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Abstract[1]

WAKIYAMA, Y. N. *Development of Skills in Mathematical Modeling Based on the Galperin, Talízina, and Majmutov Didactic System for Mathematics undergraduates.* 2022. 243 f. Thesis (Doctorate in Science and Mathematics Education) - Federal University of Mato Grosso, UFMT, Cuiabá. 2022 .[2].

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This text presents, within the limits of the abstract, the results of the doctoral work that deals with the development of skills required in the mathematical modeling process for Mathematics undergraduates at the Federal University of Amazonas (UFAM). Given that the Bachelor's/Licentite's degree courses in Mathematics should include actions aimed at developing students' skills and abilities, among others, to

> […] understanding, critiquing and use new ideas and technologies to solve problem […]; identify, formulate and solve problems in their area of application […]; establish connections between Mathematics and other areas of knowledge (BRASIL, 2002, p. 3-4).

In the words of D'Ambrosio (2009, p. 21), the fact that training in modeling is also training in mathematics, in a direct relationship: "[…] mathematical modeling is mathematics par excellence. The origins of the central ideas in mathematics are the result from a process that seeks to understand and explain

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facts and phenomena observed."

In addition, we sought to understand how knowledge is acquired and how individuals can develop mathematical skills. From the analysis of the literature review analysis, we formulated the hypothesis that Mendoza and Delgado's (2020) the Activity of Student Problem Situations (ASPD) can be a way of organizing teaching activities, leading to the internalization of knowledge and the development of skills in undergraduates, regarding the act of modelling problem situations.

In this study, the didactic system that makes up the ASPD is adopted for two main reasons. First, it considers Talízina's (1988) theory of the direction of study activity as a guide to teaching and learning activities. Second, it aligns with Galperin's (2011) perspective that the formation of skill occurs in mental stages. Additionally, Majmutov's (1983) student problem-solving framework informs the nature of the teaching tasks and the operations of the actions aimed at solving problems. The alignment of the mathematical modeling process with the Galperin-Talízina-Majmutov didactic system to be developed in the classroom, completes the research novelty.

To achieve the goal of analyzing the potential contributions of the Mathematical Modeling Student Problem Situation Activity (ASPD of MM) as a learning methodology in the formation of skills related to the construction of mathematical model, three specific objectives are established: i) diagnose the start level of students in relation to skills in the practice of mathematical modeling; ii) verify the contribution of the Complete Guiding Action Base Scheme (EBOCA) in the formation of students' skill through the ASPD of MM; iii) analyze the development of students' mental stage after implementing the didactic system.

It can be said that the first results of this work come from the bibliographical research, which provided the theoretical basis for preparing the modeling tasks and determining the EBOCA of Mathematical Modeling (EBOCA of MM), essential for carrying out ASPD of MM. as far as the tasks are concerned , Majmutov (1983) attaches a didactic-pedagogical character to

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the ascending problems that emerge from situations through the student's awareness. Once they are aware of this contradiction, i.e., what is known and unknown to them during the search for a solution, the problem (in its objective form in the task) is transformed into a student problem (subjective form) in the student's thinking.

The analysis of the types of contradictions, procedures, student problem solving and the level of independence pointed out by Majmutov (1983) allowed the classification of three types of modeling tasks: 1) in the task, the problem situation is presented with the known data, and it is enough for the student to search for the unknown, the model for the solution; 2) the task is presented, but it is up to the student to investigate additional data and procedures to build the mathematical model; 3) the task is presented, but it is up to the student to conduct a significant part of the process (WAKIYAMA; MENDOZA, 2021).

Regarding to the Mathematical Modeling EBOCA (EBOCA de MM), the skills (actions) were initially determined by studying the modeling phases proposed by different authors. Subsequently, the modeling phases converged with each other, establishing the same f invariant class of operations and actions, as described in Chart 1.

Chart 1: EBOCA of MM

1st action - Ability to formulate the student's problem		
Modeling Phases	**Control Operation**	**Action Operation**
BI - Perception and Grasping	O1. Identify the known elements from the data and/or conditions and/or concepts and/or procedures related to the problem.	C1. Did you identify the known elements from the data and/or conditions and/or concepts and/or procedures in the problem?
	O2. Define or identify unknown elements from the data and/or conditions and/or concepts, and/ or procedures of the problem.	C2. Did you define the unknown elements from the data and/or conditions and/or concepts and/or procedures in the problem?
BH - Interaction: Situation and Familiarization	O3. Recognize the contradictions generated from the problem situation	C3. Did you recognize the problem situation?
CA - Problematize; Formulate Your Own	O4. Determine or identify the knowledge to be sought.	C4. Have you determined or identified the knowledge to be

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Questions; AF - Problem Formulation		sought?
2nd action - Ability to build the conceptual and procedural core		
BU- Problem assessment; AF- Investigative process; CA- Develop through research	O5. Select the known concepts and procedures needed to solve the student's problem.	C5. Did you select the known concepts and procedures needed to solve the student's problem?
BA-Experimentation; BU-Exploratory research	O6. Update other known concepts and procedures that may be linked to unknown ones.	C6. Have you updated other known concepts and procedures that may be linked to the unknown ones?
BA-Abstraction; BH-Mathematization;	O7. Find strategy(s) for connecting known and unknown concepts and procedures.	C7. Did you find a connection strategy for connecting known and unknown concepts and procedures?
3rd action - Ability to solve the student's problem		
BI: Explicitation; AF: Mathematical representation; BH: Mathematical model BU: Problem-solving	O8. Build a mathematical model based on relationships formulated in theoretical and/or mathematical terms.	C8. Did you build the mathematical model?
	O9. Apply strategy(s) to relate the model to known and unknown procedures.	C9. Did you correctly apply the solution strategy(ies) to relate known and unknown procedures?
BA- Resolution; BH-Resolution	O10. Determine what is sought.	C10. Did you determine what you were looking for and/or objective?
4th action - Ability to analyze the solution		
BA: Validation; BH: Interpretation, Validation; AF: Response analysis	O11.Check that the solution corresponds to the objective, data and conditions of the student's problem. If ok, proceed to operation O14.	C11. Have you checked that the solution matches the objective, data and conditions of the student's problem? If yes, skip top check C14.
	O12. Review hypotheses, simplifications and data if the model found does not meet operation O11.	C12. Have you identified and correct assumptions and/or simplifications and/or incorrect data?
BA- Modification	O13.Confirm or correct the object sought.	C13. Have you confirmed or corrected the model?
BI: Significance and Expression; BA: Application; CA: Reflect and draw your own conclusions; AF: Communication of results BU: Critical analysis of solutions.	O14.Reflect on the results obtained and predict the best decisions in the face of the unknown.	C14. Was there a critical analysis of the results of the student's problem?
Caption*: BA: Bassanezi; BI: Biembengut; CA: Caldeira; AF: Almeida e Ferruzzi; BU: Burak; BH: Biembengut E Hein. *Authors cited by Cararo and Klüber (2017) and the modeling phases described by them.		

Source: Wakiyama; Mendoza, 2021.

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The four skills, formulating the student problem, building the conceptual and procedural cores, solving the student's problem and analyzing the solution, portray the actions required in the mathematical modeling process that we want to develop in students, from the perspective of Majmutov's (1983). In the formation of mental actions in stages proposed by Galperin (2011), there is the strategy of internalizing the four aforementioned actions, initially materialized in the modeling EBOCA. The theory is conceived in stages that are characterized by the changes e in each of the characteristics of the action.

Nina Talízina (1988), a prominent collaborator of Galperin, organized the characteristic qualities of action into primary ones, which concern generalized character, explained character, assimilated character and form; secondary ones, which are dependent on the primary ones and refer to the reasonable character, conscious character, abstract character and the solidity of the action.

According to Galperin (2011), the system of conditions that guarantee these parameters has five fundamental stages: elaboration of the Action Guiding Base (E1); formation of action in material or materialized form (E2); formation of action in external verbal language (E3); formation of action in external language to oneself (E4); formation of action in internal language (E5). Talízina (1988) adds to this structure the motivational stage, considered stage zero (E0).

By paying attention to the transformation of the o external actions of the object into internal ones, the effective direction of the teaching-educational process is possible when a system of requirements is developed, centered on the interaction between the object and the student. The first direction is to define the teaching objective of the cognitive activity (D1), the second is the knowledge of the starting level (D2), the diagnosis of students, and the third requirement consists of moving through the different states of the assimilation process (D3). The fourth is the feedback on the process (D4), also known as systematic feedback, and the fifth is the correction of the assimilation process

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(D5) (TALÍZINA, 1988).

The research1s methodological strategy follows the incorporated model (quali-quanti), with a qualitative emphasis, referring to the analysis based on preliminary results produced via quantitative analysis. The Chart 2 shows the four moments of the research, its objectives and the methods and instruments used by the researcher to achieve these objectives.

Chart 2: Moments Objective Application of instruments and methods

Momentos	Objective	Application of instruments and methods
1st-Initial diagnosis of the class	Determine the student's level of departure or actual development.	Application of the diagnostic pedagogical test and organization of results in a spreadsheet. Analyzing the answers to the questionnaire.
2nd-ASPD Planning	2nd-ASPD Drawing up the teaching sequence according to the initial diagnosis of the class and teaching method.	Planning the MM ASPD in a didactic sequence. Preparing the types of tasks.
3rd-Intervention	Carrying out the stages of mental actions to develop modeling skills under the direction the activity study.	Application of the modeling tasks. Checking the EBOCA actions carried out by the students. Recording the students' primary and secondary actions in the observation guides.
4th- Analysis of results	Estimate the level of mental action achieved by students after the intervention.	Checking the answers to the final questionnaire. Triangulation and analysis of results.

Source: Prepared by the author, 2022.

The research was carried out in 2021/1, 2021/2 and 2022/1 in the subjects of Mathematics Teaching Instrumentation (IEM) I, II and III, respectively. Due to the emergency pandemic, classes in IEM I and II were held remotely and, in IEM III, in person, at the Manaus Campus. Of these classes, 14 students from IEM I agreed to take part in the research, ten of whom continued to IEM II. Only three continued in IEM III and, in addition to these, seven new students enrolled in this last course.

When checking skills in the initial diagnosis, the academics did not externalize the action of analyzing (to validate) the model found, including failures

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in understanding the relationships between the known and unknown, as well as in the construction of models. Furthermore, the unsatisfactory performance in modeling tasks and the satisfactory performance in solving the proposed task with exercise characteristics endorse the need for intervention (WAKIYAMA; MENDOZA, 2021).

By triangulation the data from the tasks, observation guides and questionnaires, it was possible to analyze and estimate the stage or mental stage reached by the students and their perceptions because of MM's ASPD. Of the participants, 23% reached the materialized form of actions, i.e., the part of the action that remains related to their representations. The starting point for continuing to develop skills has been consolidated, since they have fully experienced the stage of doing and carrying out tasks with problem situations involving some of the contradictions expected for modeling practice, even with the support of their classmates s and the research teacher., remaining in the zone of proximal development.

Most of the survey participants (54%) have developed external verbal language. There is no longer a special movement from one material part to another, there is the ability to explain the actions carried out. In terms of their current level of development, the students in question can start a new stage materialized with type 3 modeling tasks. According to Majmutov's (1983) levels of cognitive independence, the academics can formulate the student's problem based on new data and new conditions that are well-defined in the task, which presupposes fewer investigative and exploratory traits, but they already presenting gradual autonomy and effective assimilation.

Furthermore, 23% of students led to deeper generalization, shortened and reached the stage of forming the action into an external language to themselves. The cognitive activity of these academics encompasses all stages of the process of solving a problem, which they already formulated, through independent analyzes of problem situations. These are characteristics of creative thinking at a high level of independence and effectiveness of problematic learning (MAJMUTOV, 1983).

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The skills of ten students are still being consolidated. The fourth action proved to be the most challenging, as mentioned in the transcribed speech of one student: "this was the one I found the most difficulty a bit more complicated". One possible reason for this is the addition of an action to the resolution scheme with no links to previous experiences, unlike the other actions that seem more familiar to the student. Another reason is that the questions concern the students' reflective and critical knowledge, which was, in fact reflected in the results regarding the action in question.

Students who have internalized the actions can improve their guiding bases with each new problem situation they face. The others can continue to solve the modeling tasks, under the guidance of the materialized BOA, until they achieve the desired skills in modeling practice.

MM's EBOCA, going through Galperin's stages, under the direction of Talízina's study activity, allowed undergraduates to gain a deeper understanding of the operations necessary to determine the mathematical model of problem situations in a progressive system of arranged contradictions, according to Majmutov. By the end of the MM ASPD, all the students acquired functional awareness of the four EBOCA actions as an operational guide for solving the student's problem.

References

BRASIL. Ministério da Educação. *Diretrizes Curriculares Nacionais para os Cursos de Matemática, Bacharelado e Licenciatura*. Distrito Federal: MEC, 2002. Disponível em: http://portal.mec.gov.br/cne/arquivos/pdf/CES13022.pdf. Acesso em: 3 nov. 2023.

CARARO, E. F. F.; KLUBER, T. E. Concepções de Modelagem Matemática na Formação de Professores em Modelagem Matemática. In: Encontro Paranaense de Educação Matemática, 14., 2017, Cascavel: UNIOESTE-PR, 2017, p. 1–16.

D'AMBROSIO, U. *Educação matemática da teoria à prática*. Coleção perspectivas em educação matemática. 17. ed. Campinas: Papirus, 2009.

DOI: http://doi.org/10.14393/OBv8.e2024-30

GALPERIN, P. Y. La formación de los conceptos y las acciones mentales. In: QUINTANAR, R. L; SOLOVIEVA, Y. (Org.). *Las funciones psicológicas en el desarrollo del niño*. México: Trillas, 2011. p. 80-90.

MAJMUTOV, M. *La Enseñanza Problémica*. Habana: Pueblo y Revolución, 1983.

MENDOZA, H. J. G.; DELGADO, O. T. Proposta de um esquema da base orientadora completa da ação da atividade de situações problema discente. *Obutchénie*. Revista de Didática e Psicologia Pedagógica, *[S. l.]*, v. 4, n. 1, p. 180–200, 2020. Disponível em: https://seer.ufu.br/index.php/Obutchenie/article/view/56482 . Acesso em: 3 nov. 2023. DOI: https://doi.org/10.14393/OBv4n1.a2020-56482.

WAKIYAMA, Y. N.; MENDOZA, H. J. G. Diagnóstico da aprendizagem por meio da atividade de situações problema discente em modelagem Matemática dos estudantes de licenciatura em Matemática da Universidade Federal de Amazonas. *Revista de Ensino de Ciências e Matemática*, *[S. l.]*, v. 12, n. 6, p. 1–25, 2021. Disponível em: https://revistapos.cruzeirodosul.edu.br/index.php/rencima/article/view/3175 . Acesso em: 3 nov. 2023. DOI: https://doi.org/10.26843/rencima.v12n6a05.

TALÍZINA, N. *Psicologia de La Enseñanza*. Moscu: Editorial Progreso, 1988.

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