# The use of the historical-dialectical method in research on the development of students' computational thinking in the context of Mathematics Education

## O uso do método histórico-dialético na investigação sobre o desenvolvimento do pensamento computacional de estudantes no contexto da Educação Matemática

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**ABSTRACT**

This article proposes and discusses the use of the historical-dialectical method as a theoretical-methodological approach for research in Mathematics Education, especially in the investigation of complex cognitive phenomena such as the development of higher psychological functions and the formation of mental actions and mathematical concepts. Anchored in the Historical-Cultural Theory, the study details how this method, by prioritizing the analysis of processes rather than objects, explanation over mere description and the unveiling of fossilized behaviors and conceptual formations, allows us to unravel the genesis and totality of psychic development. We use as a central illustration the application of this methodological framework in our investigations on the development of computational thinking and the formation of mental actions and the concept of "regular polygon" in 15- and 16-year-old students. We present how the method of double functional stimulation and unit analysis reveal the intricate relationships between cognition, sociocultural environment and technology. The

**RESUMO**

Este artigo propõe e discute o uso do método histórico-dialético como um caminho teórico-metodológico para a pesquisa em Educação Matemática, especialmente na investigação de fenômenos cognitivos complexos como o desenvolvimento das funções psicológicas superiores e a formação das ações mentais e de conceitos matemáticos. Ancorado na teoria Histórico-cultural, o estudo detalha como esse método, ao priorizar a análise de processos e não de objetos, a explicação sobre a mera descrição e o desvelar de comportamentos e formações conceituais fossilizados, permite desvendar a gênese e a totalidade do desenvolvimento psíquico. Utilizamos como ilustração central a aplicação desse arcabouço metodológico em nossas investigações sobre o desenvolvimento do pensamento computacional e a formação de ações mentais e do conceito "polígono regular" em estudantes de 15 e 16 anos. Apresentamos como o método de dupla estimulação funcional e a análise por unidades revelam as intrincadas relações entre cognição, ambiente sociocultural e tecnologia. O artigo visa,

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article thus aims to invite researchers to begin a discussion on the use of this method in research in the field of Mathematics Education.

assim, convidar pesquisadores ao início de uma discussão sobre o uso desse método em pesquisas do campo da Educação Matemática.

**Keywords:** Historical-Cultural Theory. Dual functional stimulation method. Unit of analysis.

**Palavras-chave**: Teoria Histórico-Cultural. Método de dupla estimulação funcional. Unidade de análise.

## 1 Introduction

The development of computational thinking has emerged as a research field of growing relevance, driven by the profound cultural and social transformations brought about by technological advances in society (Silva & Javaroni, 2024a). Understanding how this new way of thinking is constituted and integrated into students' conceptual formation is a challenge that requires robust methodological approaches aligned with the complexity of the human psyche.

This article aims to initiate a discussion on the use of the historical-dialectical method in research that takes cognition and the development of higher psychological functions of individuals as its object of study, within the context of Mathematics Education, presenting a theoretical-methodological framework for future research in this field. This discussion will be fostered by our investigations into the development of computational thinking and the formation of mental actions and mathematical concepts in the teaching and learning process of Mathematics, under the lens of the historical-dialectical method.

Since 2017, we have been dedicated to investigating the development of computational thinking in the context of Mathematics Education, grounded in Historical-Cultural Theory. In our initial investigations, we predominantly relied on qualitative research approaches not only as techniques and procedures of inquiry but also as a worldview and a stance assumed as researchers in the field, establishing an initial dialogue with Vygotskian assumptions regarding the formation of thought.

Our research fields have always been the teaching and learning contexts of Mathematics, involving mostly upper elementary and high school students, but

also teachers from different areas of knowledge who work in public schools in the state of São Paulo. The results of these initial studies can be found in several publications, such as Silva (2018), Javaroni & Silva (2019), Silva, Zampieri & Javaroni (2019), Gadanidis, Javaroni, Souza & Silva (2022), Silva & Javaroni (2022), and Zampieri, Santos, Silva, Souza & Javaroni (2024).

However, in seeking to transcend the merely visible and to deepen the understanding of the intricate relationships between thought and language, as well as the formation of individuals' higher psychological functions—resulting from a cultural transformation driven by technological advances in society—we carried out a four-year investigation using the historical-dialectical method. This process encompassed theoretical appropriation of Historical-Cultural Theory and its method, field research for data production, and subsequent analysis, shaping our worldview and stance as researchers within Mathematics Education. The developments and conclusions of this more in-depth investigation are presented in Silva (2023), Silva & Javaroni (2024b), and other works under review at the time of writing this article.

From this investigative journey, we realized that the use of the historical-dialectical method represents a path still in the making within research in Mathematics Education. Our aim with this article is to actively contribute to the expansion and consolidation of this path, offering not only the fundamental concepts of the method but also exemplifying its practical application and discussing its implications based on our research experience.

With regard to the context of Mathematics Education, we do not intend to limit the application of the historical-dialectical method solely to research on the development of computational thinking. We hope this will be the starting point for a broader discussion about the use of this method within this field of inquiry, contributing so that other researchers may adopt it in their investigations of different objects of study pertinent to Mathematics Education.

To make the application of the principles of the historical-dialectical method more concrete, we refer to our most recent investigation on the development of students' computational thinking from a historical-cultural

perspective and its contribution to the formation of mental actions and of the concept of the regular polygon through the production of geometric thinking (Silva, 2023; Silva & Javaroni, 2024b).

This investigation, in its field research, involved six students aged 15 and 16 in November and December 2022, from the first year of high school at a public school in the municipality of Vargem/SP, Brazil. They were organized into two groups of three for the interaction sessions with the first author of this article. It is important to note that Group 1 was composed of Ana, Paulo, and Otávio, and Group 2 of Carol, Júlia, and Carlos fictitious names assigned to preserve their real identities.

Although the students were present in the same classroom during data collection, interaction with the researcher took place virtually through the *Google Meet* platform, due to the protocols established to prevent COVID-19 contamination. The research was regulated and approved by the Ethics Committee of the Faculty of Sciences (FC) at São Paulo State University "Júlio de Mesquita Filho" (UNESP), under ruling no. 5.101.360, referring to the project entitled "The interrelations of computational thinking with the genesis of the process of mental action formation: a perspective on the internalization of external activity into internal activity in Mathematics classes."

The choice of this educational stage and the organization into groups of three were intentional: adolescence, according to Historical-Cultural Theory, is a fundamental transitional period for the development of conceptual thinking and intellectual autonomy; and the organization into trios (inspired by Vygotsky's "troika" with Luria and Leontiev) fostered dialogical interaction, peer mediation, and the externalization of thought—essential elements for the investigation of higher psychological functions. The complexity of the setting (remote teaching due to the COVID-19 pandemic) and the diversity of students' experiences stemming from their sociocultural environments enriched the application of the method, allowing for the observation of manifestations of thought as a higher psychological function.

To fulfill the purpose of this article, we begin by presenting the theme and the objectives of the discussion. We then delve into the foundations of the

historical-dialectical method and its relevance for understanding the human psyche in its totality. Subsequently, we detail the practical application of the method of functional double stimulation, exemplifying the methodological procedures used in our most recent investigation, mentioned earlier.

The way of conceiving and carrying out data analysis is also discussed, focusing on the units of analysis in the historical-dialectical method and the principles of interpretation for unveiling the cognitive process. Finally, we conclude the discussion by synthesizing the methodological contributions of the article and their implications for research in Mathematics Education. Therefore, the next section is an invitation for the reader to delve into the theoretical foundations that underpin the historical-dialectical method.

## 2 The Historical-Dialectical Method and the Investigation of Cognition

The historical-dialectical method, in the study of the human psyche, was developed by Vygotsky (2014, Vols. I, II, III, IV, V, VI) in the early twentieth century. It emerged as a response to the way scientific psychology at the time treated psychic functions in isolation. This fragmented approach, by breaking down complex psychological processes into separate elements, prevented the explanation of the totality and richness of internal relations, such as those between thought and language. Its assumptions hold that knowledge is not a simple, passive, or inert reflection of reality, but rather a historical and dialectical process, complex and governed by universal laws.

The principles and laws of Marx's dialectics (2013) became fundamental elements for the investigation of historical-cultural psychology, assisting in the understanding and explanation of the constitution and development of the human psyche. One of the most complex and central aspects of the dialectical method lies in its ability to fully encompass the totality of explanations regarding the process of knowledge production.

Vygotsky (2014, Vols. I and III) proposed a methodological approach that, rather than merely describing what is present, seeks to thoroughly understand the process of development. This means that, in order to fully grasp a given stage

or the developmental process itself, it is essential to know the outcome toward which it is directed, its final form, and the way this form is achieved. It is from this clarity about the final objective that the researcher can plan the investigative process, guiding development and analyzing it in depth. This intentional orientation makes it possible to go beyond superficial observation, uncovering the roots that generated the observed development and the transformations that occurred in the human psyche.

In constructing the historical-dialectical method, Vygotsky (2014, Vols. I, II, III) incorporated three essential categories that distinguish it and make it fundamental to Historical-Cultural Theory: labor, the material character of existence, and the historicity of facts. Labor, conceived as purposeful activity, is regarded as the element that humanizes the individual. Through the needs that emerge from reality, human beings establish goals, plan actions to achieve them, and, by transforming nature, simultaneously transform themselves in a continuous process of humanization. The conditions of social life are thus defined by this productive activity.

*The material character of human existence* is the category that elucidates the mode of production and constitution of the humanization process, indicating that the foundations of social relations are established through material production. Finally, the historicity of facts emerges as the essential dimension of the formation of the human psyche, recognizing that the development of society in concrete life entails changes in human consciousness and conduct.

The understanding of the importance of totality in the study of the human psyche requires an active, critical, and conscious role of the researcher, who must create and imagine in order to encompass the integrality and wholeness of the complex process of psychic development. This totality is not a mere sum of parts but the product of the integration of various interconnected totalities, revealing the genesis, consolidation, and development of the phenomena under study.

The historical movement in the constitution of the human being is an essential category for the method, requiring an analysis of the processes of hominization and humanization, which are made possible by the symbolic mediations elaborated by humanity. Understanding the relationship between the

categories of totality/unity and historical movement is fundamental to grasping higher psychological functions. Vygotsky (2014, Vol. II) argues that, throughout historical development, what changes is not so much the functions themselves, but rather the relations and connections among them, generating new groupings that did not exist at earlier levels.

However, identifying this central unit is not a trivial task for the researcher, requiring caution so that it does not become detached from the psychic whole and reduced to an isolated element. To aid in this search, Vygotsky (2014, Vol. II, p. 20, our translation) posed a fundamental question: "What is the unit that does not admit division and that includes properties inherent to linguistic thought?" He argues that, in the context of investigating thought and language, the meaning of the word presents itself as this primordial unit. Meaning is intrinsic both to thought and to language, constituting, in its psychological aspect, a generalization and an act of thinking in its strictest sense. Without meaning, the word is emptied of sense and loses its pertinence to the domain of language.

This perspective implies that the most appropriate method of investigation for the problem at hand is semantic analysis, focusing on the meaningful aspect of language. It is through the study of verbal meaning that direct answers can be obtained about the relationship between thought and speech. By analyzing the development, functioning, and structure of this unit, it becomes possible to elucidate the complex interconnection between thought and language, uncovering the internal dynamics that constitute and shape them.

The relevance of language for the study of thought and cognition is of utmost importance. Vygotsky (2014, Vol. II) proposed a method of investigation that would allow access to the essence of the relationship between thought and language. For him, the central aspect of this problem lies intrinsically in the relation between thought and the word. In studying this internal relation, the author argued that the value of the experience provided by the experiment resides in its ability to reveal manifestations of the subject's own activity as a participant in the investigation, who appropriates the language of their peers in the process of interaction during the study.

In this context, mediation is fundamental for the developing subject to act within their Zone of Proximal Development (ZPD), moving from what they already know to what they are potentially capable of achieving through interaction and the appropriation of new meanings. The focus, therefore, is on the development of word meanings and the creation of complex generalizations that the individual constructs for themselves, although initially without the direct influence of the already elaborated and stable sense of words present in peer interaction.

From this understanding, it becomes evident that the analysis of an experiment should not be limited to observing the individual's generalization of concepts in a cultural vacuum. On the contrary, Vygotsky (2014, Vols. I, II, III, and VI) directs his focus to initial development, since it is in this period that the foundations of the psyche are established and the process of cultural appropriation is most visible and fundamental. It is in continuous interaction with those who possess greater cultural experience that stable and culturally elaborated meanings, present in language, directly influence the individual's development.

The language of the individual's social environment predetermines and channels the development of their generalizations in a specific direction. However, it is essential to note that although more experienced individuals can guide the development of generalization and its final outcome, they cannot transfer their own way of thinking. The developing individual, in their active process, assimilates the already elaborated meanings of words, but the construction of the objects and concrete complexes that compose them is a task that must be carried out by the individual themselves.

This active process of the individual where they arrive at results that may appear similar to those of someone with more experience, yet through intellectual operations that are distinct and peculiar to their own way of thinking is what Vygotsky (2014, Vol. II) designates as the *pseudoconcept*. This phenomenon is an essential point for the researcher. The adoption of the method proposed by Vygotsky (2014, Vols. II and VI) aims precisely at enabling the researcher to recognize the delicate boundary that separates the pseudoconcept from the true concept a distinction that becomes almost imperceptible when analyzed only phenotypically.

The same challenge applies to the formation of mental actions. Galperin (1986) adopts these theoretical-methodological assumptions, considering the necessity of first defining the method for studying the intellectual development of the student in relation to their learning. In this sense, the question of method is intrinsically linked to the general understanding of the emergence of psychic processes; and it is through the study of the internalization of external material actions into psychic actions that the formation of mental actions can be investigated.

The understanding of a higher psychological function, such as an individual's thinking, is established by observing its development from a historical perspective. For Vygotsky (2014, Vol. II), each current stage of development is built upon the previous one. Thus, in order to investigate the development of thought and the formation of mental actions, experimental analysis must necessarily adopt a historical perspective.

It is within this complexity of psychological development historically and culturally shaped that our investigations on the development of students' computational thinking in Mathematics classes are situated, employing the historical-dialectical method precisely because computational thinking is not an isolated phenomenon, but a facet of human development intrinsically linked to the historical-cultural perspective of society, to the evolution of technology, and to the way individuals interact and transform themselves in this world of constant technological change. This method has allowed us to deepen the analysis of the roots of computational thinking development and of the formation of mathematical concepts in students, going beyond superficial manifestations and seeking to understand the genesis of their constitution (Silva, 2023; Silva & Javaroni, 2024b).

## 3 The Method of Functional Double Stimulation in Practice

In this section, we detail the application of Vygotsky's method of functional double stimulation (2014, Vols. I, III, IV, and VI). This method, essential for the investigation of the development of higher psychological processes, proposes the

study of the subject's activity through the interaction of two series of stimuli: one that constitutes the object of the activity and another that functions as a sign, organizing and mediating that very activity. For this approach to be effective and to reflect the complexity of cognitive development, it is fundamental to carefully design and describe the methodological procedures. Thus, the objective of this section is to clarify how this method has been put into practice in our research, demonstrating the creative and adaptive dimension of investigative work in Mathematics Education.

The assumptions of Marx's dialectical materialism (2013) were appropriated by Vygotsky (2014, Vols. I–VI) and objectified in the elaboration of his method of investigation, sometimes referred to as the genetic-experimental method. This method is manifested in practice through the functional approach of double stimulation, which explores the dialectical relationship between human beings and nature through mediation in the process of interaction. Such a method enables the observation of the process of concept formation, experimentally induced. Although the development process does not always faithfully reflect reality, the experiment "provides the key to the correct interpretation and understanding of the real process of concept development" (Vygotsky, 2014, Vol. II, p. 152, our translation).

The essence of the method of functional double stimulation lies in the analysis of the development and functioning of higher psychological processes with the aid of two series of stimuli. One series of stimuli serves as the **object** toward which the subject's activity is directed, while the other series functions as **signs** through which this activity is organized.

In Silva (2023), these two series of stimuli were materialized as follows: (i) the task of investigating the development of computational thinking and the formation of the concept of a regular polygon, which represented the first series of stimuli—the object of study; and (ii) the use of the software Scratch and GeoGebra, which, together with the researcher's guidance and questions, constituted the second series of stimuli, acting as mediating signs to organize students' cognitive activity.

In addition to the structured and intentionally planned mediating signs designed by the research, it is important to recognize that the process of concept formation is dynamic and may be mediated by a range of other unforeseen elements. Spontaneous interactions among students, discoveries made in research sources, and even discussions or materials not directly related to the task can function as unexpected mediating signs. These unintentional mediators, although not part of the formal structure of the study, are equally relevant, as they influence cognitive activity and the development of thought, revealing the complex network of mediations that shape learning and human development within Historical-Cultural Theory. Therefore, when analyzing data from field research, it is essential to remain attentive to this richness of mediations—both the planned ones and those that naturally emerge from the context of the activity.

The value of this method lies in its ability to "discover the role of the word and the characteristics of its functional use in the process of concept formation" (Vygotsky, 2014, Vol. II, p. 128, our translation). For this purpose, the study task may be presented in detail to the research participant from the very beginning and maintained as such until the end, with the means (stimulus-signs or words) being gradually introduced.

For example, in Silva (2023) and in Silva & Javaroni (2024b), it is possible to observe that, at the beginning of the formation of students' mental actions, we sought to identify their prior knowledge about regular polygons and their elements. From the students' initial responses which revealed concepts such as "quadrilateral" or "triangle" still as complexes or pseudoconcepts, that is, an appearance of mastery of the object without an understanding of the process that constitutes it we, as researchers and more experienced adults, intervened. We introduced and reinforced the correct nomenclature of the elements (vertex, side, angle) and the defining characteristics of polygons, using language and the visual resources of the software as signs organizing the students' activity. This gradual and directed intervention not only guided the activity but also allowed for a deeper analysis of the roots of conceptual development.

In our research on the development of computational thinking and students' mathematical concept formation (Silva, 2018; Silva & Javaroni, 2022; Silva, 2023; Silva & Javaroni, 2024b), we have focused on the adolescent. For Vygotsky (2014, Vol. II), adolescence represents a fundamental transitional period in human development, in which the individual moves from a stage of external dependence to the construction of intellectual autonomy and the appropriation of more complex forms of thought. This process is profoundly influenced by the external environment and by continuous interaction with it. The experience and appropriation of the signs and cultural tools of the social environment are determining factors for individual development.

This means, for Historical-Cultural Theory, that the adolescent is capable of developing conceptual thinking in its most elaborated form, transcending "pseudoconcepts" and generalizations based only on concrete perceptions. It is a stage in which inner speech becomes consolidated and the individual acquires the ability to regulate their own psychological behaviors through signs and language, becoming more conscious and reflective about their mental processes. This period is, therefore, of particular interest to us, as it offers a fertile ground for the induction and observation of significant cognitive transformations.

The study tasks proposed during this period need to go beyond everyday conceptual aspects and relate to the student's entry into the cultural, professional, and social world of adults, addressing scientific knowledge. The driving force of this process is external to the adolescent, and the study tasks must generate needs, motives, and objectives that prompt them toward resolution.

In our most recent investigation, carried out over a period of four years, we sought to achieve this by connecting the study of regular polygons with the culture of Computer Science present in students' everyday lives (Silva, 2023; Silva & Javaroni, 2024b). For example, discussions about how games and the optimization of daily processes (such as organizing house cleaning) related to the pillars of computational thinking (decomposition, pattern recognition, and algorithm) served as motivation and a starting point for more formal conceptual

appropriations. The researcher's flexibility in creating new tasks such as reflecting on symmetry and similarity when noticing the persistence of pseudoconcepts in one of the research participants regarding the concept of the regular polygon is an example of how the investigation adapts to induce development and to deepen the analysis of cognitive roots.

It is essential to recognize the profound interrelationship between the research problem, the method, and the investigative techniques, since the definition of the object of study requires delineating the problem in order to investigate the human being in their totality. The method and the problem develop jointly during the investigative process, with the method being not only a premise but also a product of the investigation itself.

In this context, the choice of methodological procedures assumes fundamental importance, since the method is conceived as an instrument that both shapes and is shaped by the investigation itself, requiring the researcher to have clarity regarding the intended endpoint. This is the aspect that distinguishes the historical-dialectical method from other research approaches in Mathematics Education. Here, the researcher must know in advance where they want to arrive, and it is this clarity of the final objective that allows the investigative process to be actively induced. The analysis, therefore, does not merely focus on the result—since it has already been predetermined as the target of development—but rather on the process that leads to it.

For this reason, the researcher's interaction with the participants is not passive; it is intentional and directed toward provoking development. This requires from the investigator a profound knowledge of the science under investigation, the premises of the adopted theoretical-methodological framework, and the specificities of the object of study. In the case of Mathematics Education, it is essential that the researcher master the mathematical concepts involved in order to identify the nuances of cognitive development. An example of such a moment is found in Silva (2023), where we presented in the methodological design the need to address the concepts of symmetry and similarity with students through interaction with them something not initially foreseen by us but

considered in light of the mathematical concepts that emerged from the interaction between researcher and participants.

In Silva (2023), we showed that when the student Paulo brought up the idea that a regular polygon must necessarily be symmetrical, this conception, although relevant, needed to be dialectically confronted and further developed. The researcher's flexibility in creating new tasks to address pseudoconcepts or partial understandings—such as the persistence of the idea of symmetry or the confusion between internal and external angles illustrates the active and induced interaction required to guide development.

The rigor in the selection of methodological procedures, aligned with the historical-dialectical method, is based on the characteristics of the relationship between what is internal and what is external to the individual. Such procedures, according to Grass (2017), must reflect the adopted concept of development, avoiding mechanistic or multifactorial approaches. In research investigating psychic phenomena, the external observation of behavior and self-observation are fundamental methodological procedures. Historical-Cultural Theory acknowledges the necessity of both avenues for psychological knowledge, considering self-observation as a valid method for producing data on self-awareness, which must be corroborated by the observation of external behavior in order to establish their correlation and achieve scientific value.

External observation is carried out through a procedure that enables close, personal contact between the researcher and the phenomenon under investigation. This observation, which must be planned, controlled, and systematic, requires preparation on the part of the researcher, considering that each person has their own history and cultural background that determine their actions. In studies that seek to investigate the development of computational thinking, for example, the stance of participant observer has proven effective, allowing the researcher to interact with the research participants while simultaneously observing them.

Such an approach is consistent with the historical-dialectical method, which requires engagement to create contradictions and analyze the participants'

higher psychological functions. Observations may include a descriptive part—detailing participants, settings, events, and activities and a reflective part, with the researcher's personal notes, ethical dilemmas, and shifts in perspective. Data are typically produced through field notes as well as audio and video recordings.

Self-observation, in turn, is produced through study tasks that lead students to reflect on the key concepts/terms, both at the beginning and during the intervention. Additionally, field research data may be produced through semi-structured interviews, which allow for in-depth dialogue with the participants. These interviews, conducted at strategic moments of the study, aim to gather information on immediate self-observation and after a period of reflection, as well as data on historicity, the characteristics of the sociocultural environment, and the meanings that research participants attribute to the object of study or to the analyzed environment.

This methodological approach recognizes that the development of higher psychological functions, mediated by culture and social interaction, requires the promotion of meaningful experiences. Such experiences, as the basic units of the Social Situation of Development, reveal the needs and motives that drive the individual's attitude, becoming motivating forces for development within the ZPD. In the next section, we will discuss data analysis, which seeks to understand these movements, transitions, and the dialectical relations between what is external to the subject and what is internalized in their psyche.

## 4. Unveiling the Cognitive Process Through Units of Analysis

The stage of data analysis in research that adopts the historical-dialectical method differs from merely descriptive or point-based quantitative approaches, seeking instead the genesis and essence of the phenomena under study. The central objective is to highlight the specific laws of psychological processes, where the research methods are intrinsically determined by the underlying theoretical framework that serves as orientation and guide. Thus, the analysis of results approaches the phenomenon under study as an ongoing process in constant movement, rather than as a static outcome.

According to Historical-Cultural Theory, the externalization of the individual through speech, writing, gestures, or actions is the only way for the researcher to access and understand internal psychic processes (Vygotsky, 2014, Vols. II and III). Data analysis is therefore grounded in the premise that thought and consciousness manifest themselves in external activity and in language, thus becoming accessible to investigation. Drawing conclusions about an individual's development is only possible through what they externalize, and the careful transcription of these interactions is essential to capture such manifestations, allowing for a rigorous analysis of each person's forms of communication.

From this perspective, which privileges external manifestation as the pathway to the internal, the creation of symbolic mediations is a fundamental requirement for the researcher seeking to uncover the genesis of development. It is through deliberate, sign-based intervention that it becomes possible to provoke the internal movements of the psyche. Vygotsky (2014, Vols. I, II, III, and VI) presents three essential principles that guide the analysis of higher psychological functions: analyzing processes rather than objects, explanation versus description, and the problem of fossilized behavior. In the practice of data analysis, these principles are applied to unveil the complexity of the cognitive development of research participants.

The first principle *analyzing processes rather than objects* requires the researcher to go beyond the immediate appearance of development or behavior, seeking its essence and temporal dynamics. What manifests externally has a relation to the essential, but it is not to be confused with it. Thus, the fundamental task of research becomes the reconstruction of each stage in the development of the process, so that the researcher may achieve their research objective.

In the practical context of our investigations into the development of computational thinking and the formation of mental actions and the concept of the regular polygon, discussed in Silva (2023) and Silva & Javaroni (2024b), the research did not limit itself to classifying whether students "knew" or "did not know" the concept of a regular polygon, but rather sought to understand how this

knowledge was being formed, starting from the first manifestations of pseudoconcepts and complexes. In Silva & Javaroni (2024b), we discuss data from this investigation, noting that Otávio's persistence in defining a polygon as a "part of the figure," or Carol's confusion between a right triangle and its form, were not taken as failures but as stages of a process in motion.

Another example that illustrates this principle is the analysis of the concept of "square" by the students Carol, Carlos, and Julia. In Silva (2023), it is possible to observe that although they could visually identify a square and even name it correctly (the object), their initial attempts at construction in the Scratch software revealed that the concept was not yet fully formed in its essence. The difficulty in applying the correct angles or in understanding that all sides should have the same measure indicated a pseudoconcept. The analysis did not stop at the "result" (the incorrect drawing) but rather examined the "process" that led to that error, seeking the roots of thinking still in complexes, without the necessary abstraction.

The second principle *explanation versus description* requires that analysis not be limited to describing observable effects, but instead delve into the underlying dynamic-causal relations of the phenomenon. A phenomenon is truly explained on the basis of its origin, not merely by its external manifestation. In our most recent investigation (Silva, 2023; Silva & Javaroni, 2024b), this principle was evidenced, for example, in the analysis of students' initial conceptions about the sum of the interior angles of polygons.

Otávio initially stated that the sum of the interior angles of every polygon was 360°. A mere description would have simply recorded this answer. However, explanation required investigating why he reached this conclusion, revealing a prior experience with the concept of a full turn or with quadrilaterals, which had "fossilized" this measure as a universal rule. The intervention and analysis sought to uncover this origin, showing how the student was operating with an inadequate generalization, rather than simply producing a "right" or "wrong" answer.

The investigation was not limited to describing students' answers but

sought to explain why they thought in a certain way. For example, when Carol, Carlos, and Julia initially calculated the perimeter of the octagon incorrectly or attributed a measure of 90º to the interior angles of various polygons (Silva, 2023; Silva & Javaroni, 2024b), the analysis delved into the foundations of their mental operations, which were often anchored in prior experiences or in meanings still in formation, rather than in the observable results alone. The confrontation of their ideas with constructions in the GeoGebra and Scratch software was intentional, aimed at uncovering the roots of their errors and advances.

Finally, the third principle *the problem of fossilized behavior* addresses the need to denaturalize psychic processes that have become automatic or stereotyped. These behaviors, although they may appear mechanical, once had a living and complex origin that must be recovered. In analysis, the researcher is often led to seek the alteration of the automatic and mechanized character of higher forms of behavior so that they may return to their origin through the experiment.

A notable example of this principle in our investigation was the persistence of the idea of symmetry as the sole criterion for regular polygons, defended by Paulo (Silva, 2023; Silva & Javaroni, 2024b). Although symmetry is indeed a property of regular polygons, Paulo's belief represented a fossilized generalization that did not encompass all the essential attributes of the concept. The research, by inducing the confrontation of this idea with examples such as the rectangle or the trapezoid, as well as with the construction of polygons in GeoGebra and Scratch, sought to "dismantle" this fossilization, offering the student the opportunity for a more complex and genuine understanding of the concept.

The problem of fossilized behavior was addressed when investigating, for example, Otávio's and Paulo's belief that the sum of the interior angles of any regular polygon was 360º (Silva, 2023; Silva & Javaroni, 2024b). This "fossilization" of a concept learned in a specific context (such as the square) needed to be denaturalized through new study tasks and interactions that revealed the origin of this rule and the necessity of extending it to other polygons, thereby fostering the correct generalization of attributes.

The application of these principles is inherent to any investigation that adopts the historical-dialectical method, and its realization occurs through the design of procedures and the constant interaction of the researcher with the object of study. There is no predetermined number of interactions; it is up to the researcher to determine this quantity depending on their object of study, starting point, and endpoint. It is through this path that it becomes possible to unveil the complex network of relations that constitute the development of higher psychological functions, grasping the genesis and evolution of cognitive phenomena in their totality and historicity, such as the development of computational thinking from a historical-cultural perspective.

According to Bernardes (2017), the phylogenetic and ontogenetic planes encompass the ontological, gnoseological, epistemological, and logical dimensions of the human psyche, requiring historicity, concreteness, and materiality in analysis. In this sense, analysis by units is proposed to capture the totality of the phenomenon, since it does not fragment the psyche, ensuring that the properties inherent to the whole are preserved in the investigation of its parts.

Unlike elemental analysis, which decomposes the phenomenon, analysis by units seeks to identify components that, even in their simplest form, carry and reflect the complexity of the whole. This approach is common in Historical-Cultural Theory because it recognizes that psychological development is a dynamic and interconnected system, where the relationships between fragments are just as important as the fragments themselves. Thus, it allows for the investigation of movements, transitions, and the reciprocal constitution between the singular and the collective, which are essential for understanding qualitative changes in development.

In the context of Historical-Cultural Theory, the selection of units of analysis is not based on a predefined or rigid list but rather emerges from the very nature of the phenomenon under study and the research problem. Vygotsky (2014, Vol. II) emphasizes that the researcher must find a unit that cannot be further divided without losing the fundamental properties of the whole of which it is a part. For him, this unit must be a "synthesis of the diverse," capturing the essence of complex relationships.

For example, in investigating the relationship between thought and language, Vygotsky identified *the meaning of the word* as the fundamental unit of analysis, since it is inseparable from both processes and reflects their dialectical unity. Therefore, these are not ready-made units required by the theory for all research, but rather a methodological demand for the researcher to construct or identify, based on their object of study and theoretical framework, the most appropriate unit to unveil the totality of the phenomenon in question.

The relevance of a unit such as *experience* or action and communication—in a specific study, such as the one we have conducted on the development of students' computational thinking (Silva, 2023; Silva & Javaroni, 2024b), is determined by its ability to reflect the dialectical interconnection between the internal and the external in the individual's psychological development.

In the first unit, experience, we focused on understanding students' reality from their own perspective, seeking to identify their awareness regarding the school and social context, the use of technologies, and the incorporation of cultural aspects of Computer Science into their actions. This allowed us to uncover the needs and motivations driving their engagement, such as Paulo's perception of technology as a pathway to professional success, shaped by his family and school (Silva, 2023; Silva & Javaroni, 2024b). This unit was fundamental for understanding the "why" of students' interactions and appropriations.

In the second unit, *action and communication*, we focused on the formation of students' mental actions and on the communication of these actions during the development of the study tasks. In this unit, we observed how students externalized their thinking through speech, gestures, and interaction with the software, and how this externalization made it possible to analyze the genesis of the process of mathematical concept formation and the development of computational thinking. For example, discussions about symmetry as a criterion for regular polygons, or the evolving understanding of interior and exterior angles, were analyzed through verbal interactions and software productions, revealing the transition between stages of mental action formation (Silva, 2023; Silva & Javaroni, 2024b).

This in-depth understanding of analysis by units is of inestimable value to Historical-Cultural Theory, as it lies at the core of accessing the most intricate dynamics of the human psyche. Far from being a mere methodological tool, the unit of analysis represents the point of intersection where historical, cultural, and social forces manifest themselves in individual development. It is within this dialectical frame that the researcher encounters the totality in miniature, grasping relations that would otherwise remain hidden in a fragmented analysis. In our investigations, the selection and organization of these units have thus been a direct reflection of the commitment to the pursuit of this processual totality, rather than merely a static outcome regarding the development of students' computational thinking. This approach is, in fact, the most robust pathway for investigating how computational thinking is constituted and developed in the context of Mathematics Education, revealing the mediations that drive or hinder the formation of mathematical concepts.

## 5. Final Considerations: Implications for Research in Mathematics Education

This article has followed a methodological path aimed at providing a solid framework for investigating cognitive development, particularly computational thinking and the formation of mathematical concepts, through the lens of Historical-Cultural Theory. This discussion proposes and explores avenues for future research in this field, especially in the context of Mathematics Education. The central focus was to demonstrate how the historical-dialectical method makes it possible to transcend the mere observation of results, directing attention instead to the complex process of transformation of the human psyche in its historicity and concreteness.

The discussion contextualized the historical-dialectical method as a fundamental response to the limitations of scientific psychology in grasping the totality of psychic functions and their internal relations, such as those between thought and language. It was argued that knowledge, from this perspective, is not a passive reflection of reality but a historical and dialectical process,

intrinsically connected to human labor, the material character of existence, and the historicity of facts. The discussion served to illustrate the imperative of analyzing units that, unlike isolated elements, preserve the essential properties of the whole. In this sense, the meaning of the word emerged as a central unit for understanding the interrelationship between language and thought.

The exploration of the practical application of the method of functional double stimulation revealed how this approach materializes in investigations of the development of higher psychological functions, such as computational thinking. It was shown that the researcher's clarity regarding the final objective the targeted endpoint of development is essential for inducing the research process and for analyzing its roots. The researcher's intentional interaction, through the use of stimuli and signs (such as programming environments and dynamic geometry software), was presented as the mechanism for provoking cognitive transformations in the participants. The flexibility in creating study tasks, adapting to the participants' manifestations, illustrated the investigative dynamic that seeks to confront pseudoconcepts and fossilized generalizations, fostering conceptual understanding.

The data analysis, detailed in the previous section, reinforced the imperative of Vygotskian principles: to analyze processes rather than objects, to prioritize explanation over description, and to address the problem of fossilized behavior. Analysis by units proves to be an essential pathway for capturing the totality of the phenomenon. It enables the unveiling of students' needs and motivations, as well as the externalization of their thinking during interaction and concept formation. This approach highlights that the ability to analyze how thought and consciousness manifest themselves in external activity and language is central to understanding psychological development.

In light of the foregoing, this article argues that investigating the cognitive development of the individual from a historical-cultural perspective requires a complex and multifaceted approach. The historical-dialectical method, with its principles and forms of analysis, offers a robust pathway for uncovering the dynamics of this development.

In this sense, the historical-dialectical method in Mathematics Education research is not limited to offering a theoretical lens; it imposes a way of conducting investigation that is inherently developmental and transformative. By requiring the researcher to know the potential endpoint of development and to act inductively to provoke this process, the method distinguishes itself by its capacity to go beyond *what* the student learns, to investigate *how* and *why* they learn, in depth.

This is particularly significant for Mathematics Education, where the formation of concepts often remains restricted to superficial results. The adoption of this method, therefore, implies a commitment to understanding the genesis of concepts, the formation of mental actions, and the complex mediations (social, cultural, and technological) that impact the development of students' mathematical thinking. Research in Mathematics Education that is concerned with the development of higher psychological functions, by embracing this perspective, gains tools to unveil the roots of cognitive phenomena, thereby proposing more effective and contextually grounded pedagogical interventions.

In Silva (2023) and Silva & Javaroni (2024b), the historical-dialectical method allowed us a unique deepening of the investigation into the development of students' geometric thinking. By focusing on the planned formation of mental actions and mathematical concepts, it was possible to follow the cognitive transitions from initial understandings based on lived experiences and pseudoconcepts to the appropriation of more complex abstractions. For example, the analysis of students' initial definitions of "polygon" and "regular polygon" revealed a stage of "chain complex" or "pseudoconcept," evidenced by the association of terms based on isolated characteristics or on the mere appearance of the figure.

Another three students Carol, Júlia, and Carlos initially defined a hexagon, for example, as "a thing with 6 or 7 sides," or a square as a "geometric shape with 4 sides in the form of a cube." The intentional intervention, mediated by the Scratch and GeoGebra software, and the induced confrontation of these

ideas through discussions on symmetry and the properties of different polygons, were essential for the students to begin abstracting the essential attributes and internalizing the concept.

Otávio's ability, by the end, to deduce the measure of the exterior angle and its relation to the construction in the Scratch software, or Carol's, Júlia's, and Carlos's perception of the rule of dividing 360º by the number of sides to determine rotation in programming, demonstrate the formation of mental actions and the appropriation of concepts that transcended the mere memorization of formulas. The analysis thus revealed not only the mathematical concepts being formed but also the very process of internalization and appropriation of cultural instruments (such as language and computational tools) that mediated this development.

Given the depth and richness that the historical-dialectical method brings to the understanding of cognitive phenomena, the methodological discussions and practical examples presented here aim to serve as a beacon for the research community. It is essential to emphasize that, in proposing the adoption of the historical-dialectical method in Mathematics Education research, we do not seek to invalidate or delegitimize other methodological approaches. On the contrary, our intention is to contribute to the richness and diversity of research in the field, offering a robust and in-depth pathway for understanding complex phenomena such as the development of computational thinking.

By emphasizing the need to go beyond the visible, to understand the genesis of cognitive processes, and to consider the totality of the relations that shape them, this article presents itself as an invitation. It offers a theoretical-methodological framework that nurtures the understanding of the complex interrelations among cognition, the sociocultural environment, and technology in the field of Mathematics Education.

# El uso del método histórico-dialéctico en el contexto de la Educación Matemática: una posible discusión

**RESUMEN**

Este artículo propone y discute el uso del método histórico-dialéctico como enfoque teórico-metodológico para la investigación en Educación Matemática, especialmente en la investigación de fenómenos cognitivos complejos como el desarrollo de las funciones psicológicas superiores y la formación de acciones mentales y conceptos matemáticos. Basado en la Teoría Histórico-Cultural, el estudio detalla cómo este método, al priorizar el análisis de procesos sobre los objetos, la explicación sobre la mera descripción y el descubrimiento de comportamientos fosilizados y formaciones conceptuales, permite desentrañar la génesis y la totalidad del desarrollo psíquico. Utilizamos como ilustración central la aplicación de este marco metodológico en nuestras investigaciones sobre el desarrollo del pensamiento computacional y la formación de acciones mentales, así como el concepto de "polígono regular" en estudiantes de 15 y 16 años. Presentamos cómo el método de doble estimulación funcional y el análisis de unidades revelan las intrincadas relaciones entre la cognición, el entorno sociocultural y la tecnología. Por lo tanto, el artículo pretende invitar a los investigadores a iniciar un debate sobre el uso de este método en la investigación en el campo de la Educación Matemática.

**Palabras clave**: Teoría Histórico-Cultural. Método de estimulación funcional dual. Unidad de análisis.

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