

The logic of thought and mathematical logic: an epistemological analysis based on the study of the historical-logical movement of the concept of measure¹

A lógica do pensamento e a lógica matemática:
uma análise epistemológica a partir do estudo do movimento lógico-
histórico do conceito de medida

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ABSTRACT

This article is the result of a literature review, with the aim of discussing the problem of the consistency of mathematics in the context of formal logic and its relationship with the logic of thought. Theoretically, the study was based on Historical Dialectical Materialism, using the Logical-Historical method. The methodology consisted of a historical-epistemological analysis, relating mathematical logic and formal logic in the context of the logic of thought in order to draw up a historical synthesis that encompasses the origin, mathematical foundations and social content underlying the concept of measure, seeking epistemological elements that make it possible to elucidate the process of constructing measures. In general, the results indicate that mathematical logic results from the logic of thought, encompasses formal logic and surpasses it, to the extent that it considers the cultural and social content of mathematical concepts, their original conditions, their

RESUMO

O presente artigo resulta de uma revisão bibliográfica com o objetivo de discutir a problemática da consistência da matemática no contexto da lógica formal e sua relação como a lógica do pensamento. Teoricamente, o estudo foi baseado no Materialismo Histórico Dialético usando o método Lógico-Histórico do conceito de medida na análise epistemológica. A metodologia consistiu em, a partir de uma análise histórico-epistemológica, relacionar a lógica matemática e a lógica formal no contexto da lógica do pensamento para elaborar uma síntese histórica que engloba a origem, os fundamentos matemáticos e o conteúdo social subjacentes ao conceito de medida, buscando elementos epistemológicos que possibilitem elucidar o processo de construção das medidas. De modo geral, os resultados indicam que a lógica matemática resulta da lógica do pensamento, engloba a lógica formal e a supera, à medida que considera o conteúdo cultural e social dos conceitos matemáticos, as condições originárias deles, os seus fundamentos, seus nexos, leis e relações que os determinam como fenômenos no mundo concreto. Além disso, veio corroborar com a tese de Kopnin (1978), segundo a qual a lógica fundamental do

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foundations, their links, laws and relationships that determine them as phenomena in the concrete world. It also corroborates Kopnin's (1978) thesis that the fundamental logic of thought moves upwards from the simple to the concrete, from the lower to the higher, reflecting the laws that determine the development of the concept in the objective world.

Keywords: Concept of Measures; Logical-historical; Epistemology of mathematics; Logic of thought; Mathematical Logic.

pensamento se dirige em ascensão do simples ao concreto, do inferior ao superior, a qual reflete as leis que determinam o desenvolvimento do conceito no mundo objetivo.

Palavras-chave: Conceito de Medidas; Lógico-histórico; Epistemologia da Matemática; Lógica do pensamento; Lógica Matemática.

1 Introdução

Through logic, the human mind can apprehend what the sense organs cannot grasp because it goes beyond the limits of perception and the senses; logic makes it possible to understand the properties of objects or phenomena, their laws, and relationships, and highlights the links between the various stages of their development, giving them historicity (KOPNIN, 1978).

Considering the above assumption and taking the study of the Logical-Historical movement of the concept of measure as a method of analysis, in this article, we seek to discuss the problem of the consistency of mathematics in the context of formal logic, relating the logic of mathematics and the logic of thought.

This article is based on the foundations of Historical-Cultural Theory, inspired by Historical Dialectical Materialism. Through Vygotsky, Kopnin, Leontiev, Radeford, Kulla, Ifrah, Pucton, Caraça, among others.

The methodology adopted consisted of carrying out a historical-epistemological analysis, addressing the issue of mathematical logic and formal logic as a means of analyzing the consistency of mathematics, and broadening the discussion by including the logic of thought as a precursor to any logical process. In the second stage, we used the Logical-Historical dialectic category as a method of analysis, to look for evidence in the development process of the concept of measurement, considering its origin, its foundations, and its social and cultural content, which would allow us to elucidate the relationship between mathematical logic and the logic of thought.

We start from the following assumption: like all mathematical knowledge, measurements did not develop as a result of the cultural environment. The concept of measurement has been defined by the culture in which it develops and is integrated with it. It was under the influence of political, economic, and scientific interests, disputes, and the search for solutions for human life, that measurements became historical and cultural knowledge.

I - Mathematical logic and the logic of thought: the Logical-Historical development of mathematical concepts

The harmonious contradiction of the logical and the historical results in the dialectical category Logical-Historical, which triggers the process of object development, the appropriation of which is reflected in the process of social signification and the attribution of the meaning of the subject. Logic, in turn, is the guiding thread of abstractions. The starting point for logical movement is the observation of phenomena (natural and social) and man's practical activity in the world, which must be analyzed in their historical and social context. But neither observation nor practice are logical relationships. Logic or logical processes are constituted to

[...] It gives us insight into the form of development in its pure aspect, which thus literally does not reproduce itself in any historical process, however, the logical form of development reflects any historical process, because it is indispensable for its understanding (KOPNIN, 1978, 186).

In the materialist conception, knowledge is not a priori; on the contrary, it results from the individual's living relationship with their social and cultural space. Since this is a general law, the analysis of epistemological issues in the field of mathematics lies in the analysis of the logical-historical development of its conceptual system, because mathematical logic is, first and foremost, the logic of human thought.

It is inconsistent, in our view, to say that mathematical concepts, develop far from the objective world, only through mentalistic processes, even if there are

concepts that originate from the logic of mathematics itself. In other words, there is no such thing as pure mathematics, when the word "pure" is used to remove its originally empirical nature.

Mathematics originated from the reflection of the material world in the human mind, starting with the most general and external aspects of the object, perceived through the senses. It was the processes of abstraction resulting from mental activity that revealed the object's laws, links, and relationships with the universal flow, following the path of logical thought, which always evolves from the simplest to the most complex, from the inferior to the superior, from the abstract to the concrete in thought (KOPNIN, 1978).

Mathematical symbolic logic is the expression of such concepts and is the basis for their generation. However, the "[...] empirical is sensory in content and rational in form; sensory content is expressed by logical means." (KOPNIN, 1979, p. 148).

Mathematics does not follow a smooth and perfect path, without ups and downs, without twists and turns, without obstacles. History shows its setbacks with some precision. For example, the crisis of the incommensurable experienced by the Pythagoreans in the 5th century B.C., when they discovered the segments of lines, whose measurements could not be given by a rational number, arose amid the logical structure of Greek mathematics.

[...] The Pythagorean definition of proportion, assuming any two similar quantities to be commensurable, meant that all the propositions of the Pythagorean theory of proportions were limited to commensurable quantities, invalidating his general theory of similar figures. So great was the "logical scandal" that for some time efforts were made to keep the issue under wraps. (EVES, 2004, p. 107)

Until then, the Pythagoreans believed that all numbers could be represented as a fraction, but it was within the very famous Pythagorean Theorem that the contradiction was revealed. It took more than a century before Eudoxus, revising the Theory of Proportional Quantities, began to resolve the situation, elaborating the theory of the incommensurable which coincides in essence with the Modern

Theory of Irrational Numbers given by Richard Dedekind in 1872. What we can infer from this is that mathematical concepts are the result of man's practical activity and develop through processes of abstraction in mental activity. As Rubinstein (1979, p. 334) so aptly put it:

Man's activity is primarily a practical activity, and only later are theoretical activity and internal mental activity separated from it. However, man's practical activity always includes within its psychic components that reflect the conditions in which it is carried out and regulate it.

Consequently, mathematical concepts are abstract, not as a negation of the concrete, because they are concrete in thought and always return to the practical concrete, nor in the idealist sense that tries in vain to separate mathematical construction from the real world, defending the thesis that mathematics is a phenomenon of pure thought. The abstract nature of mathematical concepts relates to their subjective origin, anchored in the processes of abstractions (logical processes of separation and reconstruction of the totality) of human thought. Mathematical concepts, however sophisticated they may be, however idealistic they may seem, contain sensory components and are not just rational.

The logical-symbolic structure of mathematics lacks meanings because they are mediators of intersubjective thought and thought is never equal to the direct meaning of the sign, in other words, the mathematical lexicon, because the relationship between thought and the word is always indirect and linked by the meanings constituted in the social world. Thought itself is not born from another thought, but from the field of our consciousness that motivates it, encompassing our needs, our interests and motivations, our affections, and emotions (VIGOTSKI, 2000).

We should point out that the logicists (Frege, Peano, and Russell), at the beginning of the 20th century, understood mathematical logic as a set of axioms, postulates, principles, and associative laws that generate theorems as truths and sought to characterize mathematics by privileging form over content. This way, mathematics was restricted to a rigid system of axioms with the pretension of

avoiding any inconsistency in conceptual systems. The search for the perfect language held within it the Platonism of the objective world and naturalistic mathematics, ready and available to man, who had only to discover it. In this sense, it seems that mathematics is the gift of a few privileged individuals; its development processes are phenomena of the thinking of these brilliant minds. It seems, then, that it's not a question of mathematics explaining the world, but the opposite, of a world embedded in mathematics.

By reducing mathematics to "pure" logic, the logicians sought to structure it based on the application of the axiomatic method, in other words, they sought to create "pure mathematics" because of the manipulation of laws, principles, rules, and algorithms, constituting an artificialized logical structure, without concern for the historicity of mathematical concepts and their relationship with the objective world.

Similarly, the formalist school created around 1910 by David Hilbert, advocating the thesis that mathematics is essentially made up of combinations of formal symbolic systems, where its signs are mere symbols and its propositions are just formulas derived from the connections made with these symbols, sought to create a technique aimed at proving that there were no contradictions in mathematics. By reducing mathematics to a combination of meaningless symbols, he reinforced the idea that there are no mathematical objects, and, in this way, Hilbert emptied mathematics of its concrete content.

From the formalists' perspective, the concern to demonstrate the consistency of mathematics remained. They believed in the possibility of demonstrating that there were no contradictions in the mathematical results derived from calculations carried out using the axiomatic and the finitism methods. It's like proving that within the formal logical system, it would be impossible, starting from true axioms and following the rules of axiomatic calculus, to arrive at contradictory theorems. This is analogous to proving that there can be no error in a game if the rules are respected (EVES, 2004). What's more, the calculation carried out through the set of mathematical symbols and rules cannot generate contradictory formulas.

However, Gödel (1931) managed to show that it is impossible to prove the consistency of mathematics along the lines of the Formalists, in other words, it is not possible to encompass all mathematical knowledge in a formal logical system. He showed that in the system of mathematical concepts, there are "indefinable" and the consistency of this system is one of them. Hilbert's program, based on finitistic processes, succumbed to the persistence of doubt about the completeness of mathematical systems. The scientific certainty of mathematics is limited to the internal relationships of the phenomenon itself, and the relationship of the data within the phenomenon itself, but when the relationships open to the flow of the totality, there is nothing exact.

There is always an error in observing the quantum world, which increases as the wavelength emitted by the observer or their instruments increases. There is therefore nothing "exact" in nature. No result, no science, not even mathematics, is exact. [...]. Numbers are exact about themselves, in the relationships produced by them and for them. In the relationship between mathematics and the universal totality, there is no exactness (MOURA *et. al.*, 2016, p. 233).

Perhaps the logicians' and formalists' misconception about mathematics was that it was based on an idealist, intuitionist, or rationalist view from an epistemological point of view. By emptying mathematics of its concrete content, they also emptied it of its social significance, denied its historicity, and, above all, separated logic and history, form and content, relation, and contingency, and isolated mathematics itself from the object world.

The historical Logic approach can be useful as a method of epistemological analysis, as it seeks, through history, to explain the development of mathematical concepts, considering their original conditions, their foundations, nexuses, laws, and the relationships that determine them as phenomena in the objective world, in other words, revealing their social and cultural context, unlike formal logic which, in a narrow sense, is concerned with the rules for thinking correctly.

Formal logic deals with the relationship between premises and conclusion, and no longer cares about the truth of the premises. It is interested in giving the rules of correct thinking. It is an instrument that will allow the philosopher or scientist to walk rigorously. The logical correctness or incorrectness of an argument only depends on the relationship between premises and conclusion and is independent of the truth of the premises. (MUNDIM, 2002, p. 136)

For Cultural-Historical Theory, the inseparability of logic and history is fundamental. As Kopnin (1978) pointed out, the dialectical unity of the logical and the historical is the fundamental methodological principle for constructing a system of concepts and categories for any science, but it is also an effective means of revealing its essence.

Primitive ideas found in everyday life form the foundation of the logic of thought, which precedes all logical processes, including mathematics, and moves from the abstract to the concrete, from the inferior to the superior. From historical-epistemological analysis, as a means of establishing the correlations of mathematics in its complexity, historicity, and logicity, mathematical phenomena are transformed into cognitive phenomena. Therefore, mathematical logic because of the logic of thought encompasses formal logic and surpasses it, as it considers the cultural and social context of mathematical objects, the conditions in which they originated, and respects their historicity as evidence of their evolution.

II - The Logical-Historical Development of the Concept of Measure: history and epistemology

Since the beginning, measurements have been developed surrounded by circumstances that have dictated the pace, material conditions, shape, and content. Measures that were considered social content were woven together from a variety of social struggles, needs, political, scientific, and—above all—economic interests. The logical steps involved in creating mathematical knowledge can be clarified by comprehending this fabric.

In particular, the explanation for the origin of measures can be found in the intertwining of legends, myths, and fantastic stories, almost always based on a deity. However, regardless of their origin, the Logical-Historical approach measures have contributed to the formation of human consciousness by establishing standards of ethical and moral behavior. As Kulla (1988, p. 19) points out.

If for some [the measures] were invented by Cain, we can certainly find examples where the origin of the measures is related to more positive legends. According to Greek tradition, they were invented by the sage Fidon of Argos, who was venerated by him. According to the Romans, the foundations of land measurement were given by the nymph Vergoia, who appeared to the Etruscan Aruns Veltimnus. Here we see the almost divine origin of measurements.

Symbolically, the scales represented earthly justice for some peoples, and were the representation of the Archangel Michael at the Last Judgement, "[...] that is why they are continually admired by the faithful on the façades of Gothic cathedrals, an attribute of Ammon, the incarnation of justice." (Kulla, 1988, 56).

The right measure symbolized honesty and was sacred to the Semitic peoples. This can be seen in the Mosaic Pentateuch or other biblical quotations, such as Lev. 19: 35-36, Deut. 25: 13-15, Am. 8: 5.

You won't have several weights in your bag, one big and one small. You won't have two kinds of ephah in your house, one big and one small. You shall have a full and fair weight. You shall have a full and fair ephah, that your days may be long in the land which the Lord your God will give you. (BIBLIA SAGRADA, Deut. 25: 13-15. A. R. and C.)

The ephah was a measure for liquids, flour, barley, wheat, and roasted grain. In the prophet Ezekiel's account, the ephah was the container in which iniquity, represented by the woman, was imprisoned.

In Egyptian mythology, Osiris presided over the tribunal that decided the fate of the dead in the afterlife. Who did or did not deserve to join the god Ra (Sun)?

Anubis, a jackal-headed god, used a two-pan scale to weigh the dead person's heart, taking an ostrich feather as a counterweight. If the weight of the heart was less than that of the feather, the dead person could get on the million-year-old boat of Ra of Osiris to cross the Tuat (bad place) to the luminous territory of the god Ra, where his spirit would unite with that god, also becoming a star. However, if the dead person had sinned, the scales would weigh more, and the dead person would be condemned to become a demon.

In the early days, hunting depended on the construction of traps, strategies, and tools made of stone and wood. Stone tools such as axes, scrapers, and knives were first made by Paleolithic man.

In Ancient Greece, each polis had its system of measurement. In situations of war, the victors always imposed their standards of measurement on the vanquished, unlike the Romans, who, while maintaining their policy of territorial conquest during the so-called "Pax Romana", allowed the dominated peoples to continue with their standards and ways of measuring, as a strategy to avoid possible uprisings. In the Middle Ages, cities fought over control of measurements. "[...] where it is said that there are different laws for different men, it is the same as saying: there are different measures for different men." (KULLA, 1988, p. 160).

Around 3000 BC, agricultural civilizations emerged, producing in a rudimentary way, in the form of intensive farming. The formation of denser social groups gave rise to artistic and religious manifestations, including music and dance. But it was above all the mastery of a written language that enabled the production and transmission of culture, which made it possible to build an increasingly strong identity between the different civilizations.

When life-based on extractivism became impossible, man found himself faced with many needs. Producing food was the most challenging. As soon as he discovered how to plant and produce food, he realized that he didn't need to travel great distances to find it. As a result, he adopted a new lifestyle, which favored population expansion. Even so,

Primitive man didn't need a very elaborate system of measurement. His metrological needs were certainly only for some rough indications of positions, approximate distances, and relationships of magnitudes such as greater than and heavier than or less than and lighter than. However, from the moment it became necessary to cultivate the land or move animals to more fertile pastures, there was also a need to communicate more conveniently in metrological terms, and it may have been at this time that the first units of measurement appeared (SILVA, 2004, p. 38).

Agriculture also led to the emergence of other parallel activities, such as priests, scribes, and astrologers, who were responsible for monitoring and recording the movements of water that directly affected agricultural activity. The construction of irrigation canals, for example, helped to make food production more efficient. In addition, the discovery of metal made it possible to replace wooden plows. This led to the emergence of craftsmen who specialized in forging iron and bronze to produce agricultural tools, household utensils, and weapons. The valley of the River Nile, with its floods and ebbs, was ideal for agriculture. In the same way, the Tigris and Euphrates rivers in the Middle East, the Yellow River in China, and the Indus River in India fed the populations living in their vicinity.

The first nomads, when they became sedentary, built the first cities around 6500 BC, on the banks of the Tigris and Euphrates rivers. Around 5000 BC, on the banks of the River Nile in North Africa, the oldest known civilization was formed. After developing ceramics, copper, and bronze metallurgy, man was able to create and perfect various working tools and weapons. The Egyptians and Mesopotamian civilizations are credited with inventing techniques for draining flooded areas and agricultural irrigation, for organizing work, and for the first analyses of the universe.

From the 16th to the 18th century, trade underwent profound changes caused by Europe's occupation of the "New World", which led not only to a collapse in the old trade routes related to the cities of the Mediterranean but also to strong social changes, great scientific development, and radical political and economic alterations. These transformations led to the emergence of new ventures, such as the manufacture of ships capable of crossing oceans, the creation of banks and commercial companies with an international reach, and the undertaking of great overseas voyages.

Consequently, there was technological progress with the adoption of new measuring methods, new scales, new standards, and new tools and instruments. The structure of the state grew stronger and began to control measurements, define rules, establish sanctions, as well as making and guarding standards. This led to the creation of the balance houses, which existed throughout the Middle Ages until the beginning of the 19th century.

The purpose of these standards was to guarantee the fidelity of measuring methods and the inalterability of measurements. In this sense, inspectors had the role of monitoring, recording, and checking measurements. These procedures arose as a result of conflicts between lords and vassals and between the latter and the state and fuelled the distrust of traders at the open markets. This situation deepened with the rise of the bourgeoisie in the following centuries. Care was taken not to cause major losses in business. "[...] to introduce momentary changes in measurements, could open up a vast field for fraud or generate injustice" (PAUCTON, 1780, p. 7). Despite the rules and inspections, fraud was inevitable. This led to conflicts and losses.

The period known as the "Age of Exploration" was a time of great social change. International trade intensified and European merchants, after strengthening their business in Asia, began a race to find new trade routes, in other words, new suppliers and consumers.

[...]. Trade with the Arab world stimulated a growing demand among wealthy Europeans for Asian consumer goods such as spices and fine fabrics. This demand quickly became too great to be met by the relatively small quantities of these goods that Europeans could obtain from their Muslim neighbors (EVES, 2004, p. 337).

We must not forget that commercial expansion was favored by the Renaissance and, a little later, a theoretical framework would be built that would support liberalism by bringing about considerable social, political, and economic changes. By defending private property (John Locke, 1632-1704), liberals gave prominence to capitalism as an economic system and encouraged the limitation of the powers of monarchs. Politically, liberal ideas were the source of several

bourgeois revolutions in both Europe and America, expanding the bourgeoisie's space in state affairs, the private economy, and politics. According to Pauton (1780), France, in periods before the 18th century, felt the negative effects of the changes in measurements. In the same century, France began to establish the meter as an invariable and universal measure.

Empirical measurement processes gradually became geometric processes. Various documents from the 16th and 17th centuries reveal that land measurements made by surveyors began to merge geometric concepts with the old empirical units. In the mid-19th century, interest grew in changing the way grain was measured, which had previously been based on the volume of standard containers. It would then be measured by its "weight". However, "[...] the custom of weighing grain was not imposed by decree. It became widespread little by little, very slowly, pressured not by the state but by the merchants." (KULLA, 1988, p. 70).

It was a scientific, industrial, and especially commercial activity that contributed most to metrological changes. However, it was the emergence of capitalism that stimulated commercial expansion and played a fundamental role in the evolution of measures, forging the need to obtain standardized measures as a way of facilitating market control, the transport of goods, and, above all, international business. In this way, commercial internationalization also contributed to the processes of colonization, mercantilism, and industrialization, which generated great interest in the unification of measures. In this respect, Dias (1998, p. 14) explains that:

[...] With the expansion of international trade, in addition to the question of exchange rates, the difficulty of dealing with the countless units of measurement in which the goods traded in dozens of ports in the Mediterranean, the Indian Ocean, and America were expressed also came to the fore.

As a digression, we remember that around 3500 BC, the people of Mesopotamia made observations and measurements of natural phenomena to build an empirical calendar. There are indications that, at the same time, the Egyptians used scales and had their system of measurement. Science and myth were intertwined. In medieval

times, religious dogma was stronger than scientific knowledge, as religion exercised excessive control over the population and dictated the general rules of behavior. It was under the aegis of ecclesiastical power that many scientists suffered persecution, but science found in the practice of counting and measuring the possibility of describing natural phenomena and their regularities.

The speculative science of scholasticism sought to create a middle ground between reason and the Catholic faith, but it lost strength with the Renaissance movement, which broke some religious barriers, brought new light to science, and encouraged the spirit of inquiry. Scientific methods were created that included observing, measuring, experimenting, and repeating, under certain conditions, to reveal natural laws and those interpreted in mathematical laws, as well as validate them.

[...] In all fields of knowledge, there is this tendency towards the quantitative, towards measurement, in such a way that it can be said that the proper scientific state of each field only begins when it introduces measurement and the study of quantitative variation as an explanation of qualitative evolution. (CARAÇA, 2010, p. 117).

Consequently, measurements became essential in scientific studies. Thus, for Modern Science, natural phenomena could only be understood through quantitative laws, deduced from the measurement of their components, and expressed in mathematical language. In this respect, Silva (2004, p. 25-27) states that:

The metrological needs of modern physics, chemistry, natural sciences, and engineering require definitions of units, normalizations, and much more elaborate standards than those required by commerce and production. [...]. Nowadays, science and technology have been mainly responsible for improving metrology.

The decimal metric system was developed by a commission of scientists from the French Academy of Sciences, which included researchers such as Lavoisier, Lagrange, Delambre, and others. Scientific organizations of various kinds had to be set up to cope with the growing scale and sophistication of metrological problems.

Scientific advances in the fields of electricity, optics, mechanics, and metallurgy marked the emergence of new areas in the field of physics. The theoretical studies developed, and the results of experimental research carried out in laboratories generated real conditions for redefining the meter with greater precision. The redefinition of the meter in the 20th century was based on scientific assumptions and was intended to create the conditions for the emergence of an International System of Units that could represent physical measurements. Thus, in 1952, the Renaissance ideal became a reality when the International System of Units (SI) was established, consisting of 7 basic units and with the possibility of forming derived units from combinations of different measurements. The platinum-iridium bar, which represented the meter as a fraction of the earth's meridian, gave way to a new reference for the meter, which became the equivalent length of 1,650,763.73 wavelengths in the vacuum of the radiation corresponding to the transition of the 2p6 and 5d5 levels of the krypton 86 atom (SILVA, 2004). This definition brought more precision. However, it was not precise enough for all the purposes of science, leading to a redefinition of the SI. The meter came to be defined as the length of the path traveled by light in a vacuum, during a time interval of 1/299,792,458 of a second. [...].

Finally, the reticence to indicate the continuity of history, which, under different circumstances, pace, and objective conditions, will always respond to the tensions of social struggles, and scientific, political, and economic interests, transforming itself as social content.

III - Final considerations

It is important to emphasize that the epistemological analysis based on the study of the logical-historical movement of the concept of measurement has elucidated the process of development of the concept of measurement, by highlighting some of the moments of accumulation when the objective conditions came together, whether political, economic, or scientific, which generated a new qualitative change in the concept.

The results of this study indicate that mathematical logic results from the logic of thought encompasses formal logic and surpasses it, as it considers the cultural and social context of mathematical concepts, their original conditions, their foundations, their links, laws, and relationships that determine them as phenomena in the concrete world.

The study is also in line with Kopnin's (1978) thesis, according to which the fundamental logic of thought moves upwards from the simple to the concrete, from the inferior to the superior, and reflects the laws that determine the development of the concept in the objective world.

Finally, the historical-epistemological analysis, based on the Logical-Historical movement, has provided us with interesting information about the development of mathematical knowledge within human culture, demonstrating that mathematical knowledge is not merely concomitant with the development of its cultural environment; it is defined by culture and integrated into it.

La lógica del pensamiento y la lógica matemática: un análisis epistemológico basado en el estudio del movimiento lógico histórico del concepto de medida

RESUMEN

Este artículo es el resultado de una revisión bibliográfica con el objetivo de discutir el problema de la consistencia de las matemáticas en el contexto de la lógica formal y su relación con la lógica del pensamiento. Teóricamente, el estudio se basó en el Materialismo Histórico Dialéctico utilizando el método Lógico-Histórico del concepto de medida en el análisis epistemológico. La metodología consistió en, a partir de un análisis histórico-epistemológico, relacionar la lógica matemática y la lógica formal en el contexto de la lógica del pensamiento para elaborar una síntesis histórica que abarque el origen, los fundamentos matemáticos y el contenido social que subyace al concepto de medida, buscando elementos epistemológicos que permitan dilucidar el proceso de construcción de las medidas. En general, los resultados indican que la lógica matemática resulta de la lógica del pensamiento, abarca la lógica formal y la supera, en la medida en que considera el contenido cultural y social de los conceptos matemáticos, sus condiciones de origen, sus fundamentos, sus vínculos, leyes y relaciones que los determinan como fenómenos del mundo concreto. También corrobora la tesis de Kopnin (1978) de que la lógica fundamental del pensamiento asciende de lo simple a lo concreto, de lo inferior a lo superior, reflejando las leyes que determinan el desarrollo del concepto en el mundo objetivo.

Palabras clave: Concepto de Medidas; Lógico-histórica; Epistemología de las Matemáticas; Lógica del pensamiento; Lógica Matemática.

IV- Referências

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