Ciência & Engenharia (Science & Engineering Journal) ISSN 1983-4071
22 (2): 105 – 114, jul. – dez. 2013

NUMERICAL MODEL FOR ANALYSIS OF REINFORCED CONCRETE BEAMS UNDER REPEATED CYCLIC LOADS

MODELO NUMÉRICO PARA ANÁLISE DE VIGAS DE CONCRETO ARMADO SOB CARGAS CÍCLICAS REPETIDAS

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ABSTRACT
This paper deals with the application of a numerical model for the simulation of the behavior of reinforced concrete beams submitted to cyclic repeated actions in service state. The model is based on existing models of technical literature for monotonic and reverse cyclic actions of high magnitude known as *lumped dissipation models*. An improved formulation is proposed for the limit function that controls the damage increase due to the loading cycles, adapting the model for the case of cyclic repeated loads. The efficiency of the formulation is evaluated through the comparison with experimental results from twelve reinforced concrete beams subjected to repeated loads and with the results obtained from the expression proposed by CEB-FIP Model Code 1990, which provides the increasing of the maximum deflection as a function of the number of applied cycles. Such comparisons have showed a good potential of the proposed numerical model in the prediction of the increase of deflections of reinforced concrete beams subjected to cyclic loads in service.
KEYWORDS: cyclic loads, lumped damage mechanics, stiffness loss, reinforced concrete structures.

RESUMO
Este trabalho trata da aplicação de um modelo numérico para a simulação do comportamento de vigas de concreto armado submetidas a ações cíclicas repetidas em regime de serviço. O modelo apresentado baseia-se em modelos existentes da bibliografia especializada aplicáveis para ações monotônicas e cíclicas alternadas de elevada intensidade, sendo tais conhecidos como *modelos de dissipação concentrada*. Uma formulação aprimorada é proposta para a função limite que controla a evolução do dano decorrente dos ciclos de carregamento, adaptando o modelo para o caso de cargas cíclicas repetidas. A eficiência da formulação é avaliada por meio de comparações com resultados experimentais de doze vigas de concreto armado submetidas a cargas cíclicas repetidas e com resultados obtidos a partir da expressão do CEB-FIP Model Code, a qual fornece o acréscimo de flecha em função do número de ciclos aplicados. Tais comparações mostraram o bom potencial do modelo numérico proposto na avaliação do aumento de flechas de vigas de concreto armado submetidas a cargas cíclicas em serviço.
PALAVRAS-CHAVE: cargas cíclicas, mecânica do dano concentrado, perda de rigidez, estruturas de concreto armado.

1 – INTRODUCTION

In service state, many engineering structures are subjected to cyclic actions. The traffic of vehicles in bridges, the wind loads on slender buildings and the wave actions in offshore structures are examples of loading with a large number of cycles acting during the life time of those structures. It is recognized and well known in the specialized literature that cyclic loads cause, in a general way, a progressive damage of the mechanical properties of structural materials.

In reinforced concrete elements, the repeated cyclic loads cause the loss of steel-concrete bond, the loss of the contribution of concrete to resist tensile forces among cracks and the increase of permanent strains in materials. These effects have been extensively reported elsewhere (CEB, 1996). Thus, such loadings produce a progressive loss of stiffness of the structure, which is observed through the increase of deflections and cracks width in beams. In function of the number of loading cycles and the amplitude of the stresses in materials, fatigue failure can occur in the structural element.

Important works related to the effects of the repeated cyclic loads in the steel-concrete bond behavior, among which may be mentioned the investigations of Rehm and Eligehausen (1979), Balázs (1991), Koch and Balázs (1993), Oh and Kim (2007), and Lindorf; Lemnitzer; Curbach (2009). Investigations about the contribution of tensioned concrete among cracks and the damage of compressed concrete are also found in the specialized literature, such as Zanuy; Albajar; De la Fuente (2010), and Zanuy; De la Fuente; Albajar (2007), respectively. Other researches focus the effects of repeated loading in deflections of reinforced concrete beams, such as Sparks and Menzies (1973), Lovegrove and Salah (1982), Balaguru and Shah (1982), Koh; Ang; Zhang (1997), Oliveira Filho (2005) and Braguim (1995).

Some simple analytical expressions are proposed by Pitonak (1992), CEB-FIP Model Code 1990 (1993) and ABNT NBR 6118 (ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS, 2007) to evaluate the increase of deflections due to loading cycles. However, since those expressions do not account for the loading amplitude and the flexural reinforcement ratio, that are important parameters in the cyclic behavior, they cannot satisfactorily reproduce the increase of deflections when compared to the experimental values of Oliveira Filho (2005) and Braguim (1995).

The models based on Continuous Damage Mechanics have been handled in the simulation of numerical responses of various materials. The pioneering work that introduced the concept of Continuous Damage is that of Kachanov (1958), relating to the brittle rupture in metals. At a later stage, models applied to the concrete, such as that of Mazars (1984), were presented, allowing the quantification of the process of damage due to the crack formation in the macroscopic mechanical behavior of the material, including for cyclic actions as shown in La Borderie; Mazars; Pijaudier-Cabot (1991) and Papa and Taliercio (1996).

It is important to highlight that the use of complex models can disable the simulation of reinforced concrete structures with a large number of cycles, due to the large computing effort. In this way, simplified nonlinear models can be more attractive, since they have fitting and validation with experimental results. Among such models, *Lumped Damage Mechanics* (LDM) combines the Continuous Damage Mechanics and the Fracture Mechanics with the concept of plastic hinge. The use of LDM for reinforced concrete elements has been included in many researches, such as Flórez-López (1993), Cipollina; López-Inojosa; Flórez-López (1995), Flórez-López (1995), Cipollina and Flórez-Lópéz (1995), Thomson; Bendito; Flórez-López (1998), Álvares (1999), Picón and Flórez-López (2000), Marante and Flórez-Lópéz (2002), Marante and Flórez-Lópéz (2003), Thomson *et al.* (2009) and Amorim; Proença; Flórez-Lópéz (2013).

In this paper, a numerical model is proposed for the simulation of reinforced concrete beams submitted to repeated loads in service state, based on the LDM models originally proposed for monotonic and high-cycle loads. Experimental results are used to evaluate the potential of the model in performing the effects of the loss of stiffness as function of the load cycles.

2 – PROPOSED NUMERICAL MODEL

The models proposed by Cipollina and Flórez-Lópéz, (1995), Flórez-López (1995) and Picón and Flórez-López (2000) are the base for the numerical model proposed in this paper.

The main idea of the lumped concentrated models is to consider, as a simplification, that the dissipation of energy of the reinforced concrete members (consequence of concrete cracking and reinforcement yielding) is concentrated in hinges located at the ends of the member, while the rest of the member remains elastic.

In the lumped dissipation models, the nonlinear behaviour of the structural members can be represented by only two scalar variables: (i) a damage variable at the member ends (d_i and d_j), related to the stiffness degradation due to concrete cracking, with values between 0 and 1; (ii) a plastic rotation at the member ends, a consequence of the permanent strain after the steel bar yielding. Details of the procedure to determine the evolution of damage and plasticity variables can be found in Cipollina and Flórez-Lópéz (1995), Flórez-López (1995), Picón and Flórez-López (2000), Alva (2004), Alva and El Debs (2010) and Alva; El Debs; Kaminski Jr. (2010).

The value of the damage variable at an element end represents the ratio between the slope of the moment-rotation curve in the unloading and the slope of the curve before the beginning of the cracking (Figure 1). In the same way, according to Cipollina and Flórez-Lópéz (1995), the experimental values can be obtained from the results of the load-displacement curves.

As the numerical model shown in this paper focuses the application for reinforced concrete beams in service state, only the damage variable is considered, without variable for plasticity. Therefore, a nonlinear elastic analysis was performed, since the residual rotations were not considered.

For the case of monotonic or cyclic actions without *fatigue* (term used to refer to the loss of flexural stiffness due to the cyclic loading), the limit function that controls the evolution of the damage variable, for each end, is expressed by:

$$g = G - R \tag{1}$$

With:

$$G = \frac{1}{2S}\left(\frac{M}{1-d}\right)^2 \qquad R = G_{cr} - e^{-\gamma(1-d)}.q\frac{ln(1-d)}{(1-d)}$$

$$S = 4EI/L \qquad G_{cr} = \frac{M_r^{\,2}}{2S}$$

Where:

G is the thermodynamic moment;

R represents the cracking strength;

M is the moment at the member end;

d is the value of the damage at the member end;

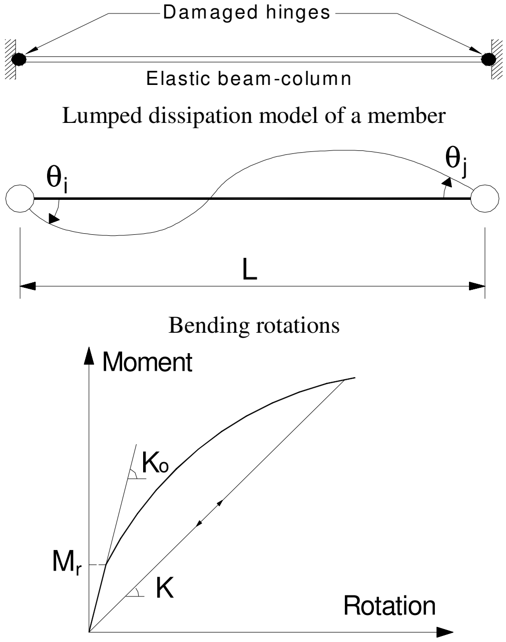
EI is the flexural stiffness of the undamaged member (without cracking);

L is the member length;

Gcr is the value of G when M=Mr (cracking moment) and d=0;

γ is a dimensionless parameter that controls the loss of stiffness after the beginning of cracking. It is a parameter of entry in the model and depends specially of the tensile longitudinal reinforcement ratio (Alva and El Debs, 2010).

Figure 1 – Lumped dissipation model



Lumped dissipation model of a member

Bending rotations

Damage variable from moment-rotation curve

The parameter q is obtained from the condition of $g=0$ and the condition that the moment is maximum when $M=M_u$ (resistant moment). The detailed resolution of this parameter is found in Alva (2004), Alva and El Debs (2010) and Alva; El Debs; Kaminski Jr. (2010). The evolution of the damage variable follows the next conditions:

$$\Delta d = 0 \quad \text{if} \quad g < 0 \quad \text{or} \quad dg < 0$$
$$\Delta d \neq 0 \quad \text{if} \quad g = 0 \quad \text{or} \quad dg = 0$$

Based on such conditions, the increment of the damage variable is calculated by:

$$\Delta d = \frac{\langle dG \rangle}{\frac{\partial R}{\partial d}} \tag{2}$$

Where:

$$\frac{\partial R}{\partial d} = \gamma . e^{-\gamma(1-d)} . q . \left[\frac{ln(1-d)}{(1-d)} \right] + e^{-\gamma(1-d)} . q . \left[\frac{ln(1-d)-1}{(1-d)^2} \right]$$

For the case of cyclic actions, in general, the increase of the damage in the materials occurs not only by the action level, but also by the loading cycles. In this regard, Thomson; Bendito; Flórez-López (1998) proposed a model applicable to the case of reverse cyclic actions, which uses a formulation similar to that presented by Cipollina and Flórez-López (1995) and Flórez-López (1995), but includes an additional parameter in the law of the damage evolution. Thus, the increment of the damage variable can be determined by:

$$\Delta d = \frac{G^z}{R^z \frac{\partial R}{\partial d}} < dG > \quad \text{if} \quad G \geq G_{cr}$$
$$\Delta d = 0 \quad \text{if} \quad G < G_{cr}$$

Where: z is a parameter which controls the increment of damage by fatigue. It can be concluded that the smaller the z values, the larger the effects of the loss of stiffness by fatigue.

Thomson; Bendito; Flórez-López (1998) suggested a constant value of $z=2$ during the analyses. Picón and Flórez-López (2000) proposed that the parameter z is variable along the numerical analysis and dependent of the damage values obtained, according to the polynomial of second degree.

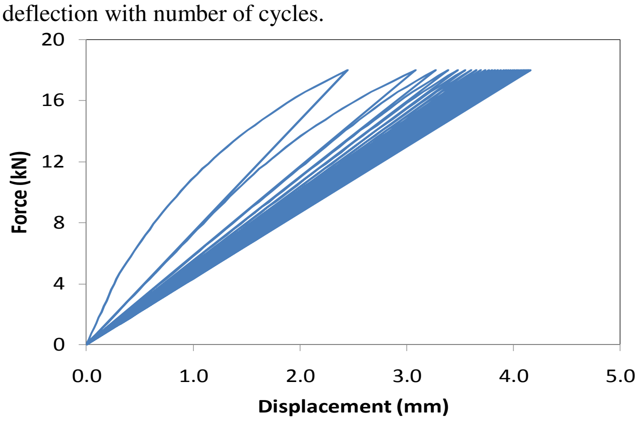
Based on the experimental results in reinforced concrete beams, it is proposed in this paper that the parameter z is determined from the following expression for the case of repeated cyclic actions:

$$z = A.d^4 \tag{3}$$

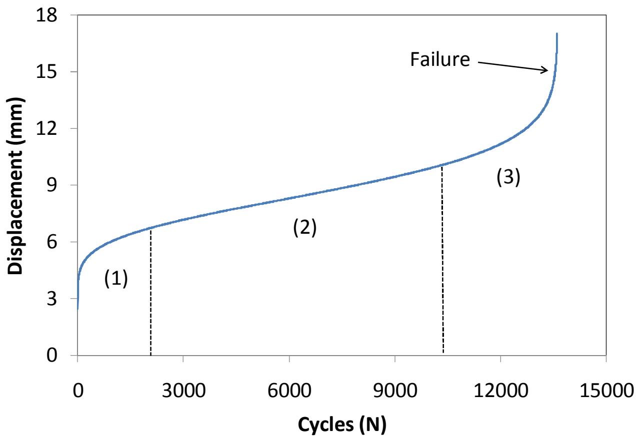
Where: A is a parameter that should be obtained by cyclic experimental tests.

In Figure 2 is illustrated an example of numerical response provided by the elastic nonlinear model of a reinforced concrete beam submitted to vertical loads concentrated at the thirds of the span. The results shown in Figure 2 indicate that the numerical model considers the loss of stiffness due to the cyclic loading. In Figure 2-b, the model provides a curve deflection *versus* number of cycles characterized by three different phases, as experimentally observed in materials subjected to fatigue processes (low-cycle fatigue and high-cycle fatigue).

Figure 2 – Typical results of the proposed numerical model for reinforced concrete beams under repeated loading with constant amplitude: a) Applied load versus deflection; b) Evolution of deflection with number of cycles.



a)



b)

The stiffness matrix of a member of plane frame with six degrees of freedom (Figure 3), considering the damage variables at the ends (d_i e d_j) is shown in Álvares (1999) and can be calculated by:

$$[K_D]=\begin{bmatrix} K_{ii}C_{ii} & K_{ij}C_{ij} \\ K_{ji}C_{ji} & K_{jj}C_{jj} \end{bmatrix} \qquad (4)$$

Where:

KD is the stiffness matrix of the damaged member;

K is the elastic-linear stiffness matrix (undamaged material);

C is the correction matrix, whose values are function of the damage at the ends and expressed by:

$$C_{ii}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \left(1-\frac{3d_i}{4}-\frac{3d_j}{4}+\frac{d_id_j}{2}\right) & \left(1-d_i-\frac{d_j}{2}+\frac{d_id_j}{2}\right) \\ 0 & \left(1-d_i-\frac{d_j}{2}+\frac{d_id_j}{2}\right) & \left(1-d_i-\frac{d_j}{4}+\frac{d_id_j}{4}\right) \end{bmatrix} \qquad (5)$$

$$C_{ij}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \left(1-\frac{3d_i}{4}-\frac{3d_j}{4}+\frac{d_id_j}{2}\right) & \left(1-\frac{d_i}{2}-d_j+\frac{d_id_j}{2}\right) \\ 0 & \left(1-d_i-\frac{d_j}{2}+\frac{d_id_j}{2}\right) & \left(1-d_i-d_j+d_id_j\right) \end{bmatrix} \qquad (6)$$

$$C_{ji}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \left(1-\frac{3d_i}{4}-\frac{3d_j}{4}+\frac{d_id_j}{2}\right) & \left(1-d_i-\frac{d_j}{2}+\frac{d_id_j}{2}\right) \\ 0 & \left(1-\frac{d_i}{2}-d_j+\frac{d_id_j}{2}\right) & \left(1-d_i-d_j+d_id_j\right) \end{bmatrix} \qquad (7)$$

$$C_{jj}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \left(1-\frac{3d_i}{4}-\frac{3d_j}{4}+\frac{d_id_j}{2}\right) & \left(1-\frac{d_i}{2}-d_j+\frac{d_id_j}{2}\right) \end{bmatrix} \qquad (8)$$

The numerical model was implemented in computational routine developed in FORTRAN language applied to the nonlinear analysis of plane frames. For the numerical solution of the nonlinear problem, the incremental-iterative Newton-Raphson method (Standard) was used, where the tangent stiffness matrix is updated for each iteration. The adopted convergence criterion was based on residual forces, with a tolerance of 0.01%.

3 – EXPERIMENTAL RESULTS AND NUMERICAL SIMULATIONS

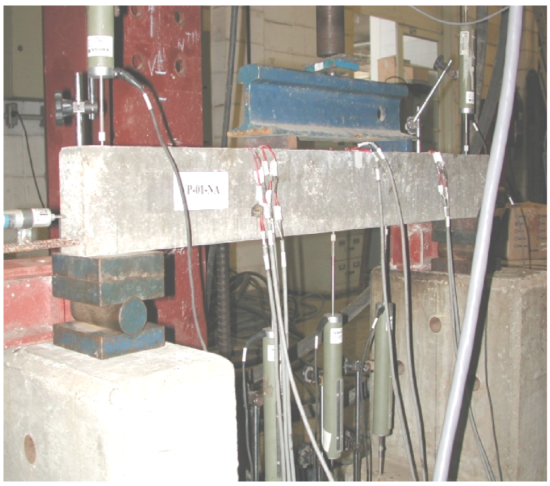
In this section are presented the experimental results obtained by Oliveira Filho (2005) and by Braguim (1995), which are compared to the results of simulations using numerical model described in Section 2.

3.1 Beams tested by Oliveira Filho (2005)

Oliveira Filho (2005) tested a series of simply supported beams with rectangular and T section, submitted to monotonic and repeated cyclic loads – the latter with an intensity simulating the service state. As shown is this figure, displacement transducers (LVDTs) were used to measure the deflections of the beams.

In Figure 4 is showed the device for the application of load, the reaction frame and the instrumentation of the beams.

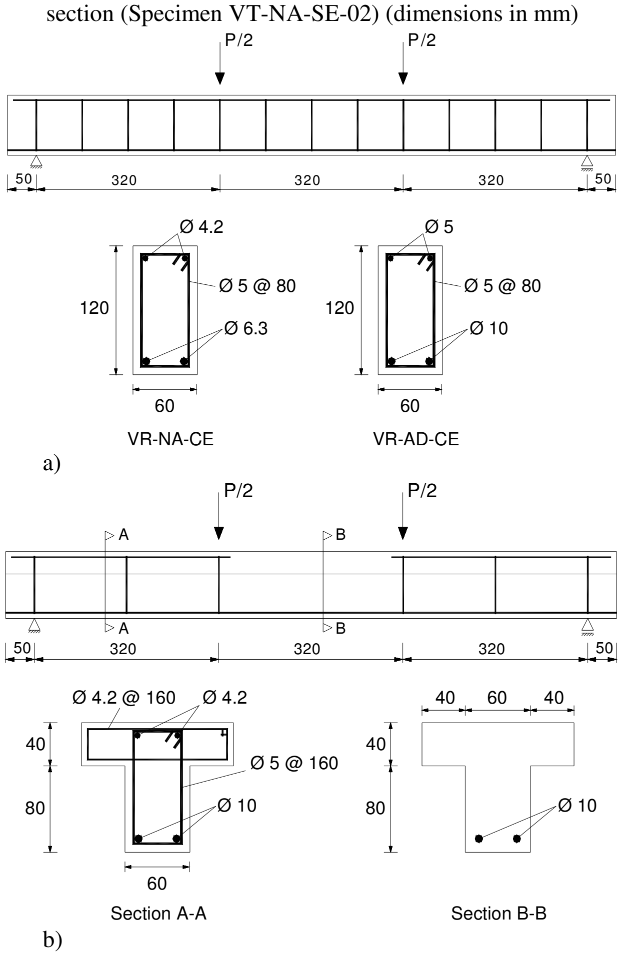
Figure 4 – Experimental test set-up - Oliveira Filho (2005)



For the comparative analyses, five beams tested by Oliveira Filho (2005) were selected: four with rectangular section (denominated VR-NA-CE-01, VR-NA-CE-02, VR-AD-CE-01, VR-AD-CE-02) and one with T section (denominated VT-NA-SE-02). The main difference between the beams of the VR-NA and VR-AD groups is the amount of flexural longitudinal reinforcement.

In Figure 5 are indicated the beam dimensions, the position of the applied loading and the reinforcements of the selected beams with rectangular and T section, respectively.

Figure 5 – General scheme and reinforcement details of beams tested by Oliveira Filho (2005): a) rectangular section; b) with T section (Specimen VT-NA-SE-02) (dimensions in mm)



In Tables 1 and 2 are showed, respectively, the main mechanical properties of concrete and steel of reinforcements of the tested beams, which were obtained from uniaxial compression tests on cylinders specimens (100 mm x 200 mm).

Table 1 – Properties of concrete

f_c (MPa)	E_c (MPa)	f_{ct} (MPa)
40.13	26781	3.20

Note: f_c: concrete compressive strength
E_c: secant modulus of elasticity of concrete
f_{ct}: tensile strength of concrete (split test)

Table 2 – Properties of steel bars

Diameter (mm)	E_s (MPa)	f_y (MPa)	f_u (MPa)
4.2 and 5.0	194330	666	756
6.3	194330	562	648
10.0	194330	515	658

Note: E_s: modulus of elasticity of steel
f_y: yield strength of steel
f_u: ultimate strength of steel

The beams were submitted to repeated cyclic loads, with maximum load (P_{max}) and minimum load (P_{min}) applied for each cycle according to the values indicated in Table 3.

Table 3 – Repeated loads applied to the specimens

Specimen	P_{max} (kN)	P_{min} (kN)
VR-NA-CE-01	18	8
VR-NA-CE-02	20	8
VR-AD-CE-01	45	20
VR-AD-CE-02	47	20
VT-NA-SE-02	44	20

The entry parameters M_r and M_u of the numerical model were determined from the classic calculation of the reinforced sections and using the mechanical properties given in Tables 1 and 2. The cracking moment M_r can be determined from the following expression (according ABNT NBR 6118):

$$M_r = \frac{\alpha . f_{ct} . I_c}{y_t} \tag{9}$$

Where:
α = 1.2 (T section);
fct is the tensile strength of concrete;
Ic is the uncracked moment of inertia of beam section;
yt is the distance from neutral axis to the extreme fiber in tension.

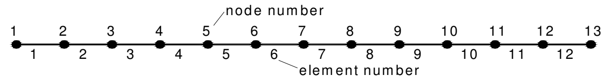
The resistant moment M_u can be determined from the following expression:

$$M_u = A_s . f_y . z \tag{10}$$

Where:
As is the area of tensile reinforcement;
fy is the yield strength of steel reinforcement;
z is the lever arm between tension and compression forces.

The value of the parameter □ was evaluated by comparing the results from the presented model with those predicted by a more precise model, with discretization of the beam in twelve elements (see Figure 6) and the use of moment-curvature diagrams. In this analysis, the expression proposed by Branson (1965) for the flexural stiffness of the sections was used. The vertical displacements of the beam were obtained by double integration of the curvatures. These procedures were also employed for the numerical simulations of Section 3.2.

Figure 6 – Discretization of the beams: evaluation of the parameter □



In Table 4 are indicated the values of entry parameters of numerical model and the number of cycles analyzed in this paper.

Table 4 – Model parameters and number of cycles analyzed

Specimen	M_r (kN•cm)	M_u (kN•cm)	γ	Cycles
VR-NA-CE-01	71	376	3.5	15000
VR-NA-CE-02	71	376	3.5	15000
VR-AD-CE-01	71	902	1.0	15000
VR-AD-CE-02	71	902	1.0	5000
VT-NA-SE-02	98	922	2.5	15000

In Figure 7 are showed the numerical results of the beam VR-NA-CE-02 and indicates the capacity of the numerical model in simulating the evolution of the damage variable along the load cycles.

As the proposed model depends on the experimental calibration of the parameter *A* shown in Equation (3), several numerical simulations were carried out in order to find the values of *A* that better adjust to the experimental results as function of the increase of deflection along cycles. In Figure 8 are indicated the results obtained in these simulations. It is also possible to observe that the model represents well the increase tendency of deflections along the cycles, as long as the parameter *A* is known.

Figure 7 – Damage evolution in specimen VR-NA-CE-02

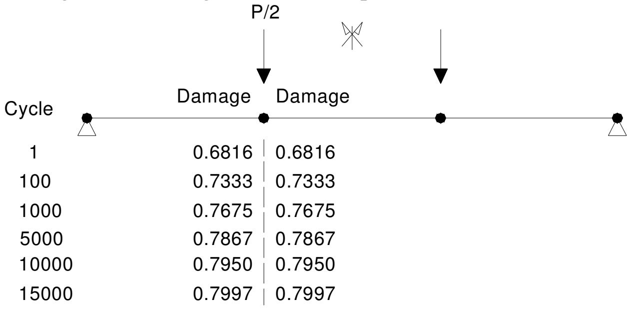
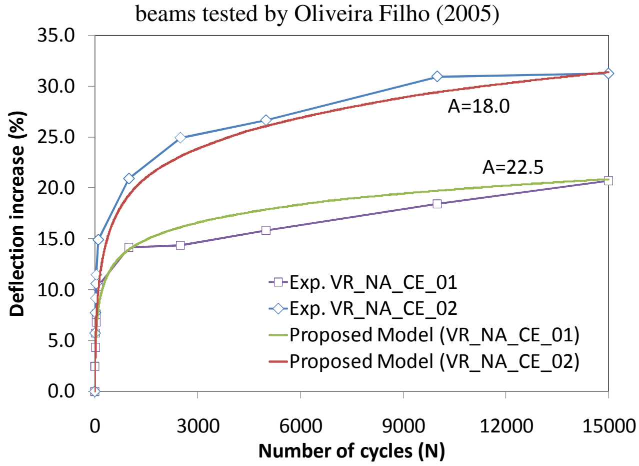
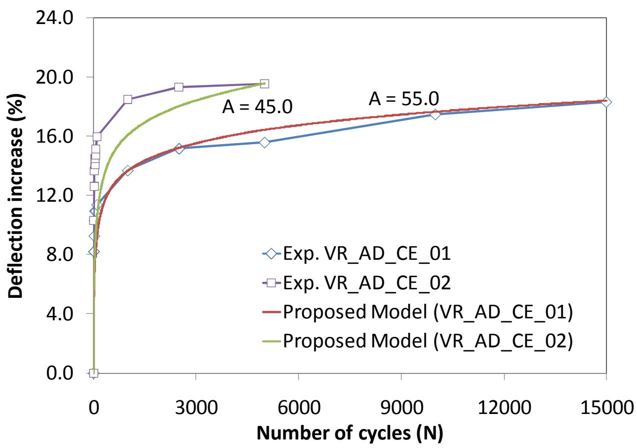
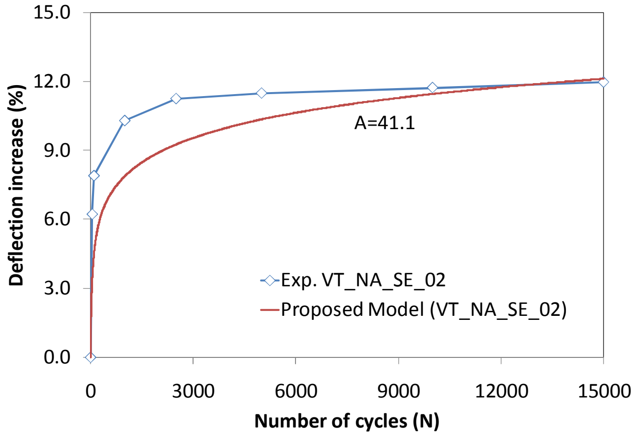


Figure 8 – Experimental results and numerical simulations– beams tested by Oliveira Filho (2005)





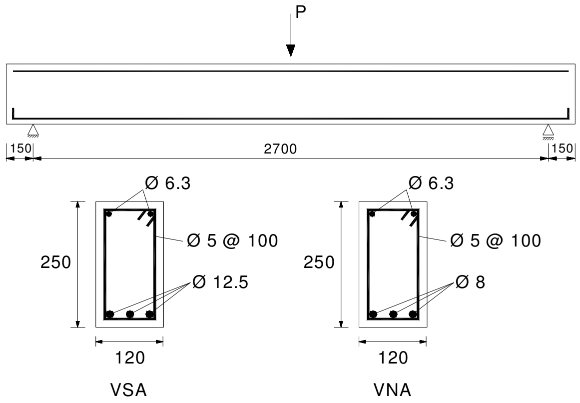


Comparing the beams with rectangular section of VR-AD group with VR-NA group, it was observed that the values of the parameter *A* are higher in the beams of the VR-AD group, which had higher flexural reinforcement ratio. Comparing the results of the beam VT-NA-SE-02 with those beams of VR-NA group, a similar conclusion is obtained regarding the influence of the reinforcement: the values of the parameter *A* increase with the increment of the flexural longitudinal reinforcement ratio. The experimental results of Braguim (1995) shown in the Section 3.2 also confirm this evidence.

3.2 Beams tested by Braguim (1995)

Braguim (1995) accomplished an experimental investigation to evaluate the effects of repeated cyclic loads in deflections of reinforced concrete beams. The geometry, reinforcements and the loading of beams are indicated in Figure 9. Three beams of VNA group and three of VSA group were selected for the numerical analysis. The average compressive strength of concrete of beams was 41.5MPa. For the reinforcements, steel with yield strength of 500 MPa was used.

Figure 9 – General scheme and reinforcement details of beams tested by Braguim (1995) (dimensions in mm)



The beams were submitted to repeated cyclic loads, with maximum load (P_{max}) equal to about two thirds of the theoretical ultimate load and the minimum load (P_{min}) equivalent to $0.6P_{max}$. In Table 5 are indicated the values of the loads used in the numerical analysis. In Table 6 are showed the values of the entry parameters of the numerical model and the number of cycles applied.

Table 5 – Repeated loads applied to the specimens

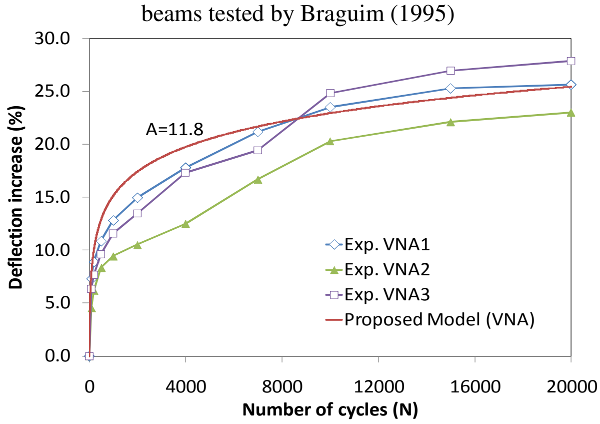
Specimen	P_{max} (kN)	P_{min} (kN)
VNA1,VNA2,VNA3	16.0	9.6
VSA1,VSA2,VSA3	33.5	20.5

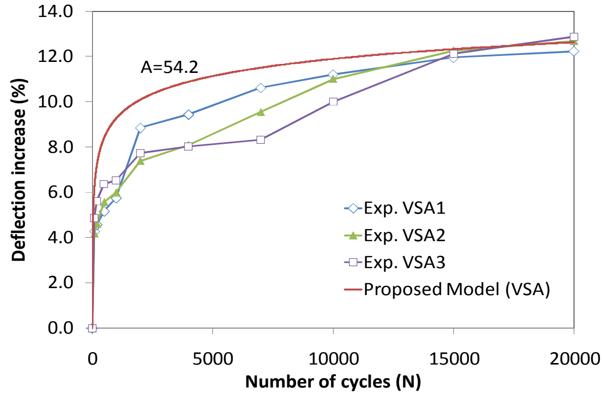
Table 6 – Model parameters and number of cycles analyzed

Specimen	M_r (kN•cm)	M_u (kN•cm)	γ	Cycles
VNA1,VNA2,VNA3	500	1650	3.9	20000
VSA1,VSA2,VSA3	540	3780	1.8	20000

In Figure 10 are indicated the values of *A* that better adjust to the experimental results in function of the increase of the deflection along the cycles.

Figure 10 – Experimental results and numerical simulations– beams tested by Braguim (1995)





According to Figure 10, similar conclusions to those of numerical simulations of the Section 3.1 can be obtained: i) the model satisfactorily represents the tendency of increase of deflections along the cycles; ii) the higher flexural reinforcement ratio, the higher the values of the parameter *A*.

3.3 Calculation of parameter *A* – Proposed Model

The analyzed beams showed differences each other, such as the flexural reinforcement ratio and the amplitude of cyclic loading related to theoretical ultimate load. The experimental results of Sections 3.1 and 3.2 indicate that, in a general way, the increase of tensile longitudinal reinforcement decreases the loss of stiffness due to the cyclic loading. Regarding the numerical model, the increase of this reinforcement implicated in the increase of the values of parameter *A* that better adjust to the experimental results.

It is also known that, in the materials in general, the larger the amplitude of loading, the larger the damage along the cycles. Thus, for a same reinforcement ratio, the larger the amplitude of cyclic loading, the smaller the value of *z* of numerical model and, accordingly, the smaller the value of the parameter *A*.

In order to improve the analysis in function of the variables and obtain an estimate of the order of magnitude of the parameter *A* for the general cases, two hypotheses are made:

a) The value of the parameter *A* is directly proportional to the reinforcement ratio

b) The value of the parameter *A* is inversely proportional to the loading amplitude

Therefore, the *A* value can be evaluated by:

$$A = k.\omega.\frac{P_u}{\Delta P} \tag{11}$$

Where:

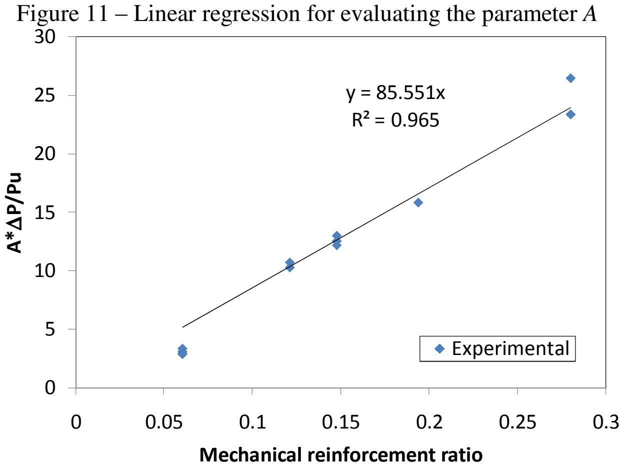
k is a parameter of proportionality, obtained from the values of A that better adjust to the experimental results (linear regression), as shown in Figure 11;

ω is mechanical ratio of the tensile longitudinal reinforcement, given by:

$$\omega = \frac{A_s . f_y}{A_c . f_c} \quad (12)$$

Where: A_s is the area of tensile reinforcement;
A_c is the concrete gross section area;
P_u is the theoretical ultimate monotonic load;
ΔP is the loading amplitude.

Figure 11 – Linear regression for evaluating the parameter *A*



In Table 7 are showed the experimental values used in the linear regression indicated in Figure 10.

The results of Figure 10 suggest that the hypotheses taken at Equation (11) for evaluation of parameter *A* are reasonable and that the *k* values about of *85* represent an initial estimate for beams with similar characteristics of those of this paper. Evidently, more experimental results of beams can contribute for a more accurate evaluation of parameter *A* and confirmation of the efficiency of proposed expression for the calculation of the parameter *z*.

Table 7 – Parameter *A* obtained from experimental results

Specimen	ω	$\Delta P/P_u$	A	$A \bullet \Delta P/P_u$
VR-NA-CE-01	0.1213	0.476	22.5	10.71
VR-NA-CE-02	0.1213	0.571	18.0	10.28
VR-AD-CE-01	0.2800	0.481	55.0	26.46
VR-AD-CE-02	0.2800	0.519	45.0	23.36
VT-NA-SE-02	0.1938	0.385	41.1	15.82
VSA1	0.1478	0.232	56.0	12.99
VSA2	0.1478	0.232	54.0	12.53
VSA3	0.1478	0.232	52.5	12.18
VNA1	0.0606	0.262	11.7	3.07
VNA2	0.0606	0.262	12.8	3.35
VNA3	0.0606	0.262	11.0	2.88

3.4 Expression proposed by CEB-FIP Model Code 1990 (1993)

In Tables 8 and 9 is showed an abstract of the percentile increases of experimental maximum deflections and of those obtained from the proposed numerical model, using Equation (11) for the determination of the parameter *A*, with *k=85.5*. Such tables also present the values originating from the expression proposed by CEB-FIP Model Code 1990 (1993), which provides, as a function of the number of applied cycles, the percentile increase related to the deflection of the first cycle of loading. This expression does not take into account the flexural reinforcement ratio.

In Tables 8 and 9 is show that the proposed numerical model provided a better prediction for the increase of the deflections compared to the expression proposed by CEB-FIP Model Code 1990 (1993), except in the group of beams with lower flexural reinforcement ratio.

Table 8 – Percentile variation of the maximum deflection: experimental results and numerical simulations

Cycle	VR-NA-CE-01		VR-NA-CE-02		VR-AD-CE-01		VR-AD-CE-02		CEB-FIP
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	
1	0	0	0	0	0	0	0	0	0
10	2.452	3.194	5.731	2.894	8.211	4.231	10.29	3.828	2.598
100	10.35	8.510	14.90	9.604	11.37	10.15	15.97	10.05	4.525
1000	14.17	14.69	20.92	18.97	13.68	15.09	18.49	15.62	7.762
2500	14.37	16.96	24.93	22.70	15.16	16.79	19.33	17.54	9.557
5000	15.80	18.77	26.65	25.64	15.58	18.11	19.54	19.04	11.15
10000	18.39	20.69	30.95	28.91	17.47	19.41	-	-	12.96
15000	20.71	21.87	31.23	30.86	18.32	20.25	-	-	14.13

(a): Experimental
(b): Proposed Model – parameter *A* obtained from Equation (11)

Table 9 – Percentile variation of the maximum deflection: experimental results and numerical simulations – Braguim (1995)

Cycle	VNA1 (a)	VNA2 (a)	VNA3 (a)	VNA (b)	VSA1 (a)	VSA2 (a)	VSA3 (a)	VSA (b)	CEB-FIP
1	0	0	0	0	0	0	0	0	0
100	7.30	4.53	6.35	2.94	4.28	4.18	4.85	6.18	4.53
1000	12.81	9.42	11.54	7.04	5.75	5.99	6.52	9.20	7.76
2000	14.95	10.51	13.46	8.37	8.85	7.38	7.73	10.01	9.09
4000	17.79	12.50	17.31	9.70	9.44	8.08	8.03	10.80	10.61
7000	21.17	16.67	19.42	10.79	10.62	9.54	8.33	11.42	12.00
10000	23.49	20.29	24.81	11.48	11.21	11.00	10.00	11.81	12.96
15000	25.27	22.10	26.92	12.28	11.95	12.26	12.12	12.24	14.13
20000	25.62	23.10	27.88	12.84	12.24	12.67	12.88	12.54	15.00

(a): Experimental
(b): Proposed Model – parameter *A* obtained from Equation (11)

FINAL REMARKS AND CONCLUSIONS

In this paper, a nonlinear elastic model applicable to reinforced concrete beams based on *Lumped Damage Mechanics* is proposed. The loss of stiffness of the structural member is considered through variables of damage in regions denominated *hinges*, where it is admitted, as simplification, that all the dissipative effects are concentrated. The numerical model is able to simulate the loss of stiffness not only as a function of the internal forces, but also as a function of the characteristics of the repeated cyclic loading.

For the repeated cyclic loads, it is proposed that the parameter which controls the evolution of damage (parameter z) is obtained by through a new parameter to the numerical model (parameter A). Numerical simulations compared to experimental results of eleven beams indicated that the numerical model has potential for simulations that include repeated cyclic actions. The experimental results also allowed the definition of an estimate for the value of this new parameter of the model.

For the purpose of prediction of maximum deflections, the proposal of this paper for a new estimate of the parameter z provided quite suitable results. The general results show the potential of the proposed model for the simulation of the effects of the repeated cyclic loading in beams

Although this model is simplified, the formulation of the numerical model takes into accounts important variables, such as the flexural reinforcement ratio and the amplitude of the repeated cyclic loading. For the evaluation of the increase of deflections, the model presents advantages on the expression recommended by CEB-FIP Model Code 1990 (1993) and other simplified expressions of Pitonak (1992) and ABNT NBR 6118 (2007), which consider only the number of cycles, independent of the amplitude of loading.

ACKNOWLEDGEMENTS

The authors would like to acknowledge FAPESP for the financial support through scholarships for the PhD's degree of the first and second authors, CAPES for the financial support through scholarships for the PhD's degree of third author and CNPq for research scholarship of the second and fourth authors.

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