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Logical-historical movement of the concepts of Area and Perimeter[1]

Movimento lógico-histórico dos conceitos de Área e Perímetro

Bruno Tizzo Borba[2]
Fabiana Fiorezi de Marco[3]

ABSTRACT
In this article, an excerpt from a thesis, we aim to present a study of the logical-historical movement of the concepts of area and perimeter, highlighting the conceptual links evidenced in the historiographies studied. To this end, we searched the literature for authors who are based on the assumptions of Historical-Dialetic Materialism, as well as the needs of different groups and their diverse cultural practices. The results indicate that the concepts of area and perimeter are broader than the external connections treated in Basic Education schools, such as the calculation of perimeter and area through formulas and representations of geometric shapes.

Keywords: Logical-historical movement; Area and Perimeter; Mathematics Education.

RESUMO
Neste artigo, recorte de uma tese, temos como objetivo apresentar um estudo do movimento lógico-histórico dos conceitos de área e perímetro, destacando-se os nexos conceituais evidenciados em historiografias estudadas. Para tanto, buscou-se na literatura autores que se fundamentam nos pressupostos do Materialismo Histórico-Dialético, bem como necessidades de grupos distintos e suas diversas práticas culturais. Os resultados indicam que os conceitos de área e perímetro são mais amplos do que os nexos externos tratados nas escolas da Educação Básica, como o cálculo de perímetro e de área por meio de fórmulas e representações das formas geométricas.

Palavras-chave: Movimento lógico-histórico; Área e Perímetro; Educação Matemática.

1 Introduction

This text is an excerpt from Borba's dissertation (2023) and aims to present a study of the logical-historical movement of the concepts of area and perimeter, highlighting the conceptual links that appear in the historiographies studied.

It is based on the assumption that cultural practices become entangled with the history of mathematics not only because strategies and concepts were formulated by humans, but also because they emerged from real-world situations and sought to meet the needs of different groups at a particular time.

[1] English version by Silvia Iacovacci. E-mail: siacovacci@gmail.com.
[2] Doutor em Educação pelo Programa de Pós-Graduação em Educação da Universidade Federal de Uberlândia. Orcid: https://orcid.org/0000-0002-5718-6366. E-mail: brunotborba@ufu.br.
[3] Pós-doutora em Educação pela Universidade de São Paulo. Docente Associada da Universidade Federal de Uberlândia. Orcid: https://orcid.org/0000-0002-7126-5626. E-mail: fabiana.marco@ufu.br.

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In order to understand the development of the mathematical knowledge of area and perimeter, a brief contextualization of the logical-historical movement of these concepts is presented within the process of organizing the teaching of geometry, and it is assumed that "[...] education is the process of transmitting and assimilating historically produced culture, through which individuals humanize themselves, inheriting the culture of humanity" (RIGON; ASBAHR; MORETTI, 2016, p. 31).

In the quest to understand the origin of these concepts, we sought to "[...] explain the need that led humanity to construct the concept in question, how human problems and needs arose in a given activity, and how people worked out the solutions or synthesis in their logical-historical movement" (MOURA et al., 2010, p. 104).

In this way, a study of the logical-historical movement was carried out to understand that different civilizations contributed to the formation of the mathematical concepts of area and perimeter, as well as to understand their conceptual connections.

According to Panossian, Moretti and Souza (2017, p. 139),

> Taking the historical and logical movement of concepts as a point of analysis, we understand that this allows us to identify essential elements inherent to a certain form of knowledge, thus constituting a "teaching object". This "teaching object", in turn, can and should be present in various "teaching contents" or "teaching topics" in the school curriculum organization.

The logic and history of a teaching object "[...] help us to understand the movement of objects and phenomena of objective reality at various historical moments in different civilizations" (PANOSSIAN; TOCHA, 2020, p. 73), recognizing the needs that triggered that form of knowledge in the historical movement of human experience.

Using Kopnin (1978), we find that

> Historical means the process of change of the object, the stages of its emergence and development. History acts as the object of thought, the reflection of history, and as content. Thought aims to reproduce the real historical process in all its objectivity,

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> complexity, and contrariness. Logic is the means by which thought accomplishes this task, but it is the reflection of history in theoretical form; in other words, it is the reproduction of the essence of the object and the history of its development in the system of abstractions. History is primary in relation to logic; logic reflects the main periods of history (KOPNIN, 1978, p. 183-184).

Therefore, the historical is understood as the process that analyzes an object of teaching and its transformations throughout the history of various social groups, and the logical is understood as the relationships between the knowledge produced (PANOSSIAN; TOCHA, 2020). To better understand the movement that interrelates logic and history, Kopnin (1978) claims that

> The problem of the interrelation of logic and history has many aspects and is not limited to the interrelation of the theory of the object and its history. Logic reflects not only the history of the object itself, but also the history of its knowledge. That is why the unity of logic and history is a necessary premise for understanding the process of the movement of thought, the creation of scientific theory (KOPNIN, 1978, p. 186).

For example, Borba (2023) thought of the logical and historical unit to develop a Triggering Learning Situation using the concepts of area and perimeter as a teaching object. It was through this logical-historical movement that the historical formation of these concepts was understood, as well as the conceptual links needed to think about Triggering Learning Situations and, consequently, a triggering problem that addressed these elements.

Regarding conceptual links, Sousa (2018, p. 51) defines them as "[...] the links between ways of thinking about a concept that does not necessarily coincide with the different languages that represent the mathematical concept. These links, in other words, the conceptual nexuses "[...] that underlie concepts contain the logic, the history, the abstractions, the formalizations of human thought in the process of becoming human through knowledge" (p. 50).

According to Moura et al. (2010), the genesis of the concept, i.e., the essence of a concept, lies in the conceptual connections necessary for the development of a learning triggering situation. Knowledge of the logical-historical movement is important "[...] to have access to the history of the knowledge of mathematical concepts, elaborated by different civilizations and

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narrated in different versions, from different points of view, in the historiographies of mathematics" (SOUSA; MOURA, 2019, p. 1083), so that we can identify the needs of some people in a certain activity that led to the construction of the concept, as well as understand the elaboration of solutions or syntheses of problems that appeared in certain social contexts.

Borba (2023), based on Sousa (2018), who sought to distinguish between internal and external conceptual links according to Davydov (1982) and Kopnin (1978), organized the characterization shown in Chart 1.

Chart 1 - Characterization of the Internal and External Conceptual Nexuses

Internal links	External links
- "[...] they arise in theoretical thought" (SOUSA, 2018, p. 50). - "[...] they make up the logical-historical movement of the concept" (SOUSA, 2018, p. 50). - "[...] they mobilize the student's movement more than external links" (SOUSA, 2018, p. 51).	- "[...] they are limited to the visible elements of the concept" (SOUSA, 2018, p. 50). - "[...] they are based on language. They are formal" (SOUSA, 2018, p. 50). - "[...] they are still a language of communication of the concept presented in its formal state, but they do not necessarily denote its history. They give little mobility to the subject to elaborate the concept" (SOUSA, 2018, p. 51).

Source: Borba (2023, p. 93)

Considering the above, we understand the need to go beyond the external nexuses of mathematical concepts that are

> [...] related to formal language because they are clean, devoid of contradictions, cultural and social practices present in the history of concepts. The external links are made explicit in the classroom, completely disconnected from the various areas of knowledge in the symbolic aspect. It's as if the symbols had a life of their own; they speak for themselves. Priority is given to the form of concepts. Form and content are disconnected (SOUSA, 2018, p. 41-42).

Therefore, in the quest to overcome the superficiality of the concepts of area and perimeter, their logical-historical movement is presented.

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2 Logical-Historical Movement of the concepts of Area and Perimeter

When thinking about human beings at the dawn of civilization, mathematicians believed that they had notions of number, magnitude and form, as there were assumptions that Stone Age men counted, measured and drew (BOYER, 1974).

However, Boyer (1974) states that neither Herodotus nor Aristotle wanted to risk the origin of mathematics, as they did not have sufficient evidence of origins before Egyptian civilization to prove such arguments, since many historical records have been lost. "For information about prehistory, we depend on interpretations based on the few artifacts that remain, on evidence provided by modern anthropology, and on retroactive, conjectural extrapolation from the documents that have survived" (BOYER, 1974, p. 4).

In this way, researchers such as Eves (2011) and Boyer (1974) have taken the Egyptians' and Babylonians' need for measurement as a more reliable historical moment to relate the constitution of the concept of area. According to the authors, in the period of Ancient Egypt, there are records of the determination of the measurement of area because, during the period of flooding of the River Nile, they needed to recalculate the portions of land administered when part of the land was flooded and the tax levied by the Pharaoh needed to be updated according to the calculation of the new area (CARAÇA, 1951).

For Herodotus (5th century BC), a Greek historian,

> [...] the need for the measure arose from the occurrence of several floods along the River Nile. The land cultivated by the farmers (landowners) was flooded at the end of each flood. They would demarcate them with ropes, hence the expression rope-drawers (ropes were used both to lay out the foundations of temples and to demarcate land). Farmers paid taxes to the king, so at the end of each flood it was necessary to ask for a tax reduction, proportional to the amount of land lost to the waters (CHIUMMO, 1998, p. 13).

According to Ríbnikov (1987), as a result of the long historical development of people's daily practical activities, the mathematical concept of area and other concepts that have abstract spatial properties of objects were formed.

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Silva (2010) describes that divergent sources are unanimous in accepting that the unit of length was based on the size of the human forearm and the distance between the knots in the rope was the size of the cubit[4]. Since the measurement of the cubit varies from person to person, they used the actual cubit as a standard, which was about 50 cm for the ancient Egyptians and about 45 cm for other peoples, such as the Romans.

It is important to note the attempt to standardize the unit of measurement by using the measurements of a single man, the ruler, in Egypt's case the Pharaoh. However, with the arrival of a new ruler, these units had to be updated and the areas resized.

According to Chiummo (1998), much of the documentary information that exists today on geometry comes from the Egyptian papyri, one of the most famous being the Rhind papyrus, which "[...] devotes twenty (20) exercises to the areas of fields and the volume of granaries, the others relate to operations with fractions, the rule of three or false positions. As far as we know, it's a schoolbook or an 'almanac' for farmers" (CHIUMMO, 1998, p. 15).

This suggests that, in this historical context, the Egyptians needed to calculate planting areas in the Nile Valley, since its floods and ebbs were conducive to agriculture. The same happened on the banks of the Tigris and Euphrates in the Middle East, the Yellow River in China, and the Indus River in India, since these were strategically chosen places for the settlement of communities (CARAÇA, 1951).

The units of human body parts, known as anthropometry, were widely used and remain a cultural heritage. Sarmento (2019) reports that in 1850, a clay tablet from about 2500 BC was found in present-day Iraq with a table of measurements whose basic units were the span, the cubit, the thumb, and the line as a multiple of the span. They "[...] did not indicate the size of the span. It was only a few years later, in the same region, in 2175 B.C., that it was established that the span unit was equivalent to about 9.30 cm" (SILVA, 2010, p. 41).

[4] In some cases, the cubit is adopted as the length from the tip of the middle finger to the elbow and, in others, from the tip of the closed fist to the elbow (Silva, 2010).

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The Babylonians were another people who possessed certain mathematical knowledge for their context and historical period, as they accurately clarified the concept of area, which can be seen in

> [...] through research dating back to the middle of the last century, from the discovery of cuneiform tablets from the Old Babylonian era of the Hammurabi Dynasty, after they were unearthed and deciphered (1800 to 1600 BC). But in both the Babylonian and Egyptian documents we find no evidence of mathematical demonstration (CHIUMMO, 1998, p. 15).

According to Eves (2011), the Greeks sought to appropriate the scientific knowledge of the Babylonians and Egyptians, expressing their respect for this knowledge available to anyone who could travel to Babylon and Egypt.

However, not even Euclid (323 and 283 BC) was able to define the concept of area in his famous work Elements because "[...] for him, two figures are called equal when they have the same length, if they are segments, and the same area, if they are plane figures, and the same volume, if they are solids. As irrational numbers were unknown, line segments were not measured. Two line segments were compared using the ratio between them, but these ratios between magnitudes were not considered numbers." (CHIUMMO, 1998, p. 22).

In 1991, Elon Lages de Lima described in "Measures and Form in Geometry" that

> [...] for Euclid, the coincidence of two plane figures by superposition was an intermediate step to infer the equality of their areas" (in fact, Axiom 4 of the Elements says: "Two figures that coincide by superposition are equal"). Thus, it was important for him to have criteria that ensured the superposition of, for example, two triangles. (The 3 known cases of equality of triangles)" (CHIUMMO, 1998, p. 22).

It is worth noting that this is a restricted conception of area, as there are figures with the same area measurement that are not superimposed due to their different shapes.

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Silva (2010) reports that the system of unification imposed by the Roman emperor Charlemagne, in the year 809, was more successful and became known to historians because it lasted until the division of the Carolingian Empire. This was because emperors sought to impose their units of measurement on the entire empire.

> An interesting example was the standard unit of length proposed by Charlemagne. At the time, he wanted to establish a universal standard and proposed adopting the "Foot of Christ", or more precisely, the marks of Christ's foot engraved on the Holy Shroud, as the standard unit of length. The value corresponds to 31.23 cm. It is not known exactly why, but another standard was eventually adopted. At the time, it was decided that the standard unit of length to be adopted throughout his kingdom would be the "King's Foot", which corresponded to 32.84 cm (SILVA, 2010, p. 65).

Furthermore, to establish the standard unit, in 1189, at his coronation, King Richard the Lionheart (reigned 1189 to 1199) ordered the unification of units throughout his kingdom, thus manifesting "[...] his power in three different ways: he owned the measurement standards; he had the power to control existing copies; and he had the power to punish forgeries" (SILVA, 2010, p. 31).

It can be seen that in the political context it was a determining factor to be able to control the system of measures, which occurred in different civilizations, such as the feudal lords who maintained control to maintain sovereignty, the Greeks who imposed their units of measurement on conquered cities, the Athenians who dedicated the standards to the gods, and the Romans who maintained local units to avoid revolt. However, they always served socio-political interests, contributing to the local economy and, in some cases, to the world stage (SILVA, 2010).

Nevertheless, to follow the historical movement of some civilizations, it is worth noting that with the growth of trade, complications arose due to the diversity of units of measurement, as each merchant wanted to use the units of measurement of his region or country (MACHADO, 1998).

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There were attempts by governments to standardize measurements through state reforms, which took place from the 13th century onwards, with the aim of collecting taxes and providing a form of control, but they proved to be "[...] inefficient, whether due to popular resistance, political issues or the lack of objective conditions" (SARMENTO, 2019, p. 91). Thus, "[...] all sovereign governments tried, in one way or another, to unify the units of measurement prevalent in the regions under their rule, but the definitive milestone for weights and measures was the creation of the metric system" (SILVA, 2010, pp. 15-16).

Throughout the Middle Ages, according to Sarmento (2019), until the introduction of the metric system, measurements were made of large or small distances, the area of cultivated regions, and the volume of grain per cultivated area.

> The units of measurement, general definitions and measuring techniques varied greatly from one region to another. In Poland, for example, at the beginning of the 17th century, as in almost all of Christian Europe, the arpent (France), Journou (Burgundy), Journal à charrú, Journal à foucher (Brittany) were used, all equivalent to the area of land that two good working oxen could plow in one day. In Germany, the area plowed by a day's work was called Morgeland (SARMENTO, 2019, p. 83).

As a result, Spain made at least five attempts at unification, the first being in 1261. According to Sarmento (2019), the unification movements unfolded in three phases: Carolingian, Renaissance, and Enlightenment. However, the greatest unification movement occurred at the beginning of capitalism, in the context of the French Revolution in 1791 and established in 1799, with the implementation of the decimal metric system (SARMENTO, 2019; SILVA, 2010).

The creation of the metric system, which for many authors divided metrology into pre-metric and post-metric (SILVA, 2010), took place with the aim of establishing standard units and the socio-political interests of the time. For example, in 1790, during the French Revolution, the French Academy of Sciences was tasked with proposing a completely new and unified system for all of France that defined the meter as the standard unit of length.

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> The Academy then decided to adopt the fraction of a meridian arc. It was decided that the tenth millionth part of the fourth part of a terrestrial meridian arc, measured between the Equator and the North Pole, would be adopted as the unit of linear measurement, called a meter. It was then established that a meridian arc of approximately 9.5 degrees would be measured between Dunkerque in France and Barcelona in Spain, and the result of this measurement would be used as the standard unit for the new unit of linear measurement (SILVA, 2010, p. 83).

The word meter comes from the Greek métron, which means "measure", and this proposal adopted the decimal numbering system for the system of measures (Silva, 2010). As the years went by and the precision of the metrology of the unit of length increased, due to the purposes of social groups in certain historical contexts, human beings had to think about how to improve this accuracy.

> The need arose to establish a unit of length that was invariable and taken from nature, i.e., a natural constant that could be reproduced at any time and, above all, that was independent of human influence [...] [A new unit of length was created] in 1960, defining it as a multiple of the wavelength of the orange ray of krypton-86 measured in a vacuum (SILVA, 2010, p. 107).

The first transition from the unit to a physical phenomenon was the use of the wavelength, but with the discovery of the atomic clock and the discovery of the laser, it was possible to improve the accuracy of measuring the speed of light which, according to Silva (2010, p. 108), "[...] is a function of the length unit (wavelength of electromagnetic radiation). Based on these new discoveries, it was established that the speed of light is 299792458 m/s".

Thus, in 1983, the chemical element krypton was replaced by any electromagnetic radiation of a stable and well-known frequency, serving as the basis for establishing the unit of length, defined as: "The base unit 1 meter is the length of the path traveled by light in a vacuum for 1/299792458 second" (Silva, 2010, p. 107).

In summary, the "[...] Decimal Metric System was duly instituted in France in 1840, and in 1875 it was adopted as an international system of measures. Since then, in 1927, 1937, 1960, 1971, and 1983, various definitions have been incorporated and others modified" (SILVA, 2010, p. 27).

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Sarmento (2019) also reports some reasons for the unification and stabilization of measures for some social segments that were served, each with their reasons:

> [...] a) the rejection by the general population and merchants of the privileges of the nobility regarding the control of measurements; b) the interests of the emerging bourgeoisie linked to commercial expansion; and c) the interests of the scientific communities in the development of science and mathematics. These points were decisive for the unification and universalization of the measurements of the decimal metric system (SARMENTO, 2019, p. 91).

Even so, it wasn't easy and quick to implement this system, as the legal requirement wasn't enough to get people on board (SARMENTO, 2019). As a result, some difficulties stood out:

> [... a) the creation of a single measure depended on social, political, and scientific conditions; b) the various attempts to unify measures, even for a small geographical area such as a village, failed, causing popular disbelief in new attempts; c) popular rejection, motivated by distrust of the nobility and the low level of education (almost all illiterate), which made it difficult to understand the proposals; d) the population was used to the prevailing standards, so breaking with this custom required a drastic change in the way of thinking about the measures, and such a change could not happen overnight. It took a long time for this process to be completed; e) popular reprisals that used the proposed changes to the system of measures to obtain currency for other interests that generated many popular conflicts, for example, the end of slavery; f) the mixed use of traditional measures with the French decimal metric system: the population used the new measures in official business but continued to use traditional measures, especially in agrarian situations; g) the issue of small traders (SARMENTO, 2019, p. 99).

It is important to remember that although many countries consider it an international system, the Anglo-Saxon countries and their colonies do not accept it as the standard for the units of the metric system. For a long time,

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France and England[5] were rivals in commercial interests, so it was inconceivable to adopt a system created by the French. This political control meant that other countries, such as the United States, although it adopted the International System, also maintained a parallel system of measures (SILVA, 2010). Therefore,

> [Some countries were slow to adopt this system, such as the former USSR, which joined in 1918. Before that, Russia had tried to unify its measures with the reforms carried out by Tsar Ivan the Terrible. However, the traditional measures remained quite strong, so these reforms did not achieve their objectives. In 1958, Japan adopted the French metrology (SARMENTO, 2019, p. 91).

There are still several Brazilian localities that use units apart from the metric system, such as inches to measure a television, palms to buy rope, feet to measure height and yard for distances (equivalent to 5.5km in Brazil). When it comes to measuring rural areas, the following units are used: hectare (10000m^2), are (100m^2) and the hectare (the São Paulo hectare = 24200m^2, the Minas Gerais and Goiás hectare = 48400m^2, and the Northern hectare = 27225m^2) (SILVA, 2010). In England and the United States, the Acre (4047m^2) is commonly used as an agrarian measure.

Sarmento (2019) endorses that even today, there are many peasants in Africa, Asia, and South America who have this way of life in which farming means opening up an area in a forest region, digging it up, and then harvesting it.

As is the case elsewhere in the world, in colonial Brazil the standards of measurement were not uniform. However, the Brazilian bourgeoisie of the 19th century felt the need to standardize measurement standards, with the growth of international trade (SARMENTO, 2019).

Under the influence of the United States' proposal, in 1834, units were linked to the metric system and the first Brazilian proposal to create a system of measurement appeared, taking as its starting point the creation of the metric system.

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[5] In the UK today, there is still a mixture of the metric system and the so-called imperial system: miles are used for distances between cities and the metric system for length and height.

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> [...] the fundamental unit for length is the rod, equivalent to eleven tenths of the French meter, i.e., 1/36363636 of the terrestrial meridian, followed by the fathom (2 rods), the span (1/5 of the rod) and the inch (1/8 of the span), completed by the mile and the league (6 km), used to measure long distances. [...] and the bushel (0.1 [vara]3 * 27.25 = 1774 [pol.]3, i.e., just over 38 liters). The quarta (0.25 hectare) was used as a submultiple of the hectare (DIAS, 1988, apud SARMENTO, 2019, p. 103).

In the midst of independence, there was already a debate about which French system to adopt. Thus, "[...] the need to exercise state control over measurements and pressure from sectors of the national and international economy led the government of Pedro II to gradually implement the metric system" (SARMENTO, 2019, p. 104) in Imperial Brazil. However, this transition was never complete, as the use of pre-metric units is still common today, especially in small Brazilian cities.

In 1921, Brazil joined the metric convention, and in the 1930s, with the creation of the National Institute of Technology and the restructuring of the system of measures under Getúlio Vargas, the use of standardized measures of the decimal metric system began (SARMENTO, 2019).

Thus, Caraça (1951) highlights some ancient and current social and economic needs to obtain a number as a result of a measurement, citing as an example the circumstances in which a person owns land and needs to carefully determine its areas:

> a) In all economic relationships between the owner and the land - to calculate the amount of seed to sow, the time the land takes to cultivate, etc., it is necessary to know its area.
> b) In relations from individual to individual, based on the land owned - any contract of sale involving land requires, among other things, as approximate a determination as possible of its area.
> c) In relations between the individual and the State, based on the land owned - the tax depends, as we know, on the area of the property as well as other elements. (CARAÇA, 1951, p. 31).

Silva (2010, p. 21) also talks about these social and cultural relationships that determine the use of measurements; since the first

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civilizations, they have always been a basic language of commerce and "[...] one of the main factors that have sustained societies through the establishment of exchange relationships in commerce, standardization to measure production and dimensional support for science and technology", even though there was chaos due to the variety of standards.

The author also points out that the social justice aspect of measurements "[...] is evident when we see that, at all times, the distribution of production has been based on measurements" (SILVA, 2010, p. 21) because

> [...] Measurement systems have always been considered a determinant of social justice and, in a sense, a criterion of civilization. The invaders of Latin America, for example, described the culture of the Mayans and Aztecs with admiration, saying: "They are a kind of people who live with justice, weights, and measures. The same thing happened with the invaders of Africa, who were often amazed at the perfection of the systems of weights and measures found in civilizations that were considered absolutely primitive (SILVA, 2010, p. 21).

This perception of social justice has always been important, both in the sense of the law and in the sense of faith, as it is still a crime to falsify a measure, and the falsification of weights and measures has always been punished rigorously; in the Bible itself, there are quotes about justice when measuring (Silva, 2010).

In this way, measuring methods also have a profound social significance, and human labor itself was a form of comparison to define a certain agricultural area (SARMENTO, 2019).

> The small differences between units didn't matter much because there were social ways to compensate. For example, a compensatory surplus was added to the calculated measure of grain. It is important to note that this type of measurement was associated with each person's daily life and therefore had a personal meaning for each of them. Thus, they were not only related to the amount of land, but also to the amount of land that could be worked, the amount of fruit harvested, and the amount of work a person did (SARMENTO, 2019, p. 83).

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It is worth noting that production, as an intermediate stage between labor and commerce, also turned out to be an activity directly influenced by metrology, so that with the Industrial Revolution, "[...] it was production that began to have its own metrological needs and to demand standards and units of measurement different from those required by commerce. Without being too strict, it could even be said that it was production that established the second stage in the evolution of metrology" (SILVA, 2010, p. 24).

Science is also another branch of human activity directly related to metrology, as measurements are "[...] the dimensional support, the guiding thread of experimental science and the normative basis for technical applications. Much scientific knowledge has been lost over time due to the simple impossibility of being able to interpret it accurately" (SILVA, 2010, p. 24); over time, acting as a watershed, the system of measurements became part of more general definitions than the action of measuring, allowing for more consistent results.

Soon, with the advancement of science, the concept of measurement was completely transformed, and its meaning

> [...] it acquired a scientific connotation, progressively moving from the quality of an empirical concept, based on everyday experiences, to become a scientific concept, developed rationally as a result of abstract thought and represented by the set of signs and symbols specific to mathematical language, present in geometry, arithmetic, and algebra. However, the evolution of measurement systems occurred in accordance with the social, political, and economic conditions of each region and resulted from the contradictions and struggles of classes with different interests (SARMENTO, 2019, p. 67).

Like science, technology is also responsible for improving metrology. According to Silva (2010, p. 27), "[...] in order to move forward, they needed coherent measurement systems and increasingly precise results, but at the same time they created the foundations for achieving these conditions", establishing standards, making changes and creating new measurement systems, proving that there have been changes in metrology up to the current standards.

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Today, there are still many needs to "[...] use the area in activities such as: designing a building, covering the rooms of a house, informing the size of an agricultural property, planning the use of a certain surface such as land, fabrics, papers, among other uses" (MUNHOZ et al., 2021, p. 686).

Thus, the emergence of measurements reveals the social value attributed to them and their representations (Sarmento, 2019).

> [...] well-defined social meanings and representations. They are part of a symbolic system that regulates social practices and defines, in addition to behaviors and values, the size of units, instruments, signs, measurement techniques, personal meanings and territorial differences, the latter evidenced in multiple forms of transformation over time and in relation to the general history of humanity (SARMENTO, 2019, p. 61).

In this sense, Borba (2023) takes up some aspects experienced in some social practices as a support for the formation of senses and the constitution of meanings of the knowledge of area, and can answer some whys in order to instigate a more critical look at the objects of knowledge.

The presentation of this panorama of the arrival of the international metric system, coming from Europe, does not deny that the movement of formation of the concepts of area and perimeter also permeates the peoples of Africa and America, the rural population, and professionals in different functions, in different social classes, with struggles of interests and rivalries of power.

This perception is in line with Ríbnikov (1987), when he describes that the development of mathematics does not take place in a harmonious process of continuous and gradual development of its truths, since

> [...] the history of knowledge of mathematical concepts only makes sense when we understand the different movements of abstractions of thought that made up the formalizations we study, which in turn make up the conceptual links (internal and external) of the concepts studied in the classroom (SOUSA; MOURA, 2019, p. 1082).

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The above shows that there are different ways of seeing and conceiving mathematics, the concept of measurement and, consequently, the teaching of mathematical concepts. In this sense, Borba (2023), anchored in Sousa and Moura (2019), points out that, despite these differences, the historiographical authors used in this research have at least one aspect in common: they conceive and view history from the Enlightenment paradigm.

Ríbnikov (1987), who describes himself as a dialectical materialist, has in his historiography an explicitly Marxist conception of the history of mathematics and is based on the assumptions of historical-dialectical materialism, whose understanding of the mathematical object and knowledge of its history is a necessary condition for understanding the place of this science in productive and social activity (Sousa; Moura, 2019).

Eves (2011), "[...] seeks to emphasize the evolution of mathematical ideas developed over centuries. In this sense, he argues that it is necessary to know the cultural panorama of each group that contributed to the development of mathematical concepts" (SOUSA; MOURA, 2019, p. 1092). In his texts, Boyer (1974) is concerned with historical chronology and mathematical rigor in mathematical definitions and demonstrations, focusing more on higher education but without delving into all subjects.

Caraça (1951), another author of historical-dialectical materialism, "[...] argues that scientific knowledge, as a living organism, including mathematical knowledge, can free people from the shackles of oppression" (SOUSA; MOURA, 2019, p. 1095). The author Silva (2010) considers it essential in his work to focus more on the evolution of units and standards over the centuries than on the unit values of measurements, relating them to their effects in specific historical contexts of different peoples.

With these considerations, we can believe that it is undeniable that the mathematical concepts of

> [...] they are constantly being reworked in different civilizations, at different times and in different contexts, as new needs arise and force us to take a different look at the fluid and interdependent

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> reality that surrounds us. Here, reality is broad and considers the movements of life and the scientific ones that are also part of our lives (SOUSA; MOURA, 2019, p. 1097).

Therefore, the aim of this study was not to exhaust the process experienced by all the civilizations and their different historical contexts that made use of the concepts of area and perimeter.

Since each of the authors cited perceives these relationships in a different way, it was necessary for this study to carry out a didactic treatment that would make it possible to establish conceptual links between the concepts studied, since each historiography indicates a part of the conceptual links evidenced in the logical-historical movement of the concepts of area and perimeter.

Thus, given the contextualization of the formation of the concepts of area and perimeter, Borba (2023) created a teaching situation from the perspective of the learning trigger situation for his empirical research with EJA students, based on the conceptual links described and evidenced in the logical-historical movement.

With regard to Triggering Learning Situations, they can "[...] create situations in which the student feels the need to appropriate knowledge that has been historically developed by humanity". (SILVA et. al., 2022, p. 56). To do this, "[...] a problem must be proposed that is capable of mobilizing the individual or the collective to solve it. This awareness of the problem is what allows the subject to enter into a learning activity that requires him to take coordinated action towards the goal he has become aware of" (MOURA; ARAÚJO; SERRÃO, 2019, p. 422).

Figure 2 shows the organized moments of the teaching situation and the conceptual links defined by the logical-historical movement of the concepts of area and perimeter.

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Chart 2 - Moments of the Area and Perimeter Teaching Situation and the internal and external conceptual links established from the Logical-Historical Movement

MOMENTS/ACTIONS	INTERNAL AND EXTERNAL LINKS
MOMENT 1: Demonstration at Uberlândia City Hall	● Non-standardized unit of measurement (realizing that you need something to measure the surface and its contour in order to create them). ● Comparing a unit with the surface and a unit with the contour.
MOMENT 2: Standardization of units of measurement	● Continuous and discrete quantities (because there is space left on the surface that has not been measured); ● Standard unit (to facilitate communication and local and cultural characteristics); ● Variety of units of measurement (created from objects in nature, created objects (rope) and anthropometric units).
MOMENT 3: Measure the delimited surface and its contour using the standardized units, without defining the concepts.	● Preferred shape and size of unit; ● Non-conventional standardized units (wood and squares - for all students to use); ● Regularity (rectangle area formula).
MOMENT 4: Correlate the standardized units with the square meter.	● Conventional units (meter and square meter); ● Numerical relationships (between the meter and standardized units in the proposal; ● Numerical relationships (between footage and the number of people).
MOMENT 5: Conceptualize area and perimeter.	● Area and Perimeter.

Source: Adapted from Borba (2023, p.108-109).

Through the studies carried out and the teaching situation organized, we can see how essential this movement is to understand the essence of these concepts and developing/proposing situations from the perspective of the Learning Triggering Situation. This can have an impact on the appropriation of concepts (area and perimeter, in this study) that are not restricted to perceptible elements of everyday life.

3 Final considerations

Summarizing the arguments of the logical-historical movement, it can be concluded, in the light of the authors studied, that the concepts of area and perimeter are broader than the external links dealt with in elementary schools, such as the calculation of perimeter and area using formulas and the representation of geometric shapes with the measurement of their sides.

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In this sense, it can be said that the external links, although also historically constructed, are limited to the perceptible elements of the concept, disconnected from the different domains of knowledge and only observing its symbolic aspect (SOUSA, 2018).

It is worth noting that we do not believe that there are only these conceptual links between the concepts of area and perimeter. However, the historiographies studied and used in the context presented here made it possible to perceive and establish the structuring elements of these concepts, as described in Figure 2.

As mentioned above, the aim was not to exhaust the study of all the civilizations and the different historical contexts that have used the concepts of area and perimeter, not least because the search for references has shown that there is much to be done in the historiographical sense.

Movimiento lógico-histórico de los conceptos de Área y Perímetro

RESUMEN
En este artículo, extracto de una tesis, pretendemos presentar un estudio del movimiento lógico-histórico de los conceptos de área y perímetro, destacando los vínculos conceptuales evidenciados en las historiografías estudiadas. Con este fin, buscamos en la literatura autores que se basen en los supuestos del Materialismo Histórico-Dialético, así como en las necesidades de diferentes grupos y sus diversas prácticas culturales. Los resultados indican que los conceptos de área y perímetro son más amplios que las conexiones externas tratadas en las escuelas de Educación Básica, como es el cálculo de perímetro y área a través de fórmulas y representaciones de formas geométricas.

Palabras clave: Movimiento lógico-histórico; Área y Perímetro; Educación Matemática.

4 References

BORBA, B. T. *Situações desencadeadoras de aprendizagem de área na EJA na perspectiva da Teoria Histórico-Cultural com o uso de tecnologias digitais.* 2023. 193 f. Tese (Doutorado em Educação) - Universidade Federal de Uberlândia, Uberlândia, 2023. DOI: http://doi.org/10.14393/ufu.te.2023.7053.

BOYER, C. B. *História da Matemática*. Tradução de Elza Gomide. São Paulo: Editora da Universidade de São Paulo, 1974.

CARAÇA, B. J. *Conceitos fundamentais da matemática*. Lisboa: Tipografia Matemática, 1951.

DOI: http://doi.org/10.14393/OBv8.e2024-38

CHIUMMO, A. *O conceito de Áreas de figuras planas*: capacitação para professores do ensino fundamental. 1998. 181f. Dissertação (Mestrado em Ensino da Matemática) - Pontífice Universidade Católica de São Paulo, São Paulo, 1998.

DAVYDOV, V. V. *Tipos de generalización en la enseñanza*. Ciudad de La Habana: Pueblo y Educación, 1982.

EVES, H. *Introdução à história da Matemática*. Tradução de Hygino Domingues. 5. ed. Campinas: Editora da Unicamp, 2011.

KOPNIN, P. V. *A Dialética como Lógica e Teoria do Conhecimento*. Rio de Janeiro: Civilização Brasileira, 1978.

MACHADO, N. J. *Vivendo a Matemática*: Medindo Comprimentos. São Paulo: Scipione, 1998.

MOURA, M. O. *et al*. A atividade Orientadora de Ensino como Unidade entre Ensino e Aprendizagem. In: MOURA, M. O. (org.). *A atividade pedagógica na Teoria Histórico-Cultural*. Brasília: Líber livro, 2010. p. 81-110.

MOURA, M. O.; ARAÚJO, E. S.; SERRÃO, M. I. B. *Atividade Orientadora de Ensino*: fundamentos. Linhas Críticas, Brasília, DF, v. 24, s/n, e19817, 2019. DOI: https://doi.org/10.26512/lc.v24i0.19817.

MUNHOZ, A. P. G. *et al*. Aportes da Teoria Histórico-Cultural em atividade de ensino de medidas de tempo e área. In: NAVARRO, E. R.; SOUSA, M. C. (org.). *Educação Matemática em Pesquisa*: Perspectivas e Tendências. Guarujá: Científica Digital, 2021. v. 1. p. 678-700. DOI: http://doi.org/10.37885/201102307.

PANOSSIAN, M. L.; MORETTI, V. D.; SOUZA, F. D. Relações entre Movimento Histórico e Lógico de um Conceito, Desenvolvimento do Pensamento Teórico e Conteúdo Escolar. In: MOURA, M. O. (org.) *Educação Escolar e Pesquisa na Teoria Histórico-Cultural*. São Paulo: Loyola, 2017. p. 125-152.

PANOSSIAN, M. L.; TOCHA, N. N. (org.). *Estabelecendo Parâmetros de Análise de Situações de Ensino de Conteúdo Matemático*: aproximações a partir da Atividade Orientadora de Ensino. Curitiba: Oficina Pedagógica de Matemática UTFPR, 2020.

RÍBNIKOV, K. *Historia de las matematicas*. Traducido del Ruso por Concepción Valdés Castro. Musco: Mir, 1987.

RIGON, A. J.; ASBAHR, F. S. F.; MORETTI, V. D. Sobre o processo de humanização. In: MOURA, M. O. (org.). *A atividade pedagógica na Teoria Histórico Cultural*. 2. ed. Araraquara: Autores Associados, 2016. p. 22-53.

DOI: http://doi.org/10.14393/OBv8.e2024-38

SARMENTO, A. K. C. *O desenvolvimento lógico-histórico do conceito de medida e o processo de significação na atividade pedagógica*. 2019. 301f. Tese (Doutorado em Educação) – Faculdade de Educação, Universidade de São Paulo, 2019. DOI: https://doi.org/10.11606/T.48.2020.tde-10122019-094300.

SILVA, I. *História dos Pesos e Medidas*. 2. ed. São Carlos: Editora da Universidade Federal de São Carlos, 2010.

SILVA, A. T. A. da. *et al*. Situação Desencadeadora de Aprendizagem. *In:* OLIVEIRA, N. M.; PANOSSIAN, M. L. (org.). *Verbetes da atividade orientadora de ensino*: grupo de estudos sobre situações desencadeadoras de aprendizagem. Capivari de Baixo: Editora Univinte, 2022.

SOUSA, M. C. O movimento lógico-histórico enquanto perspectiva didática para o ensino de matemática. *Obutchénie*: Revista de Didática e Psicologia Pedagógica, Uberlândia, v. 2, n. 1, p. 40-68, 2018. DOI: https://doi.org/10.14393/OBv2n1a2018-3.

SOUSA, M. C.; MOURA, M. O. Estudo das historiografias de Paul Karlson, Konstantin Ríbnikov, Howard Eves e Bento de Jesus Caraça: diferentes modos de ver e conceber o conceito de função. *Ciência & Educação*, Bauru, v. 25, n. 4, p. 1081-1099, 2019. DOI: https://doi.org/10.1590/1516-731320190040015.

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