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A logical-historical movement of geometry and formative actions in a Mathematics Club[1]

Um movimento lógico-histórico de geometria e ações formativas em um Clube de Matemática

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ABSTRACT

The text aims to share results of the study of the logical-historical movement on geometry and highlight contributions to teacher education processes from a collective perspective. This training is carried out at the Mathematics Club of the Federal Institute of Espírito Santo and the investigation of this process is developed in Ph.D. research, that has as theoretical-methodological foundation the cultural-historical theory, activity theory, and teaching guiding activity. Based on these assumptions, teacher education is understood as a social, dialogic process marked by the needs and demands of pedagogical work, synthesizing debates and actions that involve teaching activity, the teacher's main activity. It stands out that the formative movement in the Mathematics Club is based on the need for participants to appropriate historically accumulated knowledge, when studying a logical-historical movement of concepts, in this specific case, of geometry. The study highlights geometry as a human production,

RESUMO

O texto tem por objetivo compartilhar resultados de um estudo do movimento lógico-histórico sobre geometria e evidenciar contribuições para os processos formativos de professores em uma perspectiva coletiva. A referida formação é realizada no Clube de Matemática do Instituto Federal do Espírito Santo e a investigação desse processo é desenvolvida em uma pesquisa de doutorado, que tem como fundamentação teórico-metodológica a teoria histórico-cultural, a teoria da atividade e a atividade orientadora de ensino. Com base nesses pressupostos, a formação docente é compreendida como um processo social, dialógico e marcado pelas necessidades e demandas do trabalho pedagógico, sintetizando debates e ações que envolvam a atividade de ensino, atividade principal do professor. Destaca-se que o movimento formativo no Clube de Matemática parte da necessidade de os participantes se apropriarem de conhecimentos historicamente acumulados, ao estudarem um movimento lógico-histórico dos conceitos, neste caso específico, de geometria. O estudo evidencia a

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and, to approach its development from a logical-historical perspective, it becomes necessary to discuss the relationship between man and nature, attracting attention to the work processes and production of instruments, because the processes of perception of shape, location, and displacement in space change with the relationship between man and work. Furthermore, it is highlighted that theoretical study in the collective training process makes it possible to understand new ways of organizing teaching, approaching geometry through the perception of shapes, identification of figures, communication of their properties and different representations, and intentionally establishing an organization of actions in the formative process.

Keywords: Teacher education; Geometry; Shape; Cultural-historical theory.

geometria como produção humana, e, para abordar seu desenvolvimento numa perspectiva lógico-histórica, faz-se necessário discutir sobre a relação homem x natureza, chamando a atenção para os processos de trabalho e produção de instrumentos, pois os processos de percepção da forma, de localização e de deslocamento no espaço vão modificando-se com a relação homem x trabalho. Além disso, destaca-se que o estudo teórico no processo formativo coletivo possibilita compreender novas formas de organizar o ensino, abordando a geometria mediante a percepção das formas, identificação de figuras, comunicação de suas propriedades, diferentes representações e estabelecendo uma organização de ações no processo formativo de maneira intencional.

Palavras-chave: Formação docente; Geometria; Forma; Teoria histórico-cultural.

1. Introduction

The last decades have witnessed an increase in the number of publications on the education of teachers who teach mathematics and geometry teaching. Regarding teacher initial or continuing teacher education, we notice different concepts and formats that have been modified and expanded inside and outside Brazil. Considering this field of possibilities, we advocate education based on the connections of assumptions from the cultural-historical theory, the activity theory, and the teaching guiding activity. Dialogue with these references allows us to understand teacher education as a social process linked to the humanization of subjects and closely linked to the concept of work. In this aspect, formative processes prioritize debates and actions that involve teaching, considered the teacher's main activity.

In initial education, prospective teachers must experience formative proposals that create possibilities for developing learning and teaching activities, which is why we agree with Borowsky (2020, p. 510) when he states that “to become a teacher, the subject appropriates the cultural-historical movements that permeated the constitution of teaching work.”

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From 2020 onwards, these assumptions began to guide different formative moments developed in the context of a Mathematics Club (CluMat), located at the Federal Institute of Espírito Santo (Ifes), Vitória campus, and integrated into the Research Group on Mathematics Pedagogical Practices (Grupo de Pesquisa em Práticas Pedagógicas de Matemática – Grupem). CluMat has different researchers, mathematics undergraduates, master's and doctoral students, and basic education teachers as participants. Formative actions are structured through the following organization: collective meetings for theoretical studies of mathematical theory and concepts, especially geometry; meetings to plan teaching proposals, which unfold into the development of actions with students and take place in basic education schools or teacher education groups; and, moments of reflection and evaluation, with collective summaries on the processes developed and re-elaboration of actions.

Regarding teaching geometry, we first need to understand that we assume that teaching mathematics is a social process in which the individual negotiates, integrates, and understands different values (BISHOP, 1999). In this way, it becomes necessary to think about a way of organizing teaching that emphasizes that concepts result from a process in which, to satisfy the needs of the group, the human being, through work, improved techniques and developed new instruments. This teaching process needs to lead to the possibility of acquisition of theoretical knowledge of content, in our case geometry, which involves essential elements of historically consolidated meanings. To help the teacher understand this knowledge, we argue that teacher education processes should consider "mobilizing activities for the identification, generalization, and internalization or appropriation of a generalized way of organizing teaching" (PANOSSIAN; MORETTI; SOUZA, 2017, p. 131).

From this perspective, we think about teaching geometry in formative processes with teachers, as we agree with Silva and Lopes (2019) when they argue that teaching geometry from the perspective of cultural-historical theory contributes by addressing the logical-historical process and discussing the appropriation of concepts in the pedagogical activity dimension. The teacher or prospective teacher needs the opportunity to study geometry from this perspective to advance and improve their repertoire of knowledge.

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Considering these premises, we carried out study actions on a logical-historical movement in geometry and organized formative moments with participants from the Ifes Mathematics Club. These actions are part of a broader doctoral research and supported the preparation of this text, which aims to share the results of a study of the logical-historical movement on geometry and highlight contributions to teacher education processes from a collective perspective.

We understand that conceptual appropriation is made possible by intertwining the logical and historical aspects of the object of knowledge because, according to Kopnin (1978), the logical reflects not only the history of the object but also the history of its knowledge. The author explains that the historical concerns the stages of emergence and development of the object and is primary in relation to the logical, which is the reflection of the historical in theoretical form, reproducing the essence of the object and the history of its development in the system of abstractions.

Based on this understanding, by carrying out a theoretical discussion on geometric concepts, we seek to highlight an approach to historical data beyond facts and events that draws attention to collective needs, as well as the movement in search of meeting those needs, producing instruments, and resorting to work. The logical aspect makes it possible to show how to carry out this process, and by bringing in historical elements, we seek "the essence of the concept, its historical need, the elements and relationships that constitute them" (MORETTI, 2014, p. 37).

Regarding teacher education proposals, Moretti (2014) explains the importance of addressing "both the conceptual learning of the mathematical object in a logical-historical movement and the learning of the elements of teaching organization" (p. 40), stressing the relevance of a broad teacher education process. To know the essence of the concept, we must understand it in a broader system, in relationships with other concepts in a system of concepts, going beyond its superficial and apparent manifestations. This is a thinking level known as theoretical thinking (DAVIDOV, 1982; KOPNIN, 1978). The essence of the concept is contained in theoretical thinking, which is structured through theoretical

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knowledge. In the teaching education developed with CluMat Ifes participants, we organized different moments, one of which is called theoretical study –when the participant approaches aspects contained in the concept, they are studying beyond its immediate appearance.

To present our investigation, we organized this text by initially punctuating the formative process of the Mathematics Club and underscoring the modes of action that involve the theoretical study. Next, we pointed out part of the theoretical studies that led us to the presentation of a logical-historical movement of geometric concepts, and, to conclude, we brought our considerations to this discussion.

2. Teacher education in a Mathematics Club

The proposal for teacher education through Mathematics Club actions began in 1999 from initiatives at the Faculty of Education, University of São Paulo (FEUSP). Currently, this proposition of the Mathematics Club as a collective space for teacher education that intertwines the triad of teaching, research, and extension has been adopted by members of the Study and Research Group on Pedagogical Activity (Grupo de Estudos e Pesquisas sobre Atividade Pedagógica – GEPAPe) in different higher education institutions. It involves mainly mathematics and pedagogy undergraduates, basic education teachers, and students and professors in postgraduate programs. We participate in this process with the GEPAPe RS-GO-ES-RN nucleus, which involves other mathematics clubs with the following higher education institutions: Federal University of Santa Maria (UFSM), Federal University of Goiás (UFG), Federal Institute of Espírito Santo (Ifes) and Federal University of Rio Grande do Norte (UFRN).

Since its implementation at FEUSP, the Mathematics Club proposal has undergone reconfigurations, and, in each institution, it takes on specific characteristics of its organization and development, observing local objective conditions. However, all clubs share a way of conceiving education based on the intentional collective organization of pedagogical work. Lopes (2009) explains that

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the Mathematics Club allows students to interact with teachers and students working in basic education, thus contributing to a closer relationship between university and school. This interaction is established in a teacher education process for everyone involved: undergraduates in initial education; master's students, doctoral students, basic education teachers, and researchers/educators in continuing education. Concerning undergraduate students, we note that:

> The initial education of teachers in this space is understood as a process that allows the participant to understand their role as teachers based on their own experience; that is, an education that goes beyond the mere understanding of the content taught and the methodologies used and seeks to respond to the unique desires and insecurities of each teacher in initial education (MARCO; LOPES; ARAUJO, 2023, p. 807).

This education process occurs intentionally, organized by researchers/educators who are also educated in the process, as they study, assist, and organize situations that lead to reflection on the teaching activity. Thus, the actions developed in CluMat are intentionally organized to meet the demands of each group. In our case, the central focus of the debates is geometry and its teaching possibilities.

Regarding the development of actions in the Mathematics Club, Lopes (2009) highlights three fundamental moments: planning, interaction with students, and assessment. During this process, we argue that "the organization of teaching presupposes that teachers acquire theoretical knowledge and social meanings about their work –which is teaching" (BOROWSKY, 2020, p. 512).

Therefore, we highlight the role of human interactions because, when we assume activity theory as a theoretical basis, we agree with Leontiev (2021), who explains that, through interactions, the subjects involved share and acquire meanings. Specifically, the Mathematics Club is configured as a space for collective education. Petrovsky's (1984) propositions about the collective show that it is not a simple gathering of people, but the constitution of

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> [...] a group of united people who act together (together and not side by side), where significant social tasks are resolved and where interrelationships are characterized by responsible dependence (the success or failure of one is a condition for everyone's success or failure) (PETROVSKI, 1984, p. 60).

In that space, teachers in initial and continuing education collectively carry out actions that follow the principles of the teaching guidance activity (Moura, Araújo; Serrão, 2019). Regarding these principles, Gladcheff (2015) points out some important elements in a teacher's work: the teacher studies the process of development of human activity (study of the logical-historical movement of the concept), formulates a problem that triggers learning, discusses the triggering problem and teaching actions collectively, and reports what was experienced in the classroom, analyzing the actions collectively.

Thus, considering the elements presented by Gladcheff (2015) and the general organizational moments of the Mathematics Club (Lopes, 2009), we organized four moments that guided the development of the formative process we are carrying out at CluMat Ifes, summarized in Chart 1.

Chart 1 – Moments of development of teacher education at CluMat Ifes

CluMat Ifes moments	**Overview of the development of each moment**
Theoretical study	Studies on theoretical assumptions and a logical-historical movement of geometric concepts. Then, we also developed actions organized by researchers/educators so that participants could experience teaching proposals related to theoretical studies.
Planning and elaboration of teaching actions	Participants plan and develop teaching actions collectively to be implemented with students or in teacher education workshops. Those actions involve studying the historical synthesis of the concept to create the triggering problem, which must address the human needs for the emergence of the concept being discussed.
Interaction with students or teachers in workshops	Implementing the prepared actions is an opportunity for pre-service teachers to come into contact with the school reality, in the case of interactions with students, enabling a constant relationship between theory and practice.
Assessment and reflection	It permeates the entire process, but two specific moments are paramount: the end of the interaction with students or teachers (carried out by the subgroup that developed the actions) and the CluMat meeting with the whole group.

Source: Authors' summary

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We understand that these four moments are not isolated, and some overlap. Assessment helps participants reveal a particular understanding of the educational act, as sharing these initial understandings "contributes to a new level of understanding of the educational activity being configured" (MOURA, 2021, p. 15). These moments are articulated collectively as follows:

> The collective elaboration of teaching activities will allow the theory to be used appropriately, as it is at the service of a collective project seeking to improve learning conditions. Actions and operations are part of seeking to implement the activity. Moreover, the definition of instruments will enhance the educational action carried out consciously. The teaching strategy is the operation of the activity, and the material is the tool that will allow the optimization of actions. Hence the need for intentionality: choosing instruments appropriately for the necessary action (MOURA, 2000, p. 42).

This organization permeates the actions of CluMat, which involve the formative process. Furthermore, Esteves and Souza (2022, p. 9) highlight that implementing the teaching proposal involves choosing teaching strategies and defining instruments, which requires the teacher to acquire theoretical knowledge. This is another important moment, as theoretical knowledge is acquired given the theoretical study of concepts. Based on the historical synthesis of the concept based on a study of the logical-historical movement, participants understand the fundamental need to elaborate and develop the concept and its relationships. This process allows for the development of conditions for teaching actions based on a triggering problem that carries the identified need. This moment provides participants with the conditions to begin a process of appropriating historically elaborated cultural elements and teaching as a promoter of the humanization process (SILVA; CEDRO, 2015).

For this process to occur, educators must intentionally organize actions to guide theoretical studies with task proposals in which participants experience this process based on triggering problems. The educators carry out a broader study, aiming to substantiate this organization of education in CluMat.

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This article presents this way of organizing the theoretical study of geometry as part of a doctoral research work carried out in 2023. The research aims to understand the phenomenon of mathematics teachers' initial education, especially teacher learning. This investigation is linked to two universal projects, one from the Espírito Santo Research and Innovation Support Foundation (Fundação de Amparo à Pesquisa e Inovação do Espírito Santo – Fapes) and the other from the National Council for Scientific and Technological Development (Conselho Nacional de Desenvolvimento Científico e Tecnológico – CNPq) and uses a theoretical-empirical methodological proposal. The participants are eight mathematics undergraduates and eight postgraduate students, seven of whom also work as basic education teachers.

We understand that, through the teaching activity, participants can begin a movement to appropriate knowledge historically produced about geometry and, from this, identify some conceptual links to organize this content teaching. An important part of the teaching activity is the theoretical study of the concept so that it is possible to organize teaching intentionally and boost the movement of theoretical appropriation.

Since the study of the logical-historical movement of the concept is fundamental for the intentional organization of the formative processes that we develop, in this study, we present an excerpt from a logical-historical movement of geometry executed during the theoretical study. We show how moments of study and development of actions were organized in the CluMat Ifes to draw attention to the needs and nexuses of geometry. This movement stresses possibilities for acquiring historically produced concepts and broad and collective education.

3. The development of geometry from work relationships and its teaching process

The first movements toward the construction of geometric knowledge are marked by the observation of the surrounding space, which is expanded to a second movement, resulting from humans' active relationship with nature, in which work

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plays a fundamental role. Through it, humans modify nature to meet their needs (VIGOTSKI, 2010) and, in this process, intensify the production of instruments that incorporate use values and work processes (MARX, 2017), in addition to being the characterization of a social object in which historically elaborated work operations are incorporated (LEONTIEV, 1978).

By transforming external nature, humans transform themselves, and those continuous processes cause revolutions in the economy and society. In this regard, Childe (1971) indicates some economic revolutions, the first of which resulted from humans' beginning to control the supply of food, agriculture, and animals. This is the Neolithic revolution, when, according to Childe (1971), humans stopped collecting food and started producing it.

The relationship between humans and nature changed when humans gained control over the food supply. Organization into groups and observation of the environment were fundamental for survival. However, since then, it has not just been about perceiving the surrounding space but also substantially expanding possibilities for action on nature, stated Ghidini and Mormul (2020). This new relationship brought changes to the field of geometry, which started to have practical importance. It has expanded the idea of perception of shapes and their possible measurements, of location and displacement in space, as, with the advent of sedentary life, it was necessary to divide the land and organize space.

Food production did not immediately replace food collection and should not be confused with the adoption of a sedentary life since, as Almeida (2017) explains, for a certain period, a mixed economy between collecting-hunting and agriculture-taming was essential to ensure multiple means of survival.

In some regions, the most suitable land for cultivation was in the plains close to rivers, which, when filled and overflowed, fertilized the banks, and could be prepared for new plantations. This activity made it possible to cultivate the same area as long as it was flooded between two plantings. Thus, “agriculture may have required a more detailed observation of the seasons, a more exact division of time, of the year” (CHILDE, 1971, p. 109). Once again,

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observation of nature and the surrounding space was important, even when more advanced instruments and techniques were already available.

Starting from an approach focused on the senses and the observation of nature and the objects that compose it (sensory geometry), the development of geometry goes through different stages. The next one assumes a practical character, arising from practical human needs resulting from the production of instruments, the organization of constructions, and the development of agriculture (practical geometry). Practical geometry does not appear to replace sensorial geometry; rather, it expands possibilities due to the advancement of techniques and work processes.

Gerdes (2012) explains that geometry emerged as a science when it became deductive in Greek antiquity. The Greeks were the first to realize that mathematics could help understand nature, and geometry could also be applied to reveal, not just to describe, as Moura *et al*. (2018) explain, based on Mlodinow (2004). Thus, geometry advanced towards a formal approach (formal geometry) with the systematization of geometric ideas beyond practical needs. An important step was the Greeks' elaboration of the notions of point, line, and plane, called Euclidean geometry. Furthermore, the in-depth study of shapes, until specific geometric figures with their own properties, relationships, and representations was decisive for the development of formal geometry.

Childe (1971) highlights two other revolutions. One occurred when small farming villages were transformed into populous cities, fueled by secondary industries and foreign trade: the urban revolution. The emergence of cities contributed to a new configuration for geometry, where measurement procedures predominated. As public life acquired great importance, this also occurred with the rational and geometric organization of space, according to Roque (2012).

The other revolution (CHILDE, 1971) was called the knowledge revolution and was marked by counting processes and measurement standards. Roque (2012) explains that there was a gradual increase in awareness that most geometric knowledge should serve everything from the most practical to the most abstract applications.

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However, only after the late Middle Ages and early Renaissance did geometric knowledge reach what today we understand as scientific geometry, and this is mainly due to a qualitatively new character attributed to geometry, which occurred from the 17th century onwards with the organization of other non-Euclidean geometries and the inclusion of new subjects: analytical, differential, descriptive, and projective geometries, and works on the foundations of geometry (RÍBNIKOV, 1987).

Moura *et al.* (2018) systematized this classification into sensorial, practical, formal, and scientific geometry. We understand that it does not represent a linear development of geometric knowledge but a process that presents the relationship between human beings and nature at different moments that expand and complement each other. As historical and human knowledge, geometry underwent different modes until it was systematized.

Therefore, we understand that to approach the development of geometry from a logical-historical perspective, we need to discuss the relationship between humans and nature, highlighting the work processes and the production of instruments and their different moments. We often hear that the beginning of geometry occurred with the mediation and division of lands close to the flooded banks of the Nile River. Roque (2012) explains that this hypothesis may have its origins in Herodotus's writings. In these, he mentions the floods of the Nile and narrates that King Sesóstris sent people to inspect and measure the land to assign taxes. These actions could have been the beginning of geometry that would later migrate to Greece. Boyer (1996) also points out the practical needs of construction and land demarcation as one of the possible origins of geometry.

However, before the need to divide the land for planting and cultivation, there was the need to shelter, move around in space, look for food, and supplement the body's deficiencies with the instruments produced. The survival and maintenance needs of the species gave us evidence that elements of geometry were already being identified and used before practical needs such as land measurements. Although some practical actions were taken, such as

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the production of instruments, the basis was the apprehension of space, which resulted in other human actions essential for the development of geometry.

Therefore, based on this line of understanding of the development of geometry, we present a logical-historical discussion, highlighting the historical moment of the hunter-gatherer (CHILDE, 1971) with the proposal to identify some human actions and the geometric knowledge resulting from them.

In the hunter-gatherer phase, observation of nature and time control was vital for the species' survival, as were cooperation and collective action (Childe, 1971). The senses played a fundamental role, and geometry in this period was a way of representing and understanding space, which was made up of objects with varied shapes and relationships, with humans as the responsible for perceiving them (MOURA *et al*., 2018).

Regarding space perception, Almeida (2017) draws attention to two processes: space navigation (displacement and location) and the perception of geometric shapes (visualization of objects). Space perception is paramount in human interaction with the outside world, and Petrovsky (1980) points out some special cases, such as perceptions of objects' shape, size, depth, and distance. Still regarding perception, Leontiev (2021, p. 81, emphasis added) highlights that "those who perceive are not the sense organs, but people *with the help* of the sense organs."

Through shape perception, instruments could be created to supplement the body's deficiencies, helping humans obtain food and shelter. Transforming elements of nature into "extensions" of their own bodies, humans produced extracorporeal equipment and, thus, generated "human nature within nature and, simultaneously, artificial geometry within natural geometry" (LIMA; MOISÉS, 2002, p. 9).

Besides tool production, controlling the fire was a step towards emancipating human beings from the servitude of their environment (CHILDE, 1971), beginning a process of burning and using heat. Artistic activity also gained prominence, mainly on the part of hunters who used their different representations to carve figures in stone or ivory, model animals in clay, decorate weapons with

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representations and formal designs, create bas-reliefs on the walls of cave rocks, and engrave or paint scenes on ceilings (CHILDE, 1971).

Thus, part of the hunters' artistic action was capturing natural forms or experiences and representations through cave drawings or clay modeling. Drawing and sculpture –initially copies of models– become provocateurs of thought, affection, and ideas when they evolve beyond simple imitations (LIMA; MOISÉS, 2002). Drawing is not just the act of representing but the possibility of imagining shapes, figures, and patterns, ideas that, according to Bishop (1999), developed with drawing and representation.

Representations initially emerged in people's practical actions and were used in verbal communication. Thanks to this, elaborating representations of objects to plan practical actions became an occupation of some people who did not directly participate in the production of the material of that object (DAVIDOV, 1988).

Regarding the development of the act of drawing, Childe (1971) explains that the oldest representations were just sketches of profiles, traced with a finger in clay or sketched in charcoal, with no attempt at perspective or details. Only later did the artist or draftsman learn to shade to suggest depth and even achieve some perspective. Childe (1971) also narrates the difficulty of representing in two dimensions, despite being familiar with drawings without depth, and highlights the need to produce depth and distance on paper.

Almeida (2017) explains that, in the most primitive stages of history, there was no clear separation between art and mathematics. In this way, geometric art was characterized by lines, points, curves, spirals, and other geometric elements, which were produced only for aesthetic reasons without the intention of a mathematical movement. Thus, the search for aesthetic-mathematical pleasure indicated that transcendence surpassed survival (ALMEIDA, 2017).

According to Almeida (2017), the first symbols and the beginning of symbolic thinking seem to be the most important cognitive acquisition for mathematics during this hunter-gatherer period. Chart 2 lists some human needs and actions during the period.

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Chart 2 – Human needs and actions in the period of the hunter-gatherers

Need	Human action	What did it take?	Main geometric idea
Assist in obtaining food and shelter	Instrument creation	Shape perception.	**SHAPE**
Communication (instigate thought)	Artistic activity (representations in drawings and sculptures)	Natural shape capture	

Source: Authors' synthesis

When we discuss the development of some human needs during the hunter-gatherers' period, we identify shape as the main related geometric idea. Understanding the space and the shapes contained within it was an important step in starting the process of producing instruments that, over time, came closer to regular shapes (GERDES, 2012). Later, with the development of thought, man took over natural forms, examining the "beauty, plasticity, transformation, particularities, generalities of this figurative movement of nature" (LIMA; MOISÉS, 1998, p. 3).

Throughout the theoretical study of these concepts, we carried out a task involving the participants' perception of shapes. To do this, we provided a closed box with only side openings so that they could insert their arms where an object would be (Figure 1). This object was a representation of an unconventional geometric solid made of wood. We call this task "Which solid is this?"[5] and its performance consisted of a group participant placing their hands inside the box, exploring the representation of the solid only tactilely, and describing their perceptions to the other participants who, without any visual or tactile contact, must first represent this solid with modeling play dough (figure 2) and, subsequently, on the plane, based on the drawing.

[5]The task was based on an action entitled "Solid hidden in the box," carried out by Rovetta and Silva (2015) in a master's degree research. It had different versions and was used in actions and research guided by prof. Sandra Fraga within the scope of CluMat and Grupem (Santos; Silva, 2023).

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Figure 1 – Task box "Which solid is this?"

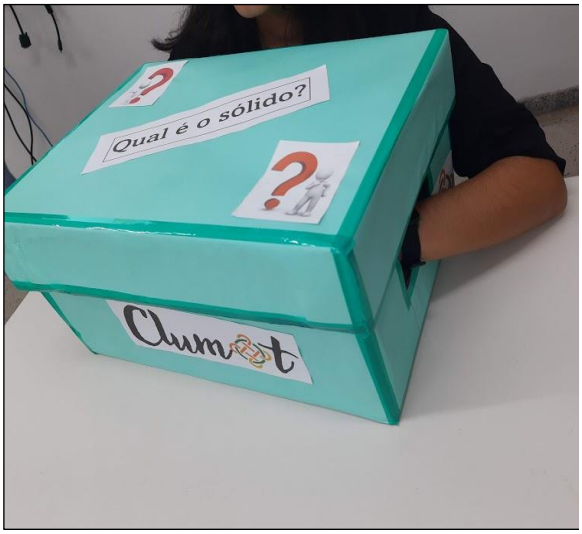

Figure 2 – Modeling with play dough

Source: CluMat Ifes Collection

Perceiving the object is linked to people's practices and leads to the organization of a psychic image, which can differ for each person. According to Leontiev (2021, p. 81, emphasis added):

> Whatever shape the perceptual activity takes, whatever degree of reduction or automation it is subjected to in its shaping and development, it is constructed fundamentally in the same way as the tactile activity of the hands, which 'photographs' the contour of the object. Like the activity of the hands that touch, *all perceptual activity finds the object where it exists, in the external world, in objective space and time.*

Leontiev (2021) states that when copying an object in a drawing or creating its spatial representation, we establish a correlation between the object (model) and the represented object (modeled or drawn) and perceive them as two different things. However, we do not establish this correlation between our subjective image of the object and the object itself or between the perception of the drawing and the drawing itself.

In addition to enabling a discussion about the senses and their combination for the perception of shapes, the task aimed to discuss different ways of

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representing solids on the plane (views, perspective) or representing them in three dimensions (thinking about other possibilities) and address characteristics of geometric solids, highlighting their types of surfaces (flat, curved, and mixed), to think about a classification based on them. Those discussions arose when reflecting on the proposal presented.

During CluMat, which we call planning and elaboration of teaching actions, participants work collectively, based on theoretical studies, preparing proposals for actions to be developed. However, even though this moment already exists, we highlight that the task "Which solid is this?" provided the opportunity for a discussion on the perception of space, relating it to important points in the historical process, such as the use of the senses, the forms of representation through artistic activity, which were essential for the development of the concept of form. Therefore, the task highlighted the possibility of intertwining the theoretical study in the face of a logical-historical movement with an action for its teaching.

For human actions of creating instruments/tools and representing objects (drawings and sculptures), we needed to perceive the shapes contained in the surrounding space. Regarding perception, Leontiev (2021, p. 58) states that, in the materialist conception, it is a process "inserted in the vital and practical relationships of the person with objective reality" and embodies the totality of human social practices, in addition to of being conditioned by the organization of the sense organs.

Petrovski (1980) also highlights the role of the sense organs in understanding and representing the object. Along the same lines, Lima and Moisés (2002) emphasize the intertwining of hands and eyes to create forms and explain that the sense organs allowed human instinct to establish correspondences between the qualities of the natural shape with the new shape created through manipulation. Thus, the authors define the shape as "the correspondence established between the quality you want to capture and the quality used to represent it" (LIMA; MOISÉS, 2002, p. 5).

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When we study the idea as part of this broader process resulting from some human needs and actions, we identify that understanding other concepts is necessary, such as the figure. We consider the figure as the representation of the object, while the shape deals with the idealization of that object based on its qualities. Oliveira (2022, p. 67) defends shape as the central idea of geometry and explains that it "corresponds to the mental representation of a figure or object." Oliveira and Cedro (2022) argue that while the figure is understood as the external aspect of an object, its contour, the shape, is seen as the internal aspect, the essence, and results in the understanding of the existence of the external and internal figure. Agreeing with the authors, we understand that the shape corresponds to the combination of the internal aspect, the qualities, with the external aspect, the figure. The shape carries the visual aspects, therefore, the figure, but is not limited to it. We do not think about the shape without thinking about the figure. Shape is a broader concept; it is related to the essence of the object, as it captures its main qualities.

According to Petrovsky (1980), when determining the shape of an object, the contour is the most informative characteristic, being the boundary between two realities: the figure and the background. Thus, we understand contour as a shape limitation both in the plane and the space. Therefore, the contour can be represented by a line when dealing with flat figures or a surface when dealing with spatial figures.

According to Petrovsky (1980), eye movement allows us to observe the contour of objects, and this is a necessary condition for forming their image. By capturing the natural shape, the individual can represent it in a two-dimensional or three-dimensional way.

4. Final considerations

In this article, we emphasized the relevance of studying the logical-historical movement of concepts, with special emphasis on geometry, as a structuring action for teacher education proposals anchored in the cultural-historical theory, activity theory, and teaching guiding activity. The intertwinement of these references

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sheds light on work as an activity that promotes human development. This concept gains centrality in the formative proposal executed in the context of the Mathematics Club's actions.

The formative movement in CluMat starts with the need for the group to acquire scientific knowledge, which, in our case, is geometric. To achieve this, we need to create formative actions so that participants can acquire historically accumulated knowledge when studying the logical-historical movement of concepts. This process must be linked to the debate on how to organize teaching to understand human needs for creating and developing particular concepts and to the choice of didactic-pedagogical instruments that help students in the learning process.

Thus, regarding the study of a logical-historical movement in geometry, we emphasize the need to seek, in the relationship between humans and nature, based on work, a broader understanding of the geometric concepts that are organized, based on the idea of shape. We verified that the perception of space and the search for its representations contributed to what we understand as shape and figure, essential elements for geometry.

However, we highlight that the appropriation of theoretical principles, whether regarding the basic theory or a logical-historical movement of concepts, must be related to teaching work so that it provides conditions for in-service and pre-service teachers to promote quality change in terms of mathematics teaching (BOROWSKY, 2020). This process occurs in all participants' education, including educators.

Concerning initial education, students must have the opportunity to discuss the profession through the experience of the main knowledge that is part of teachers' professional practice, including the pedagogical activity, which involves choosing content, organizing teaching activities, and evaluating this teaching and learning process. One possibility of effectively weaving theoretical study and educational practice is bringing undergraduate students closer to professional practice based on a theory so that they plan and evaluate the actions developed,

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establishing a theoretical-practical relationship, and advancing beyond a conception that dissociates these two moments.

In addition to providing opportunities for this theoretical-practical relationship, we understand that the theoretical study of geometry carried out in the doctoral research that supported this article is relevant to the formation of the teacher as an individual and as a social subject, who is constituted through work and can identify when it becomes alienated and non-formative. Furthermore, we point out that theoretical study from the perspective addressed can influence other important points of the formative process, including the way of organizing teaching, sharing actions with the group, the intentionality of the actions developed, the way of evaluating them and the identification of needs to return to some stage of the process.

Un movimiento lógico-histórico en geometría y acciones formativas en un Club de Matemáticas

RESUMEN
El objetivo de este artículo es compartir los resultados de un estudio de movimiento lógico-histórico en geometría y destacar las contribuciones a los procesos de formación de profesores desde una perspectiva colectiva. Esta formación tiene lugar en el Club de Matemáticas del Instituto Federal de Espírito Santo y la investigación de este proceso se está llevando a cabo como parte de un proyecto de investigación doctoral, cuyos fundamentos teóricos y metodológicos son la Teoría Histórico-Cultural, la Teoría de la Actividad y la Actividad Orientadora de la Enseñanza. Valiéndose de esas bases, la formación docente es entendida como un proceso social, dialógico, marcado por las necesidades y demandas del trabajo pedagógico, sintetizando debates y acciones que involucran la actividad de enseñar, principal actividad del profesor. Cabe destacar que el movimiento formativo en el Club de Matemáticas se basa en la necesidad de que los participantes se apropien del conocimiento acumulado históricamente, al estudiar un movimiento lógico-histórico de los conceptos, en este caso específico, la geometría. El estudio destaca la geometría como una producción humana y para abordar su desarrollo desde una perspectiva lógico-histórica, es necesario discutir la relación entre el hombre y la naturaleza, haciendo hincapié en los procesos de trabajo y de producción de instrumentos, ya que los procesos de percepción de la forma, de localización y de desplazamiento en el espacio se ven modificados por la relación hombre-trabajo. Además, el estudio teórico en el proceso de formación colectiva permite comprender nuevas formas de organizar la enseñanza, abordando la geometría a través de la percepción de las formas, la identificación de las figuras, la comunicación de sus propiedades, las diferentes representaciones y el establecimiento de una organización intencional de las acciones en el proceso de formación.

Palabras clave: Formación de profesores. Geometría; Forma; Teoría histórico-cultural.

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