

Logical-historical movement and the formative proposal of the MathClub: a look at the organization of the teaching of the concept of number¹

Movimento lógico-histórico e a proposta formativa do Clube de Matemática: um olhar a partir da organização do ensino do conceito de número

Lukas Adriel Francisco Alves²

Maria Marta da Silva³

ABSTRACT

This article's main objective is to investigate how a situation that triggers learning about the concept of number contributed to the understanding of the logical-historical movement, as a proposal for organizing the teaching of mathematical concepts. The Mathematics Club of the State University of Goiás, Campus Sudoeste, Sede Quirinópolis, was the training space that hosted the research activities. In this context, during the period from 2017 to 2023, approximately thirty mathematics teachers in training participated in a training experiment that sought answers to the following problematizing question: What are the training contributions that the organization of teaching mathematical concepts offered by the Mathematics Club, from the logical-historical movement, can it bring to Mathematics teachers in initial training? The results provide evidence that the subjects understood mathematical concepts as human productions that materialize responses to needs posed in an objective reality. They also indicate that they see the need for teachers

RESUMO

Este artigo tem como objetivo principal investigar como uma situação desencadeadora de aprendizagem acerca do conceito de número contribuiu para o entendimento do movimento lógico-histórico, como proposta para a organização do ensino de conceitos matemáticos. O Clube de Matemática da Universidade Estadual de Goiás, Campus Sudoeste, Sede Quirinópolis, foi o espaço formativo que abrigou as ações da pesquisa. Nesse contexto, durante o período de 2017 a 2023, aproximadamente trinta professores de Matemática em formação participaram de um experimento formativo que buscou respostas a seguinte questão problematizadora: Quais são as contribuições formativas que a organização do ensino de conceitos matemáticos ofertada pelo Clube de Matemática, a partir do movimento lógico-histórico, pode trazer a professores de Matemática em formação inicial? Os resultados dão indícios de que os sujeitos compreenderam os conceitos matemáticos como produções humanas que materializam respostas às necessidades postas na realidade objetiva. Também

¹ English version by Polyana Santos Moreira Nunes. E-mail: polyanamoreira@hotmail.com.

² Master's student in the Postgraduate Program in Environment and Society at the State University of Goiás. Member of the Study and Research Group on Pedagogical Activity and the Teaching and Learning of Mathematical Concepts (GEPEAM). Orcid: <https://orcid.org/0000-0002-8770-9156>. E-mail: lukasadriell1@aluno.ueg.br.

³ Teacher at the Academic Institute of Education and Degrees (IAEL), at the State University of Goiás. Leader of the Study and Research Group on Pedagogical Activity and the Teaching and Learning of Mathematical Concepts (GEPEAM). Postgraduate Program in Environment and Society, Brazil. Leader of the Mathematics Club at the State University of Goiás. Orcid: <https://orcid.org/0000-0003-3717-1439>. E-mail: mmsilva@ueg.br.

and students to have the opportunity to teach and learn mathematical concepts based on a general model of actions that prioritizes the historical elements that support the needs of their emergence and development.

Keywords: Initial training ; Mathematics Teachers ; Learning Triggering Situation ; Emergence and development of concepts.

indicam a necessidade de professores e de alunos terem a oportunidade de aprender os conceitos matemáticos a partir de um modelo geral de ações que privilegie o movimento lógico-histórico.

Palavras-chave: Formação inicial; Professores de Matemática; Situação Desencadeadora de Aprendizagem; Surgimento e desenvolvimento dos conceitos.

1 Introduction

Investigating the training of Mathematics teachers is a constant provocation. So, how is it possible to move forward and find answers to the challenges that involve the training of these teachers? How do training contexts deal with these issues? Are there other spaces capable of training this teacher from another perspective than the currently dominant one?

The dominant conceptions for the training of these teachers have disconnected the idea that mathematical concepts are responses to human needs throughout the history of man on the planet. Thus, they do not take into account the fact that these concepts are products of historical-social problems, which man has confronted (Radford, 2011; Moretti, Radford, 2015; Sousa, 2018).

Therefore, this should not be discussed in the classroom in an arbitrary way and disconnected from their history, as Borasi (1992) points out, because, if this is done, the understanding of the social decision-making process that the human communities had to do for such concepts to emerge will not be considered (Arcavi, 1991).

On the path of searching for answers and finding possibilities, this field of investigation accumulates extensive theoretical production. This is due to the growing social demand for Mathematics teachers. Although this fact contrasts with the terrible objective conditions of their training and work.

Anchored in such needs, the question is: What are the formative contributions that the organization of teaching mathematical concepts, offered by the Mathematics Club (MathClub), based on the logical-historical movement, can bring to Mathematics teachers in initial training?

MathClub is a training space that aims to conduct teacher training so that teachers can find a general way of organizing their main activity: teaching.

This formation takes place in a way that highlights the importance of the interaction between history and society, which produced the concepts to be studied. This is a prominent element within MathClub, so that teachers in training can be led to “understand conceptual developments”, placing “the knowledgeable subject and the entire mathematical activity under study within their cultural conception” (Radford, 2011, p.82).

In this way, the research that supports this article took as its object the initial training of Mathematics teachers in a special space: Mathclub.

In order for the reader to understand the entire process developed, the text is organized in such a way that, firstly, it will be clarified what the logical-historical movement of concepts is and how it interrelates with the proposal of the Learning Triggering Situation (LTS). Next, the Mathematics Club is presented and how this space embodies the logical-historical movement in its formative actions.

Next, the research methodology is presented, by didactically characterizing the formative experiment developed in its four moments.

From this perspective, sequentially, there is data analysis, in which the development of the phenomenon can be monitored based on the analytical structure that is made up of units, episodes and flashes. Finally, there are some considerations about the research carried out.

2 Logical-historical movement (LHM) of concepts: didactic proposal present in LTS

Understanding the LHM is an issue that is intertwined with the incessant search for didactic proposals for the training of Mathematics teachers, which allow for an organization of the teaching of mathematical concepts that allows the understanding of the need to "overcome the conception that it is enough to understand the context in which the concepts were developed to be able to teach them" (Leandro; Sousa; Andrade, 2020, p. 396).

This perspective ensures the defense that teacher training, which takes it as a premise, must be based on the idea that the teacher, in the process of learning how to teach, must experience situations which place him in the presence of the “movement of thought in the context in which such concepts were conceived and developed” (Leandro; Sousa; Andrade, 2020, p. 396).

In this line of thought, what can the LHM allow for the training of Mathematics teachers?

It can favor transformations in the meanings in which subjects, in teacher training, attribute to the process of emergence and development of mathematical concepts, a fact that directly interferes with the constitution of new practices in the classroom.

In this way, these practices will have theoretical-didactic conditions to establish themselves as defining the understanding of the LHM's potential in providing resources for an organization of teaching mathematical concepts that reveals the importance of social practices, linked to activity, socially, human and, historically, built.

In the search for a theoretical and methodological organization structure, which aimed to achieve the LHM, it resorted to the concept of Teaching Guiding Activity, proposed by Moura (2007). According to this author, it would be a teaching activity guided, intentionally, by those who teach and, because of this, it

has an objective, instruments and modes of action for its achievement; considers the learning possibilities of the subjects who participate in the activity and the complexity that involves the logical-historical formation of the concepts that are constitutive of the activity (Moura; Sforini; Lopes, 2017, p. 87).

In order to involve the logical-historical dimension of concepts in the Teaching Guidance Activity, it is concerned, in all its stages, with meeting its constitution. The first of these is the historical synthesis, which seeks, in the History of Mathematics, the path of emergence and development of a referred concept. Subsequently, in the second stage, the LTS (which can be offered as a game, virtual story (VH) or emerging everyday situation) seeks to materialize this logical-historical path of concepts, through the presence of triggering problems.

The triggering problems have, at their core, the need that induced humanity to construct the concept. Therefore, it is paid attention to the fact that the LTS, regarding the concept of number, which was materialized as an HV in the format of a Comic Book (CB), brought, in its plot, human primordialities, which led the man to create this concept: quantification and its consequent control of variation in quantities, the ability to measure and establish units of measurement, among others (Silva, 2022).

Ifrah (2005) highlights that, in this process of controlling the variation of quantities (represented in ‘The Agnuns’⁴), it appears that, initially, the physical objects remained closely linked to those that were intended to be counted. However, little by little, they moved away from the material indicative and represented abstract symbols, which can be linked to any object.

Caraça (1989) corroborates the conception of Ifrah (2005) when says that such distance is based on the fact of the interface existing between mathematical concepts and the needs that determined them, and “[...] mathematical concepts arise, once problems of capital, practical or theoretical interest are posed” (Caraça, 1989, p. 125).

In this sense, Caraça (1989) emphasizes that the “natural number, arising from the need for counting”, has a close connection with physical objects, however “the rational number” is born linked to the needs “of measurement”, while “the number real” is born “to ensure the logical compatibility of different acquisitions” (CARAÇA, 1989, p. 125). In this way, rational and real numbers move away from material references and the need to be linked to physical objects, as they are the product of theoretical-practical essentialities.

Thus, covering the essence of the needs that drove humanity, in the search for solutions that enable the social and historical construction of concepts up to the present moment, is a component of the movement of apprehension of the concept itself, when it materializes in the logical. Therefore, the teacher in training must deal with mathematical concepts as syntheses of the history of human work, which is embodied in activities (Leontiev, 1978).

⁴ The Agnuns (ISBN - 978-65-00-2-312-7; CDD: 741.5; CID: 21.61527) – the name given to the comic book that represents the HV designed for teaching the concept of numbers.

In this movement, the historical aspect is integrated with the logic of a particular object of study, and it is only in this dialectical unity that the knowledge of that object is admissible. According to Kopnin (1978, p. 186), the “historical aspect implies not only the history of the object, its production and development, but also the history of how humanity appropriated this object, that is, the history of its knowledge”. Therefore, “the logical-historical pair is the criterion for the systematization of knowledge to be appropriated by the student. The teacher is responsible for making this pair visible [...]” (Moura; Sforini; Lopes, 2017, p. 91).

Thus, the subjects, by appropriating these once historical aspects, understand the path taken by human thought so that such concepts are currently in a certain form, which constitutes the logical aspect. In such a way, the logical is “the reproduction of the essence of the object and the history of its development in the system of abstractions” (Kopnin, 1978, p. 183), constituting itself as the appropriation of history by man's thought. And, “for this reason, the logical is the historical freed from the casualties that disturb it” (Kopnin, 1978, p. 184), aiming at the way in which human thought carries out the task of apprehending the historical (Kopnin, 1978).

Such problems do not need to have the factual history present in the History of Mathematics. But rather the one that is saturated in the concept when considering that it embodies a historically allocated human need, “That is, a problem that brings the essence of the need that led humanity to create the concept to be taught, the core of the concept to be appropriate” (Moura; Sforini; Lopes, 2017, p. 91).

Training designed in this way takes into account that such relationships change, as teachers assimilate what has historical, social and cultural value, consequently, interpreting their social worlds in different ways. Therefore, they act on them in different ways, which, in turn, impact the dynamic relationship between the subjects' development and the historical-social situations that they experience.

Among the possible spaces to allow and encourage such interrelations, MathClub stands out, being an environment capable of maintaining the interface between the University and the School, while the teacher, in the process of learning

how to teach, learns the teaching process. This phenomenon is a process of making sense and meaning, according to the involvement with the world to which it is part (Moura, 2021). The next topic provides an understanding of how MathClub aims at this teacher training proposal, based on the LHM.

3 Mathematics Club: space for teacher training based on the logical-historical movement

MathClub is a training space that emerged in 1999, as a supervised internship project, linked to the Faculty of Education of the University of São Paulo.

The concepts defended within this space were disseminated throughout the Brazilian territory⁵ em diversas Instituições de Ensino Superior.

At this juncture, specifically, in 2017, MathClub was created within the scope of the State University of Goiás, Câmpus Sudoeste - Headquarters – Quirinópolis⁶, which continues until today.

In this training space, emphasis is placed on the importance of understanding LHM concepts. This is an element of relevance within the Club: highlight the process in which teachers in training can “understand conceptual developments”, placing “the knowledgeable subject and the entire mathematical activity under study within their cultural conception” (Radford, 2011, p. 82).

From this conception, and based on Sousa (2004), Kopnin (1978) and Saito and Dias (2013), it is highlighted that MathClub, by proposing a learning that approach to teach Mathematics linked to an organization of teaching mathematical concepts, based on the logical-historical perspective, defends the need to “understand the immutability and mutability of things and concepts [...], realizing

⁵ Currently, it is also found in the following Brazilian Higher Education Institutions: Federal University of Santa Maria (UFSM); Federal University of Goiás (UFG); State University of Goiás (UEG); Federal University of Rio Grande do Norte (UFRN); Federal Institute of Education, Science and Technology of Espírito Santo (IFES); Federal University of Uberlândia (UFU); Federal University of São Paulo (Unifesp).

⁶ CluMat - UEG is affiliated (no. 03/2021/SME-UEG) with the Municipal Department of Education of Quirinópolis. It is part of the research project "Pedagogical Activity in the Training of Teachers who Teach Mathematics through Partnerships between Higher Education Institutions and Basic Education Schools in Different Brazilian Regions". This project is funded by the National Council for Scientific and Technological Development through the Universal Call n° 18/2021.

that the logical is a reflection of the historical [...]” and that this unity “[...] presupposes the possibility of understanding the movement of thought to grasp the object [...]” (Leandro; Sousa; Andrade, 2020, p. 395).

At MathClub, the importance of concepts being studied in their production process is highlighted based on the meanings intrinsic to the historical period that emerged and developed, as well as the interrelationship of this process with human activity.

According to Radford (2006, p. 112), this activity “[...] generates conceptual objects, which become the root of changes in the activities themselves”. Still according to Radford (2006), it is important to pay attention to the explanation of how the acquisition of concepts is carried out, which are arranged at the school as content, that is, a fundamental problem in the training of Mathematics teachers, in particular, in learning in general.

Therefore, MathClub is a space that allows the understanding that the teacher in training, in this process of dealing with change, does not do so as a simple response to his behavior, which was from the student and now becomes a teacher, but it involves a process full of contradictions, which takes on new forms as it develops and generates its relationships with the historical-social situations of its development.

Thus, within MathClub, emphasis is placed on the dynamic relationship that may exist between individuals and the historical-social situations of the emergence and development of concepts so that they are capable of promoting the ability to see them as a result of historical-cultural needs, experienced by humanity.

To this end, it is necessary to understand the process of emergence and historical development of mathematical concepts and how this proposal can be possible in a training process for Mathematics teachers, which is connected with the organization of teaching mathematical concepts in Basic education.

But what can the existence of a training space, which enables the dynamics of the logical-historical movement of concepts, mean for the training of Mathematics teachers?

To find answers to this question, another question will be needed: What profile of Mathematics teachers is necessary to train?

On the way to find answers to these demands, researchers such as Cedro (2008), Radford (2011), Sousa (2018), Panossian, Moretti and Souza (2017), Moura (2017), Leandro, Sousa and Andrade (2020), among others, raise questions in their studies about the relevance of LHM in the training of Mathematics teachers. To this defense is added the perception that such a proposal could lead to another organization of pedagogical activity. To achieve this, the existence of training contexts, such as MathClub, was essential.

In this approach chosen by CluMat, “[...] concepts are understood as living productions in direct relationship with the needs of the subjects and historical times that produced them [...]”, which is why one of the objectives of this training space is allow the subjects, who are part of it, to appropriate “[...] a certain concept, understood as historical and cultural production [...]” (Moretti, 2014, p. 34).

In this way, they can be understood based on the constitution of/in man's social practice, while life itself develops and becomes more complex, allowing subjects to acquire social and cultural conditions of thinking, in addition to theorizing about this social practice, from its objects and constitutive phenomena.

This condition highlights another postulate: the understanding and the use of such a historical movement of elaboration and subsequent logical increment of mathematical concepts that shows a potential possibility for the development of teacher training activity, in this special context, called MathClub.

Thus, this space aims to overcome the training of Mathematics teachers in training that has, in recent decades, focused exclusively on the contents of the curriculum and the assessment model, which is a copy of exhaustively repeated exercises (Cochran-Smith *et al.*, 2008).

Formed based on this model, the teacher will hardly understand the Vygotskinian premise that it is not important to teach a quantity of knowledge as it is to instill the ability to acquire such knowledge and make use of it (Vigotski, 1997).

Contrary to this training, 'which trains the future teacher', at MathClub, man's search to understand and to explain the reality that surrounds him is valued, and in the meantime, he creates and acquires knowledge and, therefore, develops it.

To this end, thought follows from what already exists, but without being limited to it to arrive at something that did not exist (Kopnin, 1978). Thus, the path requires creative activity from the subject, rules and laws that govern objective reality.

In this sense, man lacks new concepts that expand his possibilities of mastering the reality that surrounds him. Only in this way, by going beyond the limits of current schemes humanity creates concepts that allow it to make choices to solve its problems and, thus, thought will not be directed in a rigid manner, but with a certain freedom (Kopnin, 1978).

Therefore, in the training proposal offered by MathClub, teaching activities that can capture the process of emergence and development of mathematical concepts are valued in a way that not only “[...] photographs the real historical process with all its casualties, zigzags and detours” (Kopnin, 1978, p. 184), but which can reflect history in theoretical form and, thus, interpret the historical process of the genesis of this concept.

However, in order to understand didactically how such actions are carried out, a methodological path is necessary to anchor this proposal. In the next topic, it will be presented highlighting the process of how MathClub's actions take place.

4 Methodological path: the formative experiment as an option

The Mathematics teacher training model, based on the ideas defended by MathClub, highlights that learning is a social process, because the interaction between subjects plays a prominent role in its development.

In this environment, it was decided to develop a formative experiment, which according to Silva (2018), supported by Davidov and Markova (1987), is an investigative structure carried out in several stages, in which all processes occur simultaneously.

In this way, the experiment allows highlighting the laws of the domain of reality of the object being researched. Therefore, for Freitas and Libâneo (2022, p. 6), the formative experiment would be the “process of identifying, understanding and explaining the historical genesis of human psychic functions in concrete conditions, revealing the movement of their emergence and transformation in social relations” (Freitas; Libâneo, 2022, p. 6).

Didactically, the experiment was organized into planning, development and collective evaluation meetings, which took place weekly and lasted 5 hours (there were 32 meetings per year). The participants were approximately 30 (thirty) undergraduates (all were students of the Mathematics course hosted by MathClub, this is the only condition to be participants, there is no selection process), who became club members from 2017 to 2023. The research actions were developed at the University (during the planning and development of activities when Basic Education students attended the Club, or when the Club members went to schools).

The data collection instruments were audiovisual recordings of all moments (planning, development and evaluation), which, later, became possible data for analysis.

The Club's activities have a very peculiar organization, which are shown in Table 1.

Table 1 – Organizational structure of the training experiment

Planning	Development	Assessment	Activities already carried out
<p>These moments are intended for the collective organization of actions that will make up the teaching activities on the mathematical concepts covered. Its purpose is to create a collaborative space where planning takes place. At this time, the historical syntheses that served as basis for planning the LTSs were prepared. This activity about the concept of numbers took approximately two years to be completed.</p>	<p>These actions take place when students go to the MathClub headquarters at the University. Club members are divided into groups, as they are held in several classrooms at the same time. The development of the activity, which supported this article, lasted 5 weeks, lasting 3 hours per week. The classes were made up of 25 to 30 students. They belonged to the final two years of Elementary School (EF) I and the initial year of EF II.</p>	<p>Parallel to the development of activities with students, weekly meetings take place, in which this development is evaluated and, when necessary, revisions are made to some aspects of the activity. In this way, actions are adjusted along the way and not at the end. These meetings are also moments for collective analysis and synthesis of what is developed and understanding the teacher training process, which takes place concomitantly with students' learning at school.</p>	<p>5 groups of activities have already been planned and developed: 1) About the geometric concept of polygons. In this activity, LTS is presented in video format. This is aimed for students in the final two grades of EF I and two initial grades of EF II; 2) The algebraic concept of equations was approached and the planning of an VH was carried out, which has the title: "Mendhi, a vizier in Luxiar". This activity is aimed at all four grades of EF II; 3) The third activity is about the algebraic concept of function, also being an VH. Its title is: Mitami's journey. This activity is aimed at the final years of Elementary School II and the initial years of High School; 4) The fourth set of activities focuses on teaching the geometric concepts of area and perimeter and has games as LTS. This activity is aimed at the four grades of EF II; 5) This set of activities addresses the arithmetic concept of number. The option made for the materialization of the LTS was an VH, which was presented in the form of a comic book.</p>

Source: Prepared by the authors (2024).

Due to the impossibility of analyzing in this article all the activities⁷ carried out by MathClub throughout the period of time that it exists, the LTS was chosen regarding the concept of number.

See in the analysis presented later how this phenomenon unfolded among the subjects participating in this training space.

⁷ The activities presented, in summary, except for 'The Journey of Mitami', are available on the website: <https://www.clumatuegquirinopolis.com/atividades>. Full access to the activities and their sets of actions can be found in Silva (2022).

5 Data analysis: understanding the phenomenon

In a scientific investigation, the production of knowledge only makes sense as long as its function is to reveal reality in its contradiction, in counterpoint and in the separation of appearance and essence, of what is not important to show what is fundamental.

Therefore, only through this process can its internal connection and, with that, its particular character emerge. “In this process, the secondary is not left aside as unreal or less real, but reveals its phenomenal or secondary character through the demonstration of its truth in the essence of the thing” (Kosik, 1969, p. 18).

In order to achieve this understanding for the analysis, a structure was used that starts from the concept of unity, proposed by Vygotski (1998), who states that it is “the result of the analysis which, unlike the elements, enjoys all the fundamental properties characteristic of the whole and constitutes a living and indivisible part of the totality” (Vygotski, 1998, p. 20).

Likewise, in the necessary search for singularities that make up the universality of the phenomenon investigated, it is looked for episodes, which would be, according to Moura (2004, p. 267), the moment, which “can reveal interdependence between the elements of a formative action”.

Continuing the explanation of the proposed data analysis structure, there are flashes, which, for Silva (2018, p. 150), would be the parts of the episodes that would configure “the signs of the conscious and internalized reflection of reality”, which “they implicitly embody the motives and needs, the meaning and meaning that are expressed in language, but, above all, they are not reduced to it” (Silva, 2018, p. 150).

However, the analysis, through units, episodes and flashes, strives to understand, through the captured flashes, the movement that portrays reality as they see it. “This movement was captured here by the subjects’ speech, understood as a mediating instrument of thought and an element that supports the development of all higher functions” (Silva, 2018, p. 154). In this way, the analysis is based on the understanding of human thought, based on verbal language. To

this end, and in accordance with Luria (1991, p. 17), “[...] conditions to overcome the limits of immediate sensory perception of the external world are necessary”.

From this procedural movement, the following composition of the analysis presented in Table 2 arises.

Table 2 – Structure of data analysis

Unit: the training structure, offered by MathClub, seen from the organization of teaching concepts.	
Episode 1: LHM's formative contributions.	Episode 2: LTS - materialization of the training and teaching proposal.

Source: Prepared by the authors (2024).

These episodes were selected among many others possible, because they are able to reflect, make new connections and relationships between the actions of the formative experiment, allowing conditions to form concepts and carry out new analyzes and syntheses about the formative process that took place with Mathematics teachers in initial training who participated in the Mathematics Club.

5.1 Episode 1: LHM's formative contributions

The use of the LHM of concepts, as a didactic proposal for the organization of teaching, which is offered in the teacher training process at the Mathematics Club, should not be confused with the custom of making the History of Mathematics a source of contextualized problems, which are adapted to teach certain content. The proposal, which is defended, differs from this approach.

It is proposed that the LHM, offered by the study of historiographies present in the History of Mathematics, seek consideration of human-social learning pertinent to the historical-cultural production of concepts.

This process allowed teachers in training to understand the limits and the qualitative changes of these learnings, which can provide theoretical thinking about human practice, without which the elaboration of the concept would not exist. See the signs of such understanding in the following flashes: “I remember there was a time when the life stories of mathematicians that had something to do

with the content contained in that chapter began to appear in the footnotes of Mathematics books. But that was very different from what we do here at the Club. We don't bring the life story of the guy who invented the content, we look for the history of the society, the people, the culture that created the concept" (**Isa⁸, Flash 1⁹, Episode 1**); "I remember that, but in my book it was in a box at the end of the subject. It was of no use, the teacher told us to read it, but it didn't explain anything, and in fact we didn't even see a relationship between what we were studying, we didn't do it like here where the activities are linked to the history of how it emerged and developed" (João, Flash 2, Episode 1).

Taking the formative process they experience as a basis, it will be very likely that these teachers, when they are leaders in the classroom, will have the ability to develop activities that "provide students with the possibility of understanding the world that surrounds them, from the moment when abstractions become concrete content for thought" (Sousa, 2018, p. 53).

Such a condition for planning teaching activities is capable of materializing, based on the knowledge of the LHM, which has the possibility of being carried out in the search for the genesis and the development of these concepts, based on the study of the History of Mathematics, which allows understanding that, for any knowledge, there is always a possibility that has already been constructed historically, at some point (Moretti; Radford, 2015).

However, this does not constitute reproducing the faithful history of the concept. It means that, even knowing this, one must seek to study this trajectory, its genesis and its elaboration, knowing and valuing its "history, the study of its conditions of possibilities, transformation, generalization, refinement [...]" (Moretti; Panossian; Moura, 2015, p. 253).

Such a choice can allow the planning and development of LTSs, which confront subjects with the need for the mathematical concept they intend to teach.

⁸ The names given to the subjects are fictitious in order to preserve their real identity.

⁹ The speeches materialized as flashes are transcribed in full, as the subjects prepared them. Therefore, there are no syntactic, semantic, cohesion or coherence corrections.

By doing this, the emergence of needs of a cognitive or material nature can be allowed, and subjects can be mobilized in the search for a solution so that the actions that are developed have as their objective the resolution of the proposed situation and not just respond to a demand placed. What constitutes, in fact, a LTS.

In this way, the LHM can contribute to the process of training Mathematics teachers by allowing an organization of teaching, which they can, in the future, carry out in their classrooms, when they are actually teachers from a historical perspective, cultural, which criticizes the crystallized ways of teaching mathematical content in schools.

There are signs of the contributions, in the next flashes, of this formative movement: “Without the opportunity we are having here at the Club, we would never do, when we are teachers, activities like this, which show students where the beginning of mathematics things” (Fernando, Flash 3, Episode 1); “I think it's a really cool idea to have how to organize the teaching of concepts from a historical perspective, and even more so that it doesn't just stay there where and how it was created, which has continued to this day (Wilson, Flash 4, Episode 1); “When I saw the proposal to use history to answer questions like: where, by whom, how, why did they create this content, I was excited, because I don't know myself, I only know how to do the math, but I don't know these answers, but I want my students when I'm a teacher, I want them to know” (Junior, Flash 5, Episode 1).

Furthermore, the option for the LHM of concepts, in training contexts, provides conditions for questioning and criticizing theoretical-didactic stances, which defend the existence of correct and adequate mathematical problems to make learning emerge, as if for each concept there existed a situation, corresponding mathematical problem, which will guide the subject to make use of the concept being taught. The following Flash corroborates this discussion: “I think it's very important that we don't use ready-made situations, we don't have a ready-made proposal, that's why we're having the freedom to create a VH to teach numbers and learn too, because there's a lot of things we didn't know, we had no idea about all these historical and social relationships that exist” (Jaqueline, Flash 6, Episode 1).

In this sense, at MathClub, the perception of the LHM is that it clarifies the process of emergence and development of concepts, as well as representing “the mutual relationship [...] and the modes of action for the appropriation of this” (Araujo; Moraes, 2017, p. 55).

Thus, during the experiment, it is clear that the LHM allowed teachers in training to understand mathematical concepts as human productions that materialize responses to needs posed in objective reality (Leontiev, 1978; Vigotski, 1998; Fraser, 1998; Moura 2007, Radford, 2011; Sousa, 2018).

Such needs belong to subjects in a given place and historical time. This perception corroborates the defense that Mathematics “is just part of the incessant process of analysis and synthesis generated in the dynamics of constructing responses to problems generated in the search for improving life in the collective” (Moura, 2007, p.45).

Thus, the LHM of concepts meets such an understanding of Mathematics, in this teaching learning process, carried out at the MathClub, being an essential element for these subjects to seek transformation of the meaning, attributed by them, to the teaching organization process. In this process, they found themselves faced with the need to plan a LTS around the concept of number. This action will be covered in the next episode and the flashes that make it up.

5.2 Episode 2: LTS - materialization of the training and teaching proposal

It is believed that teacher training should be characterized in a space-time of activities, intentionally, oriented towards teaching, particularly, promoting learning situations for two subjects: teacher and student.

In the LTS that supported the writing of this text, it was sought to highlight that this conception has guided the understanding of how Mathematics teaching has been taught in a peculiar training space: MathClub.

This LTS was materialized in a VH, which presented itself as a comic book. As such, it has triggering problems, which require the search for a solution by the subjects who participated in the development of this teaching activity.

The activity to be teaching requires the teacher to guide actions in the analysis and synthesis processes, which result in the solution of problem situations present in the virtual story. “Virtual, because it must have the possibility of 'de-reifying' the concept fixed in word-concepts that, throughout history, have become abstract syntheses whose relationship with materiality is apparently distancing itself [...]" (Moura; Sforini; Lopes, 2017, p. 93).

As fiction as it may seem, VH, according to Silva (2018, p. 77), “is not just unreality alien to reality”. Therefore, in Kopnin (1978, p. 204), it is stated that “fiction in relation to reality is to this or that extent, of this or that aspect, the incorporation of its content”.

Therefore, in the VH elaborated from the historical synthesis, it was sought to reconstruct the main moments of the process of genesis and development of the concept of number until the current moment, when it incorporates reality and materializes in the logical.

This organization allowed the subjects involved to appropriate the concept based on the understanding of human experiences throughout its history.

Therefore, the historical synthesis brings with it and takes to LTS the core of the need that led humanity to create the concept to be taught, that is, it has its essence (Moura, 2017). See how the following flashes that confirm this understanding: “Without having done the historical synthesis first, we would never have been able to create a LTS that had in it the real needs that ended up making people create the idea of number and then it was improving, as things went on” (Ana, Flash 1, Episode 2). “And we don’t need to put everything that happened over thousands of years into the comic, but reconstruct the most important parts, which will make them learn how numbers emerged and developed” (Renata, Flash 2, Episode 2).

In this way, it is believed to be impossible to reconstruct history, but it is possible to retrace the logical path taken by man and this means, according to Sousa, Panossian and Cedro (2014, p. 11), that “the history of the concept must be seen not as an illustrator of what should be taught. It is the true guide of educational activities.” Thus, the subjects externalize their understandings: “The

comic book drawings that Wallace is going to make have to be beautiful, the illustration has to be cool, but that alone is no use, the most important thing is how we are going to tell the story of the emergence of numbers, why they created it, what it was going to be used for, that's the key point, the historical synthesis gave us the path to follow, so we wouldn't get lost" (César, Flash 3, Episode 2).

In this sense, the didactic-methodological structure chosen by MathClub is aimed at the construction of teaching activities, which have in their essential features the understanding of thinking in concepts.

This is, in Vygotski's conception (1997, p. 79), the most appropriate way to imagine the

reality, because it penetrates the internal essence of objects, since their nature is not revealed in the direct contemplation of one or another object in isolation, but in its development linked to the rest of reality.

From this perspective, the LTS, prepared and presented as a VH, shown as a comic, has its setting set in the prehistory of man and has, as its main characters, a family characterized by qualities concerning this historical period.

Due to this theoretical-methodological support, the comic privileges historical elements, which support the needs for the emergence and development of the concept of number. In this way, the teaching of such a concept is based on the theoretical premise that mathematical concepts have a generic character, supported by relationships.

Along this path, it was sought to understand the conditions for the formation of these relationships and the understanding that they arise from the moment that the real subject considers knowledge as a part of their real life and not an external condition to this reality.

In this path, the way in which the LTS was organized comes from the understanding that the concept of number is a human production constituted, historically, based on social needs. Thus, this concept, although it has its presence in everyday life, "had its beginnings in prehistory, [when] human beings used different strategies to control, record and communicate quantities, as well as to

carry out calculations and operations” (Moura, 1996, p. 11). The next flashes confirm the understanding of such theoretical propositions: “It seems shameful for us to be in a degree without knowing that the concept of number was not just used to count, that it also had the need to measure and that is why when we arrived at the part of the story that they had to measure the path, we had more difficulty creating the triggering problem” (Paula, Flash 4, Episode 2); “Understanding that there is a movement throughout the history of humanity was very important, but there would be no point in knowing this if there wasn’t a proposal like LTS, which you can take to the classroom” (João, Flash 5, Episode 2).

Thinking about the concept of number, based on this understanding, is completely different from appropriating the number mechanically, through memorization, as is commonly offered in the school process.

The planning and development of a LTS allowed the teaching of this mathematical concept to involve the participation of the teacher, as the subject responsible for organizing teaching in its complexity, both in the cognitive and corporal aspects and, therefore, transcends its motor function exercised simply, by choosing exercises that require the spelling of numerals resulting from counting sets of objects.

Thus, as the formative experiment unfolded, the training teachers showed signs of embracing the idea that, although they already had a certain knowledge about numbers, this did not mean the appropriation of this complex concept. The same, according to Ifrah (2005), is, nowadays, the objectification of the numerical record of all human mental activity, in the search for control of small and large quantities.

Thus, it is argued that, although the idea of what a number is its social function and usefulness is abundant in the daily life of practically all societies, it is the role of the spaces responsible for training Mathematics teachers to expand this daily experience and provide it with objective conditions of appropriating such a concept, as an objectification of access to historically constructed cultural assets, so that, when you are at school, you are able to teach what you have learned.

6 Considerações finais

This article was established with the aim of investigating how a LTS, based on the LHM theoretical proposition of concepts, in a particular training space, contributed to the training process of Mathematics teachers.

This training space is the MathClub, which considers that the LHM is able to cooperate in understanding the important role of teaching intentionality in organizing teaching, with this organization being the starting point for establishing “the action plan, through knowledge about the idealized object” and, to this end, he needs to make use of “theoretical assumptions, define actions supported by these assumptions, seeing them as mediating instruments of these actions and, when acting in a process of analysis and synthesis, he aims his activity” (Moura; Sforini; Lopes, 2017, p. 84).

Thus, at the Club, the logical-historical conception adopts the function of a didactic training proposal “that takes history as a provider to the extent that it makes it possible to perceive the movement of thought in conception and development” (Leandro; Sousa; Andrade, 2020, p. 395) of the concepts and the way they are presented in school: as content.

Thus, when planning the LTS – The Agnuns –, it was expected that it would be embodied in the very idea of the emergence of the concept of number. The mathematical concept is understood here as the synthesis of human knowledge up to the current historical moment. The VH, which was presented as a comic, served as a springboard for new syntheses of the subjects, involved in a perennial movement of analysis and synthesis about the laws that govern objective reality.

In this way, the proposed teaching organization, offered at MathClub, presented them with mathematical concepts as ideas, through which man creates a “subjective image of the objective world” (Kopnin, 1978), unifying the subject and of the object (Moura, 1996). Such an approach is rare in other training contexts.

In this theoretical line, the requested teaching learning in Mathematics allowed a conscientious organization of individuals' training processes, through the purposeful organization of teaching that allows subjects to appropriate knowledge and ways of thinking.

El movimiento lógico-histórico y la propuesta formativa del Club de Matemáticas: una mirada a la organización de la enseñanza del concepto de número

RESUMEN

El objetivo principal de este artículo es investigar cómo una situación que desencadena el aprendizaje del concepto de número contribuyó a la comprensión del movimiento lógico-histórico, como propuesta para organizar la enseñanza de conceptos matemáticos. El Club de Matemáticas de la Universidad Estatal de Goiás, Campus Sudoeste, Quirinópolis, fue el espacio de formación que acogió la investigación. En este contexto, de 2017 a 2023, aproximadamente treinta profesores de matemáticas en formación participaron en un experimento formativo que buscaba respuestas a la siguiente pregunta: ¿Cuáles son las contribuciones formativas que la organización de la enseñanza de los conceptos matemáticos ofrecida por el Club de Matemáticas, basada en el movimiento lógico-histórico, puede aportar a los profesores de matemáticas en formación inicial? Los resultados muestran que los sujetos entendieron los conceptos matemáticos como producciones humanas que materializan respuestas a las necesidades planteadas por la realidad objetiva. También indican la necesidad de que profesores y alumnos tengan la oportunidad de aprender los conceptos matemáticos a partir de un modelo general de acciones que favorezca el movimiento lógico-histórico.

Palabras clave: Formación inicial; Profesores de matemáticas; Situación desencadenante del aprendizaje; Surgimiento y desarrollo de conceptos.

7 References

ARAUJO, E. S.; MORAES, S. P. G. Dos princípios da pesquisa em educação como atividade. In: MOURA, M. O. (org.). *Educação Escolar e Pesquisa na Teoria Histórico-Cultural*. São Paulo: Edições Loyla, 2017. p. 47-70.

ARCAVI, A. The benefits of using history. *For The Learning Of Mathematics*, v. 11, n. 2, p. 11, 1991.

BORASI, R. *Learning Mathematics through Inquiry*. Portsmouth, Ueinemann Press, 1992.

CARAÇA, B. J. *Conceitos Fundamentais da Matemática*. 9 ed. Lisboa: Livraria Sá da Costa Editora, 1989.

CEDRO, W. L. *O motivo e a atividade de aprendizagem do professor de matemática: uma perspectiva histórico-cultural*. Tese (Doutorado em Educação). Universidade de São Paulo, São Paulo, 2008.

COCHRAN-SMITH, M.; et al. *Handbook of research on teacher education* (Third edition). Routledge: New York, 2008.

DAVYDOV, V. V.; MARKOVA, A. K. La concepción de la actividad de estudio en los escolares. In: SHUARE, M. *La Psicología Evolutiva en la URSS: Antología*. Moscú: Editorial Progreso, 1987. p. 156-178.

FRASER, I. *Hegel and Marx: the concept of need*. Edinburgh University Press, 1998.

FREITAS, R. A. M. M.; LIBÂNEO, J. C. O experimento didático formativo na perspectiva da teoria do ensino desenvolvimental. *Educação e Pesquisa*, v. 48, p. 1-19, 2022. DOI: <http://dx.doi.org/10.1590/s1678-4634202248246996>.

IFRAH, G. *Os números: uma história de uma grande invenção*. 11 ed. São Paulo: Globo, 2005.

KOPNIN, P. V. *A dialética como lógica e teoria do conhecimento*. Rio de Janeiro: Civilização Brasileira. 1978.

KOSIK, K. *Dialética do concreto*. 2 ed. Rio de Janeiro: Paz e Terra. 1969.

LEANDRO, E. G.; SOUSA, M. C.; ANDRADE, J. A. A. Organização do Ensino de Matemática: o papel do grupo de estudos na significação dos sujeitos. *Educação Matemática Pesquisa*, [S.L.], v. 22, n. 2, p. 390-424, 27 ago. 2020. DOI: <http://dx.doi.org/10.23925/1983-3156.2020v22i2p390-424>.

LEONTIEV, A. N. *Actividad, consciencia, personalidad*. Habana: Pueblo y Educación, 1978.

LURIA, A. R. *Curso de psicologia geral: introdução evolucionista à psicologia*. 2ª ed. Rio de Janeiro: Civilização Brasileira, 1991.

MORETTI, V. D. O problema lógico-histórico, aprendizagem conceitual e formação de professores de matemática. *Poiésis*, [S.L.], v. 8, p. 29-44, 20 mar. 2014. DOI: <http://dx.doi.org/10.19177/prppge.v8e0201429-44>.

MORETTI, V. D.; PANOSSIAN, M. L.; MOURA, M. O. Educação, educação matemática e teoria cultural da objetivação: uma conversa com Luis Radford. *Educação e Pesquisa*, [S.L.], v. 41, n. 1, p. 243-260, mar. 2015. DOI: <http://dx.doi.org/10.1590/s1517-97022015410100201>.

MORETTI, V. D.; RADFORD, L. Culturally Meant Concept's History And The Organization Of Mathematics Teaching Activity. In: VI Seminário Internacional de Pesquisa em Educação Matemática, novembro de 2015, p. 1-12, Pirenópolis, Goiás, Brasil.

MOURA, M. O. (org.). *Educação Escolar e Pesquisa na Teoria Histórico-Cultural*. São Paulo: Edições Loyla, 2017.

MOURA, M. O. A atividade de ensino como unidade formadora. *Bolema*, v.11, n. 12, p. 29-43. 1996.

MOURA, M. O. Atividade de formação em espaço de aprendizagem da docência. *Ridphe_R Revista Iberoamericana do Patrimônio Histórico-Educativo*, Campinas (SP), v. 7, p. 1-22, 29 dez. 2021. DOI: <http://dx.doi.org/10.20888/ridpher.v7i00.16028>.

MOURA, M. O. Matemática na infância. In: MIGUEIS, M. R.; AZEVEDO, M. G. (Org.). *Educação Matemática na infância: abordagens e desafios*. Vila Nova de Gaia: Gailivro, 2007. p. 39-64.

MOURA, M. O. Pesquisa colaborativa: um foco na ação formadora. In: BARBOSA, R. L. L. B. (org.). *Trajelórias e perspectivas da formação de educadores*. São Paulo: Editora Unesp, 2004. p. 257-284.

MOURA, M. O.; SFORNI, M. S. F.; LOPES, A. R. L. V. A objetivação do ensino e o desenvolvimento do modo geral da aprendizagem da atividade pedagógica. In: MOURA, M. O. (org.). *Educação Escolar e Pesquisa na Teoria Histórico-Cultural*. São Paulo: Edições Loyla, 2017. p. 183-210.

PANOSSIAN, M. L.; MORETTI, V. D.; SOUZA, F. D. Relações entre movimento histórico e lógico de um conceito, desenvolvimento do pensamento teórico e conteúdo escolar. In: MOURA, M. O. (org.). *Educação Escolar e Pesquisa na Teoria Histórico-Cultural*. São Paulo: Edições Loyla, 2017. p. 125-152.

RADFORD, L. *Cognição matemática: história, antropologia e epistemologia*. São Paulo: Livraria da Física, 2011.

RADFORD, L. Elementos de una teoría cultural de la objetivación. *Revista Latinoamericana de Investigación en Matemática Educativa*, núm. Esp., p. 103-129, 2006.

SAITO, F.; DIAS, M. S. Interface entre história da matemática e ensino: uma atividade desenvolvida com base num documento do século XVI. *Ciência & Educação (Bauru)*, [S.L.], v. 19, n. 1, p. 89-111, 2013. DOI: <http://dx.doi.org/10.1590/s1516-73132013000100007>.

SILVA, M. M. *A apropriação dos aspectos constituintes da Atividade Pedagógica por professores de Matemática em formação inicial*. Tese (Doutorado em Educação Ciências e Matemática). Universidade Federal de Goiás. Goiânia, 2018.

SILVA, M. M. *Clube de Matemática: espaço de formação docente e produção compartilhada do ensino e da aprendizagem de conceitos matemáticos*. Curitiba: CRV, 2022.

SOUSA, M. C. *O ensino de álgebra numa perspectiva lógico-histórica: um estudo das elaborações correlatas de professores do ensino fundamental*. Tese (Doutorado em Educação) Universidade Estadual de Campinas. Campinas, 2004

SOUSA, M.C. O movimento lógico-histórico enquanto perspectiva didática para o ensino de matemática. *Revista Obutchénie*, v. 2, n. 1, p. 40-68, 30 ago. 2018. DOI: <http://dx.doi.org/10.14393/obv2n1a2018-3>.

SOUSA, M. C.; PANOSSIAN, M. L.; CEDRO, W. L. *Do movimento lógico e histórico à organização do ensino: o percurso dos conceitos algébricos*. Campinas: Mercado das Letras, 2014.

VIGOTSKI, L. S. *Obras Escogidas*. (Tomo V). Madrid: Visor, 1997.

VIGOTSKI, L. S. *Psirrologuia Iskusstva*. Minsk: Sovremennoie Slovo, 1998.

Received in February 2024

Approved in April 2024