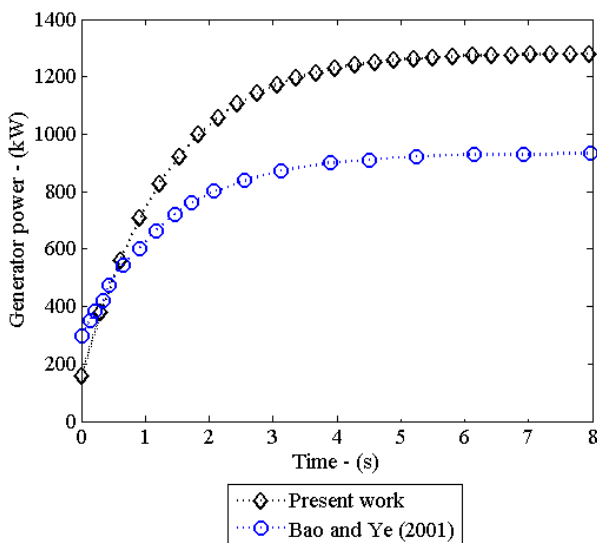
useful tool to be applied by engineer and designers. Moreover, the results showed in Figure 7 ensures that in HAWT design, transient studies need to be further investigated, and the present approach appears as an alternative.

Figure 7 – Generator power and the turbine rotation as a function of time



In Figure 8 is shown the comparison between the wind turbine designed in this work, using the classical Glauert’s optimization, and the one studied by Bao; Ye (2001). Note that the generator power produced using the proposed methodology is 37% higher than the Bao; Ye (2001) one. Such result depicts the good behavior of the approach described here, yielding relevant physical consistency. Thus, this study can be further improved considering some relevant aspects, such as: (i) the running friction torque on the driveline, (ii) starting condition considering the influence of the static friction torque, and (iii) “overspeed”, when the electric load is taken out from the system, leading the rotor speed to very high value.

Figure 8 – Generator power as a function of time



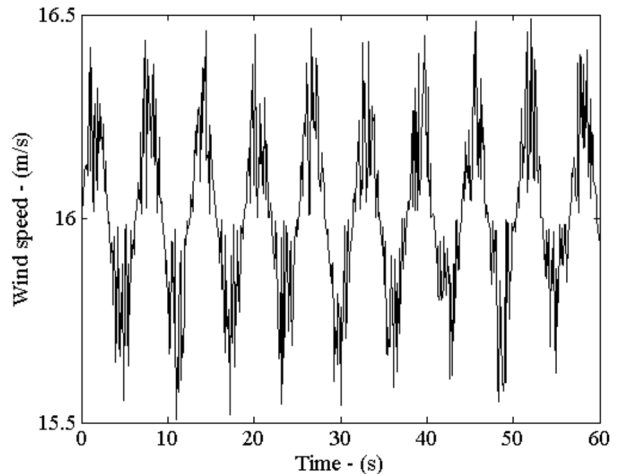
In order to assess the impact of variable wind velocities on the behavior of the model, a synthetic function (artificial) for $V(t)$ is used, as in Equation (23).

$$V(t) = \frac{V_{max}-V_{min}}{2} [\sin(t)random(N) + 1] + V_{min} \quad (23)$$

Where: $V_{max} = 16.5$ m/s, $V_{min} = 15.5$ m/s and $N = 60$. Such artificial $V(t)$ is used only for the purpose to show that the methodology ensures variable wind velocities.

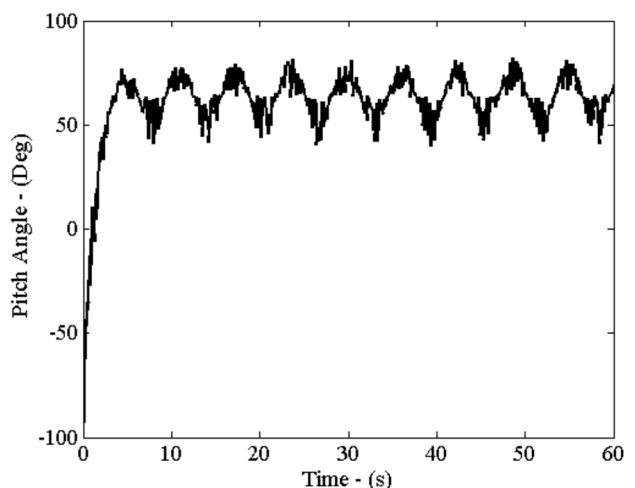
The profile of Equation (23) is presented in Figure (9).

Figure 9 – Wind velocity profile dependent on time



Previously, a fixed pitch angle was considered, since for constant wind velocity such control is not necessary, and obviously the turbine performs smoothed response. However, for realistic winds, the result always states an oscillatory behavior, which may compromise the quality of the electric energy. In Figure 10 is shown an artificial pitch angle profile obtained from Rueda (2012). This profile will be used to improve the behavior of the generated energy. Otherwise, is not objective in this work to conduct any assessment on pitch control.

Figure 10 – Variable pitch angle



In Figures 11 and 12 is shown the comparison between fixed and variable pitch angle on the turbine performance. Note that one form to stabilize the oscillation is to include a pitch angle control. Certainly, such control requires a complex assessment of the power train and also the wind

velocity acquisition, since to control the pitch angle precise equipment to measure the wind velocity is necessary, as well as, a good knowledge of the rotor aerodynamic and friction on the driveline.

Clearly, the oscillation of the rotor speed might be heavily attenuated under pitch control, as shown in Figure 11, leading the generated energy to a more stable behavior (Figure 12). In addition, such result shows in a consistently way, that the present approach is able to endure variable wind velocity dependent on time.

Figure 11 – Rotational speed for variable pitch angle

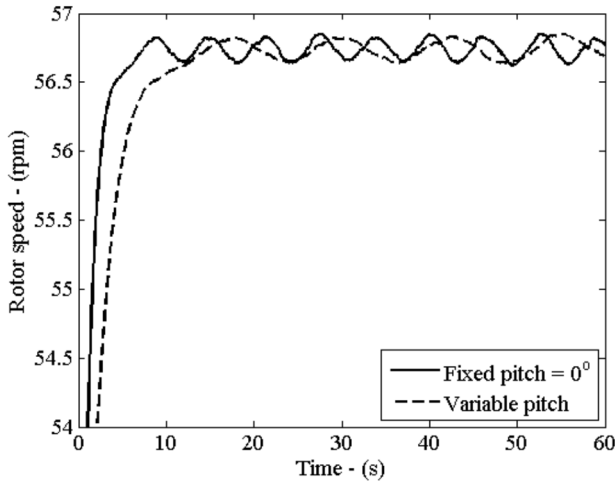
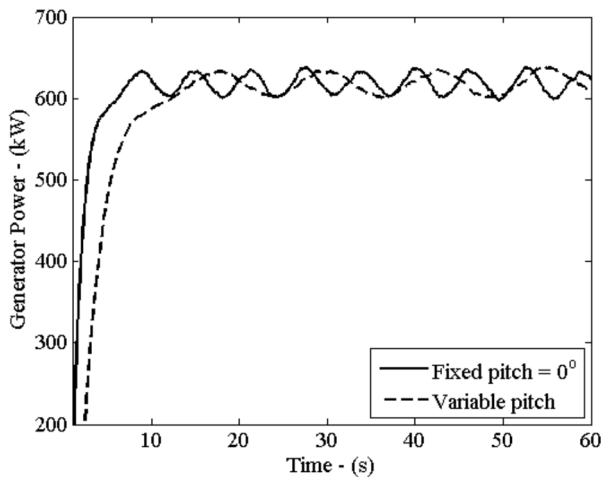


Figure 12 – Generator power for variable pitch angle



CONCLUSIONS

The proposed methodology represents an alternative approach for the efficient design of HAWT, considering the inertial effects and energy loss in the whole power system. An important contribution of this method is to solve the dynamic equation of the power train using a scheme with lower computational cost and easy implementation. In all simulations the time processing for the solution did not exceed five minutes. An important aspect in this regard, it is the use of classical BET method applied at each time step, in order to calculate the turbine aerodynamic torque. This methodology is not current in the literature, being also an important contribution. It is noteworthy that in general,

functional mathematical formulas are used for calculating the power coefficient, as given by Bao; Ye (2001), which use an approximated expression regarded as the polynomial of the tip-speed ratio and pitch angle. In the same way, Slootweg *et al.* (2003) use a numerical approximation developed to calculate the power coefficient for given values of tip-speed ratio and pitch angle. The methodology described here extends the dynamic equation for power coefficient provided by BET, since the model need only the turbine aerodynamics information, in this case, the power coefficient as a function of tip-speed ratio. The results were verified through a comparison with the model described by Bao; Ye (2001), showing good agreement. However, some limitations need to be highlighted in this work, as comparisons with experimental data, since to model the whole system two major aspects are necessary: (i) a good knowledge of the rotor blade geometry and aerodynamic behavior, which most of the references available in the literature omit such information; and (ii) detailed information of each element of the driveline, since bearings, coupling and gearbox impose relevant friction loss, changing the equation for the dissipative torque, T_D .

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NOMENCLATURE

a axial induction factor
 a' tangential induction factor
 B number of blades
 C_n normal force coefficient
 C_p power coefficient
 C_t tangential force coefficient
 F Prandtl's tip-loss factor

h step size for the numerical procedure, m
 i_{sq} electric current, A
 J_r mass-moment of inertia of the turbine, kgm²
 J_g mass-moment of inertia of the generator, kgm²
 p generator pole pair number
 P_r output power, W
 r radial position at the rotor plane, m
 R radius of the wind turbine, m
 t time, s
 T_D dissipative torque, Nm
 T_e electromagnetic torque, Nm
 T_m torque on the second shaft, Nm
 T_r rotor torque, Nm
 V free stream velocity, m/s
 v_i transmission ratio
 x local-speed ratio

Greek symbols

α angle of attack, degree
 β twist angle, degree
 η_g generator efficiency
 η_i transmission efficiency
 λ tip-speed ratio
 ω_g generator angular speed, rad/s
 ω_r wind rotor angular speed, rad/s
 ω₀ initial wind rotor angular speed, rad/s
 φ dimensionless time
 ρ air density, kg/m³
 σ solidity of the wind rotor
 θ pitch angle, degree
 Ψ magnet flux, kgm²/(s²A)

Subscripts

e electromagnetic
 g generator
 i transmission ratio
 n normal
 p power
 r wind rotor
 t tangential
 0 initial value

Superscripts

i iterative loop