Obutchénie
Revista de Didática e Psicologia Pedagógica

DOI: https://doi.org/10.14393/OBv8.e2024-46

On psychological issues in math problems

Questões psicológicas em problemas de matemática

Yakov I. Abramson[1]

ABSTRACT
Some psychological data based on results returning back from solving math problems which include psychological component such as self-restriction is sometimes irrelevant due to different reasons. Sometimes the possible error in great deal is caused by ill posed question and sometimes, especially regarding preschoolers and elementary school students it is mixed with their insufficient proficiency in operating with math concepts pertaining to the problem. In the article are also offered samples of fixing these issues by introducing set of questions preceding statement of the question.

Keywords: Math problems. Geometrical concepts.

RESUMO
Alguns dados psicológicos baseados em resultados obtidos na resolução de problemas matemáticos que incluem componentes psicológicos, como a autorestrição, às vezes são irrelevantes devido a diferentes razões. Ocasionalmente, o possível erro em grande parte é causado por questões mal formuladas e às vezes, principalmente em pré-escolares e alunos do ensino fundamental, se confunde com sua insuficiente proficiência em operar com conceitos matemáticos pertinentes ao problema. No artigo são oferecidos exemplos de solução desses problemas, introduzindo um conjunto de perguntas que precedem a formulação da questão.

Palavras-chave: Problemas matemáticos. Conceitos geométricos.

It was somewhen around 1960, when I was 6 years old and did not go to school yet and P. Ya. Gal'perin in cooperation with his doctoral student L.S. Georgiev, Bulgaria had been publishing works on education of basic math for preschoolers. I was convenient person for experiments in this field. I barely knew how to count and how to read (back then it was normal for boys of my age) but was considered as "apt" by my grandparents may be due to my soft spot for different sort of jokes. So, sometimes Piotr Yakovlevich and his colleagues would call me

[1] Yakov Abramson graduated in 1976 from Moscow State University, College of Mathematics and Mechanics, with a major in mathematics. From 1976 through 1989, worked as a programmer and IT engineer in computer centers. In 1989, began a teaching career, initially in after-school math circles, and later in both private and public schools. Developed his own core curriculum and pedagogical approach. Published three manuals and over 20 articles on education, all but one in Russian. Currently developing an online math course corresponding to the one published in the manuals, also in Russian. Has been teaching young educators to use P. Ya. Gal'perin's method of step-by-step formation of mental activities. Two websites provide more information, both in Russian: https://yabramson.ru/ and matematika-abramson.com Orcid: https://orcid.org/0009-0007-1204-2960. E-mail: yakovabramson@mail.ru.

DOI: https://doi.org/10.14393/OBv8.e2024-46

and offered me a question. They were testing the role of psychological factor in decision making in the process of solving logical problems. As a sample, I remember the following episode.

I was given two identical boxes and told that one of them is made of some sort of stone and the other – from pressed cotton. And then followed the question: "Which one is heavier?"

I hesitated a bit and irresolutely pointed to the stony box. My grandfather was delighted as well as his guest. He explained me that they were of the same weight, that stone was pumice stone and cotton was pressed to the degree when it becomes so stiff that can compete even with some kind of stone. The idea behind the experiment was that the prior knowledge about materials given before this question predetermines the following answer.

I remember the feeling I've got then. It was not the surprise from the discovery that in some case cotton can be as heavy as some kind of stone (and even heavier). It was feeling that there was something wrong in the question itself. To me it looked like sort of cheating. Of course, I was too young to understand why I had had that feeling and I was too playful boy to keep it in memory for long.

There were too many other funny things to do. But many years afterwards, when I graduated from math and physics high school and Moscow state university with major in math, it came down again and I realized what was the reason. The reason was simple: the question itself as it was posed, did not admit that answer! The question was posed as dichotomy with choice of two options and the right answer was out of these two options!

Another sample of the same sort was the question, what is heavier, 1kg of iron piece or 1kg of the wooden one? In this case the question put up this way is dubious by itself: do we mean their mass or their weight? If it were the weight, then where (it depends!) – in the air on the ground of earth, in the water or in vacuum? In first two cases the right answer for two pieces of equal mass would be different from expected as physics teaches us: the one which has greater volume will weigh less. But suppose we neglect all these effects, the weight of the air, etc. and admit that both things weigh the same.

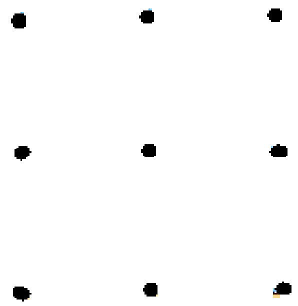
DOI: https://doi.org/10.14393/OBv8.e2024-46

But then again, this answer is out of offered choices! And that is exactly the main reason of wrong answers. The right statement of problem would be:

1. Is the 1kg of iron heavier than 1kg of wood?
2. Is the 1kg of iron lighter than 1kg of wood?
3. Are they both of the same weight?

Imagine the test where the correct answer would be neither of those, offered in the list! Less obvious are math problems where real difficulties young children face with are connected not only with the ones which adults do (and suppose kids have the same), but in great deal also with the fact they did not get firm understanding of concepts used in the problem. Let's turn to the one well-known problem, which also was among those I was tried on.

Figure 01 – Crossing the dots.



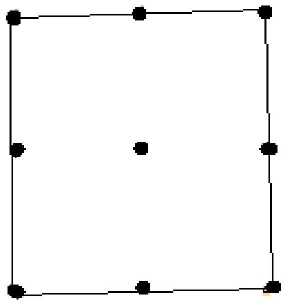
Source: the author

Here we see 9 points arranged in a square 3x3 with 3 rows and 3 columns, each has 3 points in it, lying on same straight lines.

The problem invites to cross them all with 4 straight lines not taking your pen from paper. In other words, with 4 consecutive moves, each move begins in the point where previous had ended. The idea is that in this case the solver collides with self-inflict psychological restriction: to search the solution within the box (Figure 2), like the attempt on Figure 3 which often occurs in pedagogical practice.

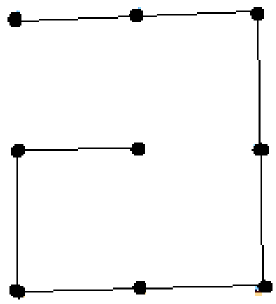
Figure 02 – The first try. The central dot is left out.

DOI: https://doi.org/10.14393/OBv8.e2024-46



Source: the author

Figure 03 – The second try. Dots are crossed, but with 5 lines.

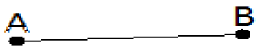


Source: the author

The conclusion about psychological restriction is correct, it does take place in this case. But when we offer this problem to children who did not get systematic education of Geometry, this outcome may be confused with another problem.

In standard math courses taught in Russia, pupils start to study Geometry only in 7th grade and one barely can expect any proficiency before 8th grade what corresponds to teenagers of 14 years old. So, when this problem is placed before 7-12 years old, one can keep in mind that the very concept of "straight line" is not yet developed. The stumbling block is the idea of line as stretched indefinitely in both directions. If pupil is asked to draw straight line through points A and B, he almost certainly will draw it the way we can see in Figure 4.

Figure 04 – Straight infinite line presented as segment



Source: the author

It will not stretch farther over A or B. So, in this case that very restriction

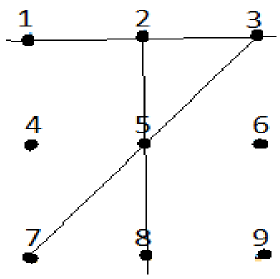
Obutchénie
Revista de Didática e Psicologia Pedagógica
DOI: https://doi.org/10.14393/OBv8.e2024-46

has not only psychological origin but comes from another reason – they are not being familiar with basic geometric concepts like infinite lines on a plane. To clear this problem from this obstacle and make it fair for younger kids, we have to preface its offer with some carefully selected work. As a sample of it, I would recommend several exercises, designed as I always prefer it, in form of questions.

Again, the formal definition of (straight) line as stretched infinitely in both directions, pupils may formally know, but, nevertheless, this idea of infinity lies too far from their everyday experience and too abstract to grasp and to apply it in real problems. We need to talk a little bit about parallel lines on a plane before returning back to problem. Here is the sample of corresponding sequence of questions, pertaining to the problem.

1. Draw 2 straight lines through some of points numbered from 1 to 9 which will be parallel to the line 1-2-3.
2. Draw 2 straight lines through some of points numbered from 1 to 9 which will be parallel to the line 2-5-8.
3. Draw 2 straight lines through some of points numbered from 1 to 9 which will be parallel to the line 3-5-7. (Figure 5).

Figure 05 – draw lines parallel to the 3 given lines through numbered points

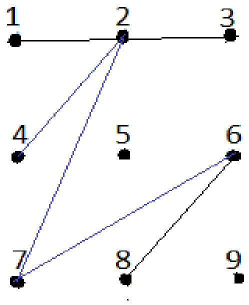


Source: the author

The answer on the third question differs from first two. First of all, this time these lines come just through 2, not 3 of numbered points like before. But then comes another set of questions:

Figure 06 – Which lines are not parallel and where do they meet?

DOI: https://doi.org/10.14393/OBv8.e2024-46



Source: the author

1. Are lines 1-2-3 and 6-8 also parallel?
2. We don't see them crossing, do we?
3. If, nevertheless, they aren't parallel, they should cross somewhere, yes?
4. If so, then where?

This time they will have to take pencil and ruler and to extend both lines till their meeting point. And afterwards there will be appropriate to add some other questions to fix this understanding:

5. Are lines 7-6 and 1-2-3 parallel? If not, where is their meeting point?
6. Are lines 2-7 and 6-8 parallel? Do they cross anywhere?
7. How about lines 6-7 and 2-4? Find the place where they cross each other.
8. Find out other pairs of lines which are not parallel and their meeting points.

Now, after this preparation, one can present the initial problem and it becomes "fair game", being cleared of insufficient elaboration of corresponding geometrical concepts as to parallel lines and lines at all. Now all difficulty will, indeed, be connected to just psychological issue mentioned above and not mixed with other issues.

Another well-known sample of the sort is the next problem: "make 4 equal triangles out of 6 sticks of the same length". One assumes that here psychological obstacle lies in restriction oneself to 2-dimensional solutions instead of considering building it in 3D. Actually, it is accepted in math to include whole setting into problem formulation, in this case it would be exactly dimension of the space in which this problem is supposed to be solved.

For example, when we ask students to solve some equations, we define explicitly or implicitly what set of numbers we bear in mind. Say, if pupils know

DOI: https://doi.org/10.14393/OBv8.e2024-46

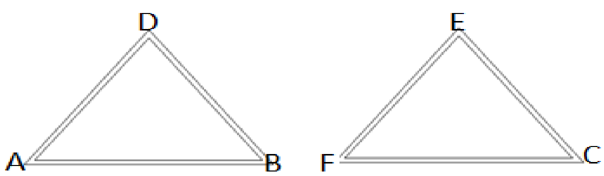
just natural numbers (or the problem itself by its content implies answer as natural number), then equation $x^2 – 4 = 0$ has just one root. If they acquainted with whole numbers but not with rational ones (or the problem itself by its content implies answer as an integer), then equation $3x - 1 = 0$ has no roots and equation $2x^2 - 3x + 1 = 0$ has just one root. If they acquainted with rational numbers but not with irrational ones, then, for example equation $x^3 + x^2 - 2x – 2 = 0$ has just 1 root, though in set R of real numbers it has 3 roots. Similarly, when students in public school not enrolled in AP math class deal with, say, equation $x^3 – 1 = 0$, they absolutely accurately come with just one and single root $x = 1$, because they are limited to just set R of real numbers, though in set C of complex numbers this equation has 3 different roots.

The same situation takes place in higher math. When, for example, we look for solution of some variation problem we define from the beginning in what class of functions do we look it for. In other words, definition of settings in which solution is sought for is part of problem situation, and so it has to be included in the very task conditions. But on the other hand, in this very case any direct reminder about 3D is a prompt to solution.

How on the one hand not to mislead students by encouraging them to look for the solution on plane and not to directly forward them to look in 3D. Very important that we mention sticks, not segments, for example. We should give them those sticks, moreover we have to provide them with modelling clay to fasten them together to make rigid triangles.

From 6 sticks they can form 2 triangles:

Figure 07 – 2 triangles made of 6 sticks.

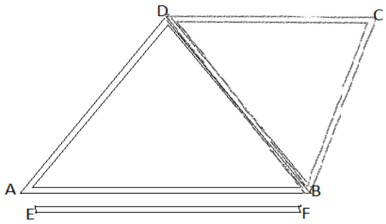


Source: the author

Obutchénie
Revista de Didática e Psicologia Pedagógica
DOI: https://doi.org/10.14393/OBv8.e2024-46

What can one do? After all, we need two more! We can attach one to another and spare one stick since one stick will serve a side to both triangles:

Figure 08 – eliminating 1 stick



Source: the author

I would recommend glue them together by common side BD. What should we do with the only one left stick EF? We would get two more triangles ADC and ABC if it could connect A and C. But it cannot because it is too short. One cannot stretch it but may be one can make A and C get closer to each other? We now can leave students at this point to find solution by themselves. At least we can say then that we did our best to play fair game in this case. Now only that very psychological self-restriction can play its role.

Discussing this and the previous problem we closely approached to the question of role and place of so called "insight" in problem solving. I remember how much did this question interest my grandfather. But that would be quite different topic and would require special and thorough investigation.

Closing this brief review of math problems which interested P. Gal'perin in connection with role of psychology in the process of solution, I would mention also another kind of geometrical constructions which also were subject of his keen

DOI: https://doi.org/10.14393/OBv8.e2024-46

interest. They were constructions which cause optical illusions, which in their turn, had also psychological side.

Cuestiones psicológicas en problemas de matemáticas

RESUMEN
Algunos datos psicológicos basados en resultados obtenidos en la resolución de problemas matemáticos que incluyen componentes psicológicos, como la auto-restricción, a veces son irrelevantes por diferentes razones. Ocasionalmente, el posible error se debe en gran parte a preguntas mal formuladas y, en algunos casos, especialmente en preescolares y alumnos de educación primaria, se confunde con su falta de competencia en el manejo de conceptos matemáticos pertinentes al problema. En el artículo también se ofrecen ejemplos de cómo solucionar estos problemas introduciendo un conjunto de preguntas que preceden a la formulación del problema.
Palabras clave: Problemas matemáticos. Conceptos geométricos.

Recommended bibliographies[2]

ABRAMSON, Y. On the content and methods of teaching math in elementary school for gat students in the Proceedings of the PME and Yandex Russian conference (p. 243). *Technology and Psychology for Mathematics Education* [Electronic Resource]. ed. by A. Shvarts; Nat. Research Univ. Higher School of Economics. Moscow: HSE Publishing House, 2019.

GAL'PERIN, P. Ya. Learning and intellectual development (from 5 to 8 years). *Proceedings of the 19th International Congress of Psychology*. London, British psychological society, 1971, p. 201.

GAL'PERIN, P. Ya. *Intellectual capabilities among older preschool children*: *on the problem of training and mental development*. The University of Chicago Press, 1982, v.6, p. 526-46.

GAL'PERIN, P. Ya. *Psychology*: *Subject and method*. Selected psychological works. (In Russian). Moscow, 2023.

OBUKHOVA, L. F. *Fundamentals of general psychology. Theory of P. Ya. Gal'perin and modelling experiment* (In Russian). Moscow, 2022.

[2] See also on website https://education.yandex.ru/pme/en/

DOI: https://doi.org/10.14393/OBv8.e2024-46

Recebido em 20 de agosto de 2024
Aprovado em 20 de setembro de 2024