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MATHEMATICAL MODELING OF TRANSFORMERS BASED ON THE FREQUENCY RESPONSE OF MAGNETIZING PARAMETERS

MODELAGEM MATEMÁTICA DE TRANSFORMADORES BASEADA NA RESPOSTA EM FREQUÊNCIA DOS PARÂMETROS DE MAGNETIZAÇÃO

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ABSTRACT

A mathematical modeling procedure for single phase low power transformers is presented in this paper. It is based on the frequency response of the shunt parameters, taken from the electric model. The open circuit test was performed for different frequencies in four transformers, and the magnetizing parameter frequency responses were determined. It was verified that the sum of exponential functions can be employed to represent the behavior of the magnetizing parameters as a function of frequency, with notable accuracy. The results contribute to the improvement of mathematical models and circuit equivalents of single phase transformers.

Keywords: Frequency response, magnetizing parameters, mathematical modeling, transformers.

RESUMO

Um procedimento de modelagem matemática para transformadores monofásicos de baixa potência é apresentado neste artigo. Este procedimento é baseado na resposta em frequência dos parâmetros de derivação do modelo elétrico. O teste de circuito aberto foi realizado para diferentes frequências em quatro transformadores, e a resposta em frequência dos parâmetros de magnetização foi determinada. Foi verificado que a soma de funções exponenciais pode ser empregada para representar o comportamento dos parâmetros de magnetização como uma função da frequência, com boa acurácia. Os resultados contribuem para o melhoramento de modelos matemáticos e circuitos equivalentes de transformadores monofásicos.

Palavras-chave: Resposta em frequência, parâmetros de magnetização, modelagem matemática, transformadores.

1 – INTRODUCTION

The transformer is one of the most important equipment of electrical systems. Its working principle is based on electromagnetic induction due to the coupling between two or more coils though the closed core, which is made of ferromagnetic material. The main function of the transformer is to transfer energy from the input circuit to output circuit, increasing or decreasing the values of voltage and current, keeping the frequency constant. Like any real equipment, transformers have losses arising from the electrical resistance of windings, the dispersion of magnetic flux and the cyclical process of core magnetization.

With that in mind, in order to analyze the transformer and determine its model, methods of equivalent circuits based on physical principles and mathematical modeling based on the classical theory of electrical circuits can be employed. These methods can also be used to estimate and to predict alterations in operational characteristics of a transformer.

Techniques related to transformer modeling are employed in several researches and their applications are diverse, such as: transformer design, in order to operate at different frequencies, power quality, improvement of mathematical models, simulation and prediction characteristics, as cited in the next paragraphs.

Roshen (1991) studied the influence of the frequency on the modeling and design of transformers, since it affects the losses, the parameters, and the indexes of power quality equipment. The author presented a method for core loss calculation at high frequencies. It is observed, that the losses in magnetic circuits are important design parameters, and in many high frequency models the core magnetization is limited by their losses.

Similarly, Petkov (1995) provided a model for transformer design considering the influence of frequency, which is considered the major contributor to the reduction of volume and mass of switching power sources. Additionally, Duffy and Hurley (1995) performed studies of transformer operation at different frequencies with respect to the skin and proximity effects. These effects cause increase of losses in the conductors and changes on the values of resistance and inductance for each frequency.

The influence of the frequency can also be used as a transformer diagnostic technique, such as in Abu-Siada and Islam (2007), where frequency response analysis was used to detect deformation in the windings.

The electrical parameters of transformers are also the focus of several studies, and are important for design modeling, monitoring and simulation. Other studies focus on the determination of the transformer electrical parameters as a function of frequency, which has a direct impact on power quality and transformer operation. In Meister and Oliveira (2009) the open circuit test was performed at different excitation frequencies. The objective was to make a curve adjustment to determine mathematical expressions that represent the magnetizing parameters as a function of frequency. They showed that a third degree polynomial can be adjusted by the magnetizing reactance, and a polynomial of second degree represents the core loss resistance.

In Mitchell and Welsh (2013) a methodology for obtaining initial estimates and constraints for several fundamental transformer parameters was verified. The paper aimed to show the importance of an accurate transformer model for use in the study of power transformers, since the manufacturer's proprietary restrictions will generally make access to the detailed design of a transformer difficult.

Moreover, the exact real time identification of transformer winding parameters is favorable for a new transformer protection and winding deformation detection. In Wang (2012), a new model for the calculation of transformer winding parameters was presented using the least squares method. The method was not affected by the operating conditions and the difference among the three phase winding parameters, and had fast convergence and high precision and stability.

As shown in the last citations, modeling the influence of frequency and magnetizing parameters is an issue associated to the design, diagnosis, behavior analysis and simulation of the equipment.

Hence, this study was performed with the objective of contributing to the development of mathematical and circuit equivalent models in low power single phase transformers, by analyzing the magnetizing parameter frequency responses. The proposed mathematical models are analytical expressions that represent the magnetizing parameters as a function of the frequency. The method for designing the models is based on an algorithm that is efficient and has low computational cost.

In this work, the open circuit test was performed in the transformers, applying a sinusoidal voltage with variable frequency between 60 and 2,580 Hz. As the frequency was swept, the voltage and excitation current signals were measured and recorded. The voltages (primary and secondary) and the current were the routine inputs for the magnetizing parameter determination in different frequencies. Once the magnetizing parameter frequency responses were found, the analytical expressions were determined using the Recursive Least Squares (RLS), and its adherence to the experimental data was evaluated by the coefficient of determination (R^2).

2 – THEORETICAL FRAMEWORK

The theoretical foundation on which the method is based is presented here and in the next sub-sections. Thus, the information about the open circuit test, the recursive least squares method, the functions used for representing the magnetizing parameters as a function of frequency and the transformer modeling by equivalent circuits for different excitation frequencies are discussed.

2.1 Open circuit test

The open circuit test is performed to determine the transformer excitation current and the magnetizing parameters, applying the rated voltage and frequency on the secondary side, while the primary side remains open. The voltage must be sinusoidal (IEEE STD., 1995).

Sometimes the voltage contains harmonics, thus, a source that provides a less distorted voltage ensures a more reliable measurement. In this paper, sinusoidal voltages were applied on the transformers with different fundamental frequencies. The frequency range applied was between 60 Hz and 2,580 Hz.

2.2 Recursive Least Squares

The recursive least squares is a technique that estimates, iteratively, unknown parameters. The method monitors the estimation of parameters, reaching the expected value before the stopping criterion, which results in reduced computational effort (ASTROM and WITTENMARK, 1990).

In (1), (2) and (3) a set of expressions which may be used to implement the algorithm are presented.

$$\alpha(k+1) = \alpha(k) + K(k) \cdot [y_{k+1} - \varphi^T(k+1) \cdot \alpha(k)], \qquad (1)$$

$$K(k) = P(k) \cdot \varphi(k+1) \cdot [\lambda + \varphi^T(k+1) \cdot P(k) \cdot \varphi(k+1)]^{-1}, \qquad (2)$$

$$P(k+1) = \frac{[I - K(k) \cdot \varphi^T(k+1)] \cdot P(k)}{\lambda}, \qquad (3)$$

Where: α is the unknown parameters vector; K is the gain matrix; P is proportional to the covariance matrix of α; y represents measured values; I is the identity matrix; λ is the forgetting factor; and φ corresponds to the approximation function that represents the model.

The forgetting factor used in this paper was 1, which represents the classic recursive estimation, wherein the actual estimative is determined by the previous estimative.

2.3 Modeling for determination of analytical expressions

In the modeling for the determination of the analytical expressions, two types of functions were used, the polynomial and the sum of exponential functions. Furthermore, to evaluate the quality of the analytical expressions that represent the magnetizing parameters as a function of frequency, the coefficient of determination (R^2) was used.

The coefficient of determination (CAMERON and WINDMEIJER, 1988) can be calculated by (4).

$$R^2 = 1 - \frac{SSE}{SST}, \quad (4)$$

Where: R^2 is the coefficient of determination; SSE is the summation of the squared differences of the model data and the experimental data; and SST is the summation of the squared differences between the model data and the experimental data mean.

Additionally the analytical expressions that represent the magnetizing parameters as a function of frequency were determined using the functions considered in (5), (6) and (7).

$$Y(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d, \quad (5)$$

$$Y(x) = a \cdot x^2 + b \cdot x + c, \quad (6)$$

$$Y(x) = a \cdot e^{b \cdot x} + c \cdot e^{d \cdot x}, \quad (7)$$

Where: Y is a function of x and a, b, c and d are parameters unknown, which can be estimated by the RLS. Y represents the magnetizing parameters and x corresponds to the frequency.

These functions were chosen to compare the polynomial modeling to the exponential sum modeling.

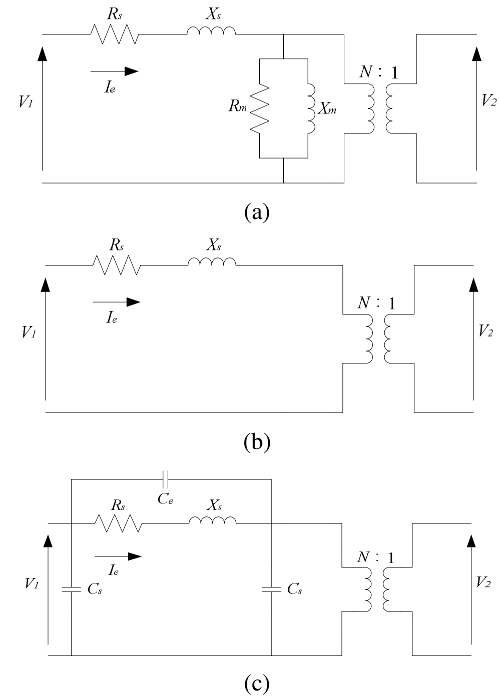
2.4 Equivalent Circuits for Different Frequencies

To characterize a transformer in a wide range of frequencies, it is necessary to write the representative equations of circuit parameters as a function of frequency. If the behavior of the equivalent circuit parameters as a function of frequency is known, several simplified circuits can be modeled, and each one is valid for a determined frequency range (SLEMON, 1994) and (FALCONE, 1979).

Three equivalent circuits of a transformer, characterizing the behavior in different frequencies, are shown in Figure 1 (WANG, LEE and ODENDAAL, 2003).

In Figure 1, R_s is the equivalent winding resistance referred to the primary side; X_s is the leakage reactance referred to the primary side; R_m is the resistance which represents the active core loss; X_m is the magnetization reactance; N is the transformation ratio; V_1 and V_2 are the primary and secondary voltages, respectively; I_e is the excitation current; C_s and C_e corresponds to the capacitances between windings and between windings and ground. The excitation current is, in concept, separated into two components: a magnetizing current and a core loss current.

Figure 1 – Transformer equivalent circuits (SLEMON, 1994) for: (a) low frequencies, (b) medium frequencies and (c) high frequencies



It is known that in low frequencies (50 to 200 Hz) the nonlinear effect of the magnetization branch is more significant. In medium frequencies (200 to 1,500 Hz) this effect is reduced. In high frequencies (above 1,500 Hz), the capacitance effects should be take into account (FALCONE, 1979).

Consequently, the equivalent circuit modeling was used to study the transformer physical phenomena and they were sufficient to assist in the representation of transformer behavior used in this work.

3 – MATERAL AND METHODS

In this section, the characteristics of the transformer and details of the performed tests are presented.

3.1 Material

The transformers employed in this paper are characterized in Table 1.

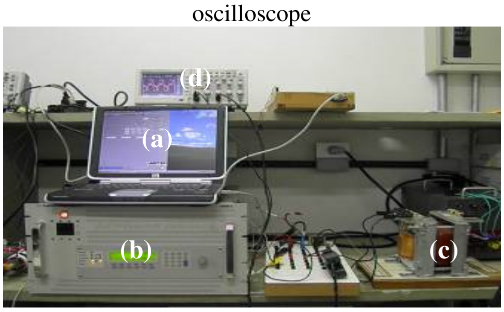
The source used to feed the transformers has the following specifications: zero to 400 V, 40 to 5,000 Hz and 3 kVA of maximum power output.

The experimental platform confectioned to measure the voltage and current signals is presented in Figure 2.

Table 1 – Transformers characteristics

Tag	Rated Power (VA)	Voltage Ratio (V/V)	Core material
T1	1,000	220/110	Amourphous
T2	1,000	220/110	FeSi
T3	100	220/110	FeSi
T4	60	127/12	FeSi

Figure 2 – Experimental platform, containing: (a) computer that controls the source, (b) source, (c) transformer and (d) oscilloscope



There are seen in Figure 2: (a) computer that controls the source; (b) a programmable frequency source; (c) the transformer; and (d) a four-channel digital oscilloscope (sampling rate of 1 GS/s). On the experimental platform other components were used, such as: voltage probe to measure the voltage, a current sensor with ratio 100 mV/A to measure the excitation current and a contact matrix to make the connections between the components

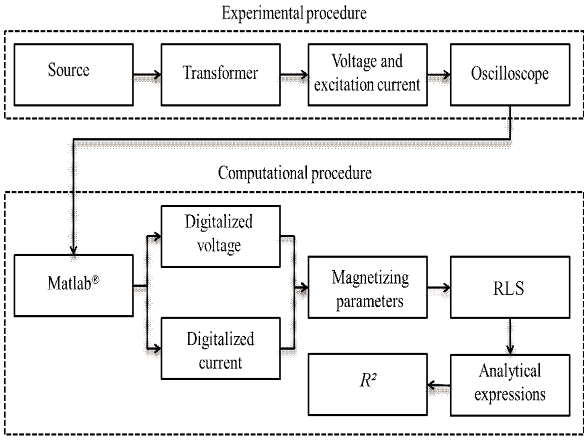
3.2 Experimental and computational procedure

The open circuit test was performed in the transformers, applying a sinusoidal voltage with variable frequency from 60 to 2,580 Hz.

As the frequency changed, the voltage and excitation current signals were measured and recorded in the oscilloscope. The recorded signals were subsequently processed by a dedicated Matlab® routine. The voltage and the current were the routine inputs for the magnetizing parameter determination in different frequencies. Once the frequency response of the magnetizing parameters was defined, the analytical expressions were determined using RLS, and its fidelity to the experimental data was evaluated by R^2.

It is shown in Figure 3 a block diagram that represents the experimental and computational procedure.

Figure 3 – Block diagram that represents the experimental and computational procedure



3 – RESULTS AND DISCUSSIONS

With the objective of contributing to the development of mathematical and circuit equivalent models in low power single phase transformers, the magnetizing parameters frequency response was determined for the four transformers, according to Sections 2.1, 2.2 and 2.3. In Figures 4 to 7 the frequency response of the resistances (R_m) are shown, which represent the active core loss, and in Figures 8 to 11 the frequency response of the magnetization reactances (X_m) are shown. In Figures 4 to 11 the curves determined by RLS are also presented, which correspond to the analytical expressions that represent the magnetizing parameters as a function of frequency.

Figure 4 – Frequency response of the resistance which represents the active core loss and approximation curves for T1

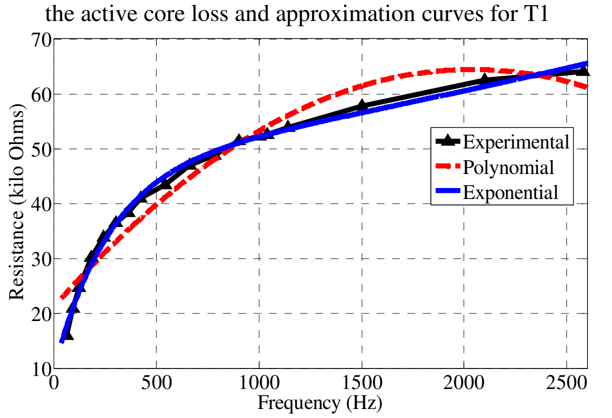


Figure 5 – Frequency response of the resistance which represents the active core loss and approximation curves for T2

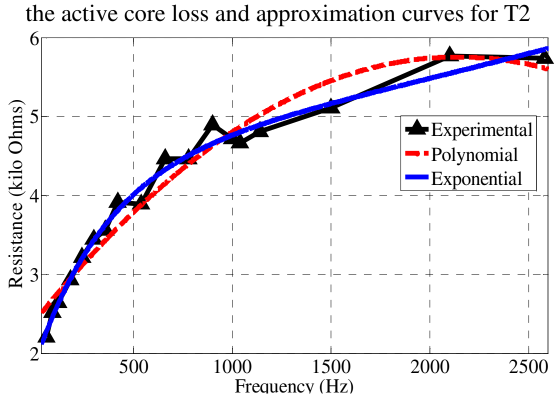


Figure 6 – Frequency response of the resistance which represents the core loss and approximation curves for T3

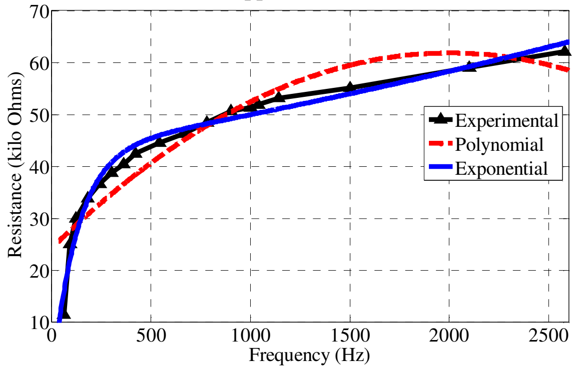


Figure 7 – Frequency response of the resistance which represents the active core loss and approximation curves for T4

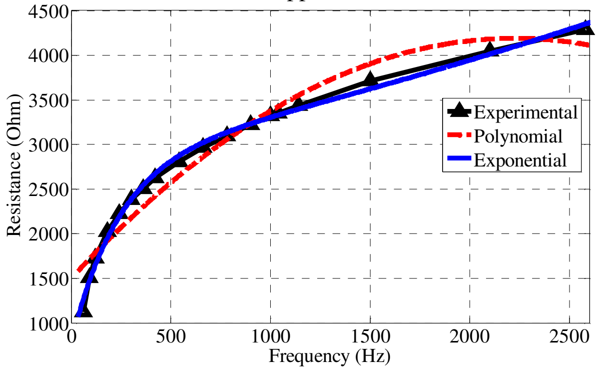


Figure 8 – Frequency response of the magnetization reactance and approximation curves for T1

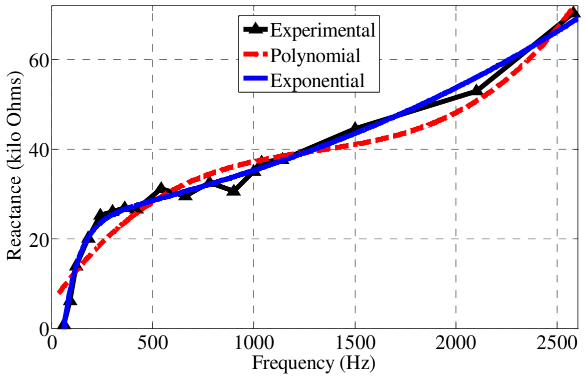


Figure 9 – Frequency response of the magnetization reactance and approximation curves for T2

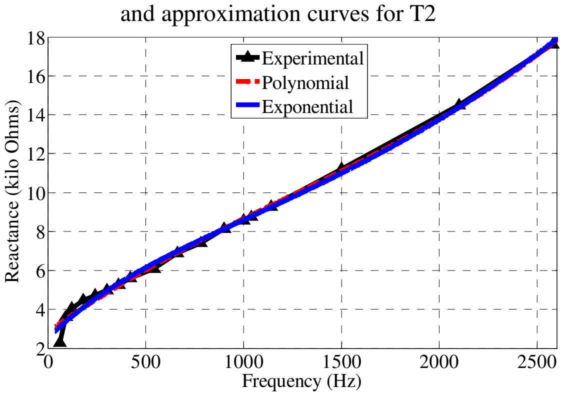


Figure 10 – Frequency response of the magnetization reactance and approximation curves for T3

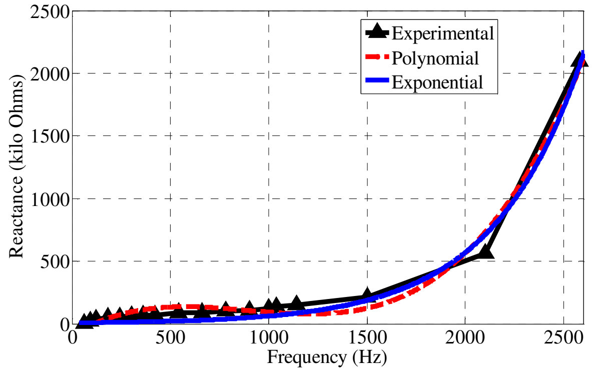
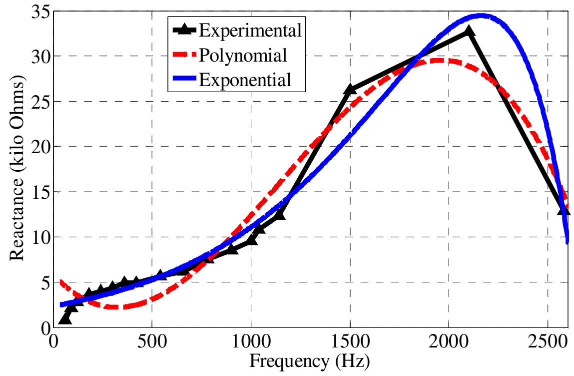


Figure 11 – Frequency response of the magnetization reactance and approximation curves for T4



In Figures 4 to 11 it was shown that the magnetizing parameters increase with the frequency elevation. Thus, the equivalent circuit modeling presented in Figure 1 fits the presented results, since tin the Figure 1 the magnetizing parameters tend to those of the open circuit, showing an increase with the frequency. The resistance that represents the active core loss (R_m) increases with the frequency elevation because of the decrease of current flow area. Moreover, the magnetization reactance (X_m) increases with the frequency elevation because of the proportional ratio between the reactance and frequency.

Wang (2012) shows that the use of cores with a smaller size can increase the capacitive effect. In this manner, in Figure 11 a decrease of the magnetization reactance is presented at 2,580 Hz for T4. It occurred because the capacitance effect in this transformer was enough to decrease the value of the magnetization reactance, since the size of T4 is the smallest compared to the other transformers. Once the magnetizing parameter frequency response curves were defined, the analytical expressions could be determined. The RLS was used to determine the polynomial and exponential sum functions, as seen in Figures 4 to 11.

In Figure 11 it can be observed that in the frequency range between 60 and 1,250 Hz, the frequency response of the magnetization reactance is better represented by exponential sum functions. In the frequency range between 1,250 and 2,580 Hz, the polynomial functions represent better the behavior of the magnetization reactance as a function of frequency.

In Table 2 the analytical expressions that represent the magnetizing parameters as a function of frequency are shown. The polynomial functions, as proposed in Meister and Oliveira (2009), were used to represent the magnetizing parameters as a function of frequency, and the results were compared to exponential sum functions.

Table 2 – Analytical expressions that represents the magnetizing parameters as a function of frequency

Analytical Expression	R²
$R_m^{T1}(f) = -0{,}0113 \cdot f^2 + 42{,}39 \cdot f + 21230$	0.9480
$R_m^{T1}(f) = 46530\, e^{0{,}0001 f} - 36580\, e^{-0{,}0036 f}$	0.9956
$R_m^{T2}(f) = -0{,}0014 \cdot f^2 + 6{,}256 \cdot f + 2806$	0.9620
$R_m^{T2}(f) = 6913\, e^{0{,}0001 f} - 5191\, e^{-0{,}0026 f}$	0.9880
$R_m^{T3}(f) = -0{,}0093 \cdot f^2 + 37{,}57 \cdot f + 24230$	0.8692
$R_m^{T3}(f) = 42820 \cdot e^{0{,}0001 \cdot f} - 44380 \cdot e^{-0{,}0079 \cdot f}$	0.9740
$R_m^{T4}(f) = -0{,}0005 \cdot f^2 + 2{,}419 \cdot f + 1500$	0.9511
$R_m^{T4}(f) = 2818 \cdot e^{0{,}0001 f} - 2038 \cdot e^{-0{,}0041 f}$	0.9950
$X_m^{T1}(f) = (2{,}821 x10^{-5}) \cdot f^3 - 0{,}1105 \cdot f^2 + 161{,}3 \cdot f + 13860$	0.9403
$X_m^{T1}(f) = 57780 \cdot e^{0{,}0004 \cdot f} - 133300 \cdot e^{-0{,}0133 \cdot f}$	0.9907
$X_m^{T2}(f) = (5{,}422\, x10^{-7}) \cdot f^3 - 0{,}0019 \cdot f^2 + 7{,}259 \cdot f + 2827$	0.9945
$X_m^{T2}(f) = 5791 \cdot e^{0{,}0004 \cdot f} - 3262 \cdot e^{-0{,}0022 \cdot f}$	0.9951
$X_m^{T3}(f) = 0{,}0004 \cdot f^3 - 1{,}2 \cdot f^2 + 934{,}2 \cdot f - 89840$	0.9840
$X_m^{T3}(f) = 39060 \cdot e^{0{,}0011 \cdot f} + 3{,}494 \cdot e^{0{,}0049 \cdot f}$	0.9993
$X_m^{T4}(f) = (-1{,}252\, x10^{-5}) \cdot f^3 + 0{,}0428 \cdot f^2 - 23{,}84 \cdot f + 5871$	0.9187
$X_m^{T4}(f) = (-1{,}501\, x10^{6}) \cdot e^{0{,}0020 \cdot f} + (1{,}504\, x10^{6}) \cdot e^{0{,}0020 \cdot f}$	0.9666

It is important to observe that the estimated parameters on the analytical expressions do not have any physical meaning.

CONCLUSIONS

In this paper a mathematical a model based on the frequency response of the magnetizing parameters of a transformer was presented. The open circuit test was performed in transformers, applying sinusoidal voltages with variable frequency.

The analytical expressions that model the magnetizing parameters as a function of frequency were determined employing the least square method. The R^2 of the exponential sum functions are very close to 1, showing that the exponential sum functions model more satisfactorily the magnetizing parameters behavior, when compared with the polynomial functions.

The results contribute to improve the mathematical models of transformers, since that is achieved by using the frequency response of magnetizing parameters. Moreover, the results allow characterizing the transformers in a wide band of frequencies, since used analytical expressions are a function of frequency.

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