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Development of algebraic thinking in early years teachers in the context of continuing education[1]

Desenvolvimento do pensamento algébrico de professores dos anos iniciais no contexto da formação continuada

Fernanda Cristina Ferreira Santos[2]
Vanessa Dias Moretti[3]

ABSTRACT
Discussions on the teaching of algebra in the early years of Elementary School have been the subject of research in recent decades and have intensified in recent years, especially with the new National Common Curricular Base. Investigations related to the training of teachers who teach mathematics in the early years become relevant as the teacher is responsible for organizing the teaching, being directly related to the students' learning process. In this article, we present a segment of a research, grounded in the Cultural-Historical perspective, which aimed to investigate the development of algebraic thinking in early years teachers, considering algebraic thinking as theoretical thinking mediated by algebraic concepts. The methodological aspects were based on historical-dialectical materialism, and data production was carried out through a formative experiment organized based on the Teaching Guiding Activity, with Learning Triggering Situations that considered the logical-historical movement of algebra, aiming at approaching the essence of algebra and algebraic conceptual nexus. The analysis segment presents scenes from the formative movement of one of the participating

RESUMO
As discussões sobre o ensino da álgebra nos anos iniciais do ensino Fundamental têm sido tema de pesquisas nas últimas décadas e se intensificaram nos últimos anos, sobretudo com a nova Base Nacional Comum Curricular. Investigações relacionadas à formação de professores que ensinam matemática nos anos iniciais tornam-se relevantes na medida em que o professor é o responsável pela organização do ensino, estando diretamente relacionado ao movimento de aprendizagem dos estudantes. Neste artigo, apresentamos o recorte de uma pesquisa, fundamentada na perspectiva Histórico-Cultural, que teve como objetivo investigar o desenvolvimento do pensamento algébrico de professores dos anos iniciais, considerando o pensamento algébrico como o pensamento teórico mediado por conceitos algébricos. Os aspectos metodológicos pautaram-se no materialismo histórico-dialético e a produção de dados se deu por meio de um experimento formativo organizado com base na Atividade Orientadora de Ensino, com Situações Desencadeadoras de Aprendizagem que considerassem o movimento lógico-histórico da álgebra, com vistas à aproximação da

[1] English version by Leonardo de Gusmão. E-mail: Leonardo.maciel@tikinet.com.br

[2]Graduate Program in Education, Doctoral degree student in Education at the Universidade Federal de São Paulo (UNIFESP), Brazil. Orcid: https://orcid.org/0000-0001-8293-4592. E-mail: santos.fernanda@unifesp.br.
[3]Graduate Program in Education, Associate Professor at the Department of Education Universidade Federal de São Paulo (UNIFESP), Brazil. Orcid: https://orcid.org/0000-0003-2435-5773. E-mail: vanessa.moretti@unifesp.br.

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teachers in the experiment, allowing the identification of possible improvements in the way the teacher defines what algebra is, in addition to her understanding of fluency, variation of quantities, variable, and range of variation, in a movement from the general to the particular, that is, from the search for general rules that solve specific situations. Such movements of approaching conceptual nexus and the essence of algebra by the teacher refer to the process of developing algebraic thinking.

Keywords: Algebraic thinking; Theoretical thinking; Algebraic conceptual nexus; Ongoing training for early years teachers; Teaching Guiding Activity.

essência da álgebra e dos nexos conceituais algébricos. O recorte de análise traz cenas do movimento formativo de uma das professoras participantes do experimento, permitindo a identificação de possíveis superações no modo de a professora definir o que é álgebra, além de sua compreensão sobre a fluência, variação de grandezas, variável e campo de variação, num movimento do geral para particular, isto é, da busca de regras gerais que resolvam as situações particulares. Tais movimentos de aproximação dos nexos conceituais e da essência da álgebra pela professora remetem ao processo de desenvolvimento do pensamento algébrico.

Palavras-chave: Pensamento algébrico; Pensamento teórico; Nexos conceituais algébricos; Formação continuada de professores dos anos iniciais; Atividade Orientadora de Ensino.

1 Introduction

The incorporation of algebra as a central unit during the early years of Elementary Education in the new National Common Core Standards (BRASIL, 2018) has sparked intensified discussions surrounding the teaching of this mathematical domain, bringing research agendas—addressed since the 1980s—closer to the realities of the classroom. In this sense, research by Fiorentini, Miorim, and Miguel (1993); Lins and Gimenez (2005); as well as Lanner de Moura and Sousa (2008) had already contributing with international research (Kaput, 2007; Canavarro, 2007; Kieran, 2004; Kieran *et al.*, 2016) centered on the concept of Early Algebra, which advocates for nurturing algebraic thinking during the early stages of Elementary Education. More recently, additional scholars have delved into this subject, providing substantial contributions to this field (Sousa, 2004; Panossian, 2014; Radford, 2018; Moretti; Radford, 2021).

Considering the intrinsic connection between teaching and learning processes, it is imperative to underscore the importance of studies pertaining to the training of educators responsible for teaching mathematics during the foundational years, given the imperative to devise instructional frameworks that prioritize the cultivation of

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algebraic thinking from this early stage. Within this context, we undertook a master's degree research project (Santos, 2020) aimed at investigating the development of algebraic thinking among educators teaching mathematics during the initial years within the framework of continuous professional training.

The research was grounded in the Historical-Cultural perspective, particularly drawing upon the insights of Vigotski[4] (2004, 2007, 2010) regarding human development, senses, meanings, and activity, alongside Activity Theory (Leontiev, 1978, 1983, 1988) and Developmental Theory (Davidov, 1983, 1988; Davidov; Márkova, 1987).

Considering this theoretical framework, which views education as the process of acquiring scientific and cultural knowledge historically produced by humankind through theoretical thought, the objective was to establish an intentionally planned and deliberate formative environment to support both the instructional activities of educators and the enhancement of their theoretical cognition. In essence, the objective was to examine the progression of teachers' algebraic thinking within teaching contexts, integrating continuous training in mathematics, specifically in algebra, with instructional design for the initial years of Elementary Education.

The investigation was rooted in historical-dialectical materialism, employing a formative experiment approach (Davidov, 1988; Vigotski, 2004; Cedro, 2008). The Teaching Guiding Activity (Moura, 1996; Moura *et al.*, 2016) served as the theoretical and methodological foundation, leading to the proposal of Learning Triggering Situations designed to foster collective actions aimed at grasping algebraic conceptual nexus and nurturing algebraic thinking, aligning with theoretical cognition mediated by algebraic knowledge concepts.

In this article, we aim to provide a synthesis of our understanding of algebraic thinking as a form of theoretical cognition mediated by algebraic concepts. Then, we will underscore several methodological contributions of the Teaching Guiding Activity to research into the structuring of teacher training

[4] Although there are more than one spellings for Vygotsky's name, we have adopted that one used by the consulted bibliographic sources.

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endeavors, aimed at grasping conceptual nexus and fostering the development of algebraic thinking. The analysis section will follow the training process of one of the educators examined in the study, elucidating her involvement in various collective initiatives and highlighting the dialectical progression in her thinking as evidenced by her engagement with algebraic conceptual nexus and the evolution of her algebraic thinking. We will conclude by offering reflections on the research findings and the continuous training processes aimed at nurturing algebraic thinking among educators responsible for teaching mathematics during the early years of Elementary Education.

2 Theoretical thinking mediated by algebraic concepts

According to Davidov (1988), it is understood that human thought enables the rational comprehension of reality through the conceptualization and planning of the object, followed by its restructuring after practical experiences, reflecting a synthesis of the concrete and the abstract. In light of this understanding, thinking can either be grounded in empirical observations of reality, focusing on its observable, external, and sensory aspects, termed empirical thinking, or it can manifest as a quest to establish internal and external connections to uncover the essence of the object, from which its particularities and specifics emerge, thus constituting theoretical thought.

Empirical thinking begins with the observation of specific cases, repeated and transformed into general representations based on patterns in the abstract field, leading to empirical generalization derived from sensory experiences of the object.

Conversely, theoretical thinking begins with the apprehension of concrete and disordered cases when abstracted to the conceptual field to conduct mental experiments through reflection and rationality, aiming to approach their essential components—namely, the conceptual nexus of this object of knowledge—to unveil its essence, the overarching law governing such an object. Subsequently, these relationships ascend once more to the concrete field, namely thought, wherein the overarching relationship unveils the object within its comprehensive system, facilitating the manifestation of its essence in

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resolving particular cases related to this general law. This process is exemplified by the concrete-abstract-concrete triad (Romeiro; Moretti, 2016), a progression enabling the mental reproduction of an object within its comprehensive system, thereby forming the theoretical concept regarding this object (Davidov, 1988). In short, the merging of abstraction, generalization, and concept formation, transitioning from the general to the specific, that constitutes the evolution of theoretical thought.

> The establishment of conceptual nexus, as a means of implementing theoretical thinking, facilitates the comprehension of the logical (pertaining to the thought process) and historical (concerning the progression of phenomena in the objective world) examination of concepts, through various forms of theoretical thinking (abstraction, generalization, and concept development), analysis, synthesis, and the dialectical logical progression from abstract to concrete, as articulated in dialectical logic (Panossian, 2014, p.110).

Interconnected with the process of grasping conceptual nexus and the essence of the object, it is understood that the logical-historical progression allows for the unveiling of the trajectory of human experiences throughout history, reflecting the logical actions of human thought concerning the constituent path of the object (Kopnin, 1966). Given the impossibility of fully replicating history, the logical-historical movement enables the identification of essential elements within a concept, thus crucial for the advancement of theoretical thought.

Similarly, just as theoretical thinking facilitates the rational comprehension of objects in reality "by discerning the essence of the object and forming mental connections regarding the properties constituting its universal form" (Santos, 2020, p.34), algebraic thinking is oriented towards understanding the dynamics of reality and its phenomena (Sousa, 2004; Lanner De Moura; Sousa, 2008), which arises from establishing relationships among quantitatively varying quantities, constituting the core of algebra (Panossian, 2014).

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From this standpoint, algebraic thinking also evolves through recognizing and internalizing algebraic conceptual nexus, involving the comprehension of overarching relationships concerning variable quantities. In the early years, this progression may involve understanding and manipulating operational structures analytically, ensuring the inseparability of arithmetic, geometry, and algebra.

Throughout the developmental journey of algebraic thinking, rooted in concrete and initially disorderly scenarios, overarching relationships are abstracted and then reintegrated into specific cases related to each phenomenon, enabling their reinterpretation and generalization, thereby expressing algebraic knowledge through concepts. Given this understanding, algebraic thinking is perceived as theoretical thinking facilitated by algebraic concepts (Moretti; Virgens; Romeiro, 2020; Santos, 2020; Moretti; Virgens; Romeiro, 2021).

It should be noted that arithmetic, from this perspective, is not merely seen as the empirical aspect of algebra, as there are theoretical generalizations within the field of arithmetic. Instead, arithmetic is regarded as another domain of knowledge inherently intertwined with the teaching of algebra, particularly in the early stages. Comprehending arithmetic structures supports the progression towards understanding algebraic relationships, just as the exploration of conceptual nexus and the cultivation of algebraic thinking can imbue new significance into arithmetic calculations, which were once rigid and mechanized. Therefore, in the interaction between arithmetic and algebraic knowledge, one does not precede the other, as their instruction can and should coexist.

Building upon these principles, Learning Triggering Situations were formulated throughout the training experiment with teachers, aimed at familiarizing them with certain conceptual nexus, thereby enabling them to internalize elements of algebraic knowledge and foster the development of algebraic thinking. Thus, we draw upon the insights of Sousa, Panossian, and Cedro (2014), who, grounded in the logical-historical progression of algebra, underscored fluency, the variance field, and the variable as the principal

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algebraic conceptual nexus, which were embraced in our research during the investigative period.

In short, fluency encapsulates the notion of continual transformation and evolution of the world and its phenomena; the "variable represents the potential values for a given quantity" (Santos, 2020, p. 112), while the variance field delineates the movement of the variable's variance. As our research progressed, our perspective on links and algebraic conceptual elements has evolved, leading to a broader understanding. We now recognize links such as "movement, unknown value (question), pattern recognition and generalization, variables, relationships between variable quantities, and functional algebraic relationships" (Moretti; Virgens; Romeiro, 2021, p.1469).

3 On the methodology of the research

In alignment with the theoretical underpinnings of our research, we adopted the historical-dialectical materialist method, which, grounded in dialectics, seeks to transcend formal logic, recognizing that "the phenomenon under study must be presented in a manner that enables its apprehension in its entirety" (Cedro, 2008, p.96). Thus, the researcher, informed by a theoretical understanding of the reality of the phenomenon being studied, endeavors to track its dynamic movement to unveil its essence (Gamboa, 2000; Frigotto, 2000). This approach entails surpassing mere descriptive analysis of the data produced, taking into account dialectical pairs and the inherent contradictions within this process.

To undertake such an investigation, a formative experiment was devised with deliberate planning on the part of the researcher, focusing on the teaching activities of educators, their learning processes, and their psychological development, as "one of the fundamental aspects of this perspective is that it assumes the active involvement of the researcher in the psychological processes he investigates" (Cedro, 2008, p.105).

The formative experiment was conducted as an outreach initiative by Universidade Federal de São Paulo (UNIFESP), in collaboration with a public

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school in Guarulhos. One of the researchers served as a multi-disciplinary teacher in Early Childhood Education and the initial years of Elementary Education. A total of 23 teachers from the same school participated in the experiment, engaging in 17 scheduled meetings, each lasting one hour.

The Teaching Guiding Activity (TOA) served as the theoretical and methodological foundation for planning and organizing the experiment. It is perceived as the amalgamation of teaching and learning activities and can guide both the teacher's instructional organization (Moura, 2010) and the researcher's investigative trajectory, allowing for "the use of its structure to identify motives, needs, actions triggered, and meanings ascribed by subjects in the teaching process" (Moura et al., 2016, p.125).

Promoting elements such as playfulness, interaction, and collectivity as integral components of the learning process, the TOA envisions the creation and execution of Learning Triggering Situations (LTS) (Moura, 1996), which aim to confront individuals with a need bound to the logical-historical progression of a concept, thereby fostering an approach to the essence of knowledge through the appropriation of its conceptual nexus.

LTSs utilize virtual narratives (fictitious stories related to human needs in the evolution of knowledge production), games that transcend mere surface engagement and evoke a theoretical necessity to be addressed within the community, and real-life scenarios that raise questions pertinent to the learning context. It is imperative that LTSs, besides encapsulating the historical synthesis of the concept, facilitate group discussions and collective synthesis with the guidance of the teacher (or researcher) and peers, stimulating a thirst for learning and reinforcing the engagement of participants, all aimed at cultivating theoretical thinking (Moura et al., 2016).

Throughout the sessions of the training experiment outlined in the research, educators were presented with Learning Triggering Situations pertaining to elements of algebraic knowledge, challenging them to grapple with "elements to potentially engage individuals in solving a problem related to the

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contents [...]" (Moura, 1996, p.4), particularly focusing on specific conceptual nexus within algebra.

The experiment unfolded in three modules: the first aimed to ascertain the group profile and the initial senses of participants regarding algebra and its instruction; the second endeavored to familiarize teachers with algebraic conceptual nexus to nurture algebraic thinking; and the third sought to integrate the teacher's instructional and learning activities, aligning their training process with the organization of teaching centered on fostering algebraic thinking among students in the early stages of Elementary Education.

Various instruments were employed to produce data, including questionnaire responses, audiovisual transcriptions, field diaries, and collective records. Subsequently, the analysis aimed to transcend mere description and integrate the collected data, enabling a dynamic understanding of the investigated phenomenon—the development of teachers' algebraic thinking, seeking to discern signs of progress in thinking patterns, particularly regarding the alignment with algebraic conceptual nexus and the progression of algebraic thinking.

4 Development process of algebraic thinking of early years teachers in the context of continuing education

In this article, we will present an analysis of the training journey undertaken by one of the three teachers who participated in the research. Specifically, we will offer a summary of Teacher Bia's (alias) progression towards establishing conceptual nexus, aiming to uncover indications of her evolving algebraic thinking. Our emphasis will be on the data produced during the second module of the experiment. Nonetheless, we will incorporate select scenes from the first module to explore the teacher's initial comprehension of algebraic concepts. Additionally, excerpts from interactions between Teacher Bia and her peers will be included to validate her developmental trajectory.

At the outset of the formative experiment, the participant teachers were prompted to articulate and share their understanding of algebra with their peers. Teacher Bia, much like her colleagues, highlighted a challenge in defining what

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algebra entails, illustrating a lack of clarity regarding its distinction as a separate mathematical field, unrelated to arithmetic.

> *Bia:* To be honest, I saw very little of this algebra discrimination, everything for me was mathematics. Discrimination had to do with geometry… I never heard of "algebra" during my training. So, I find it difficult to define exactly what algebra is. Is it the games? The problem situations? Maybe everything? […].

Bia's observation highlights a problem inherent in her teacher training, which aligns with research indicating a deficiency in both initial and ongoing education for teachers instructing mathematics in early years (Nacarato *et al.*, 2023), which often results in a lack of understanding and integration across various mathematical domains, leading to a predominantly technical approach that fails to address the resistance many educators harbor from their early educational experiences.

In the fourth session, teachers were prompted to contemplate the concept of the equal sign and its instructional significance in mathematics. While not explicitly tied to algebraic concepts, recognizing equality as a representation of equivalence and balance, rather than merely a static indicator of a solution, is crucial for fostering the development of algebraic thinking.

> *Bia:* Equal means… equals to? (laughs) So… it's meant to show the result (laughs).

Neither Teacher Bia nor any of the other teachers recognized equality as equivalence or balance, but rather as an operation—a static outcome for problem scenarios, as seen in textbooks, which typically involve placing a known value on one side, adding it to another unknown, and equating it to the result on the other side of the equation. In said instances, students are tasked with finding the missing value, which can be achieved solely through discrete counting, without necessitating a broader grasp of manipulating known or unknown quantities on both sides of the equation—to discern structure beyond surface appearances. Expanding comprehension beyond the superficial notion of equality entails understanding equivalence, thereby recognizing

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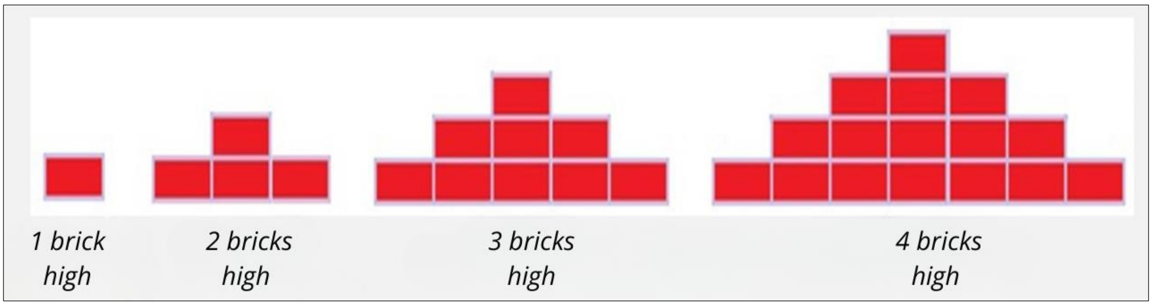
that regardless of the operations performed or quantities manipulated, the principle of equality ensures balance on both sides consistently.

Overall, during the initial meetings, Teacher Bia's notes and statements suggested that her understanding and thought processes were confined to the field of arithmetic. Similar to her colleagues, at no point did Teacher Bia introduce elements indicative of algebraic knowledge, such as variable quantities or the variance field. Additionally, she did not employ typical algebraic language, such as using letters to denote variables or unknowns. Reflecting on Davidov's (1988) assertion that theoretical thinking hinges on comprehending the essence of the subject and its conceptual nexus, it appears that at that juncture, Teacher Bia did not engage in theoretical contemplation of the algebraic field due to a lack of established concepts regarding the subject. Consequently, she was not yet engaged in algebraic thinking.

As the second module progressed, teachers were presented with an LTS in the form of a virtual narrative titled "The Young Builder," which introduced a pattern and several problem scenarios linked to said sequence. In this virtual tale, Pedro, a budding builder, embarks on a project to build a staircase—a structure meant for ascending on one side and descending on the other—without knowing the total amount of bricks needed.

Image 1 — Pattern from LTS's "The young builder"



Source: Santos, 2020, p.82.

When tackling one of the given scenarios, which queried the number of bricks in height and the total brick count for a staircase wall with a base of 39 bricks, Teacher Bia emphasized the necessity of an overarching rule to facilitate solving specific instances. This approach aimed to circumvent the need for resolving each scenario

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individually through arithmetic means, given the impracticality or even impossibility of doing so in cases with notably large values.

> *Bia:* [...] when the question brought up base 39, we did it manually, you know, outlining everything. We began with a base of 39, adding two more from the first position until reaching it, thereby determining a height of 20 bricks. However, I raised a concern that this method was only feasible for a small number of bricks; dealing with larger quantities would be impractical. If there were hundreds of bricks, manual counting would be unmanageable. This dilemma underscored the need for a systematic approach to determine quantity based on the base, without resorting to manual calculation.

The teacher's emphasized necessity refers to the process of algebraic generalization, wherein a general rule is extracted from a specific (often chaotic) situation and subsequently applied back to concrete scenarios in a deliberate manner. In this case, it involves deriving a general rule for determining the height and quantity of bricks required for any stair base. The initial prerequisite for this upward progression of thought, as expressed by Teacher Bia, is the recognition of the imperative to transition from the abstract to the specific (Davidov, 1988). In her case, this is manifested by the acknowledgment of the necessity to "conceptualize something that could serve as a basis for determining the quantity, eliminating the need for individual calculations."

Within the proposed LTS, two inherently linked variables emerge: the base and the height of the wall. From this relationship, a third variable—the total amount of bricks—arises. In the excerpt provided and throughout the session, Bia unmistakably identifies two variables: the height, expressed by the word "quantity," and the base. She also acknowledges the necessity of establishing a functional correlation between them through a general rule, thereby hinting at an understanding of the concept of variables, which constitutes an algebraic conceptual nexus. Although she may not explicitly name it at this point, her recognition points towards an approximation of the variable concept.

The depicted scenario pertains to an LTS titled "Height of the Pyramid" (Sousa, 2004), where an ancient Egyptian contractor grapples with

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determining the height of the main column of a pyramid in the absence of the Pharaoh's specifications. All he knows is that 12 stones have been utilized in the master column, leaving 60 in storage.

Through discussions surrounding the LTS within a smaller group, Bia seemingly awakens to the potential for variance and functional relations among quantities.

> *Alice:* If he used it all, for example, the height would be 60 plus 12.
> *Bia:* But then again, what is he asking for?… Is it the height he wants to find? See… the Pharaoh hasn't chosen yet.
> *Alice:* So, h is going to be x plus 12. H is the height… x is how many more stones he will need.
> *Bia:* Yes… 'cause he doesn't need them all… he may use 10, 15, 35 more… depending on how high the Pharaoh wants it.
> *Alice:* Yeah!
> *Bia:* Got it!

In dialogue with Teacher Alice, who holds a mathematics degree, Teacher Bia suggests potential values that the variable could encompass, highlighting the variability in magnitude associated with the quantity of stones still available for use. Despite specifying these values, throughout the discussion, she conveys an understanding that these values must be confined within a defined range, expressed by the variability ranging from zero to sixty, mirroring the height variance range from 12 to 72. This approach further aligns with another aspect of algebraic conceptualization.

Additionally, within the same discourse, it is observed that the quantity of stones required to complete the main column of the pyramid "depends on the height requested by the Pharaoh," establishing a correlation between the two variable quantities involved, albeit initially, reflecting the essence of algebraic comprehension.

Another LTS, "The Fantan game" (Panossian; Moura, 2010), is described as a board game in each four players are represented by a distinct color. The board is square, with its four corners marked by numerals from zero to three. Each player is given 20 chips of their respective color and wagers zero to three

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chips per round in any of the corners on the board, ensuring that the number of chips wagered does not coincide among players. One of the players picks up a random amount of beans and spreads them out on the table. Groups of four beans are assembled, and the participant who wagers on the corner where the quantity aligns with the remaining beans after grouping wins the round, collecting chips equal to their own wager from each player, or the opponent's wager if it is lower.

In discussions about the game, Bia's dialogue introduces points that seemingly hint at algebraic conceptual nexus.

> *Bia:* I recognize the concept of possibilities once again. It helps to know how far they can go. The remaining beans range from 0 to 3, as do the chips we could potentially win per round… because if I won the top prize from everyone—which is 5—, I'd end up with 20… then there's the remainder, the quantity of beans, the quantity of chips, and so forth… we need to understand what might be happening during the game.

Teacher Bia begins to acknowledge the connection between algebra and the ability to foresee and regulate the movement of quantities, emphasizing both the fluidity and the variance range of the quantities involved, highlighting two algebraic conceptual nexus. Similarly, in collaboration with her group, she verbally formulated a rule that could control or calculate the total quantity of any handful of beans based on the groupings made, thereby leveraging and imparting new significance to the arithmetic concept of division from an algebraic perspective.

> *Jonas:* I think we could use letter q to represent the total amount of beans.
> *Researcher:* And how much will q be worth?
> *Jonas:* There are plenty of possibilities. We don't know how many beans will be caught in each handful.
> *Bia:* With this in mind, the number of groups in each round could also be represented by another letter. But then it has to be multiplied by 4, right, Jonas? Just like you said, multiplying the group amount by four.

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> *Researcher:* So how would we record this?
> *Alice:* Q equals 4g plus the remainder. The remainder could be r.
> *Researcher:* And would this be the only way to register things?
> *Elisa:* We could write it down instead of using letters. The total amount of beans equals the number of groups multiplied by 4, plus whatever remains.

Besides devising a general rule to address all specific cases, the teachers seemingly acknowledge variables and articulate them through various languages, including alphanumeric representations. This collaborative effort reflects a moment of collective production and assimilation, further aligning with the trajectory of Teacher Bia's thinking.

An adapted version of the Jackstraws game was also employed as LTS, aiming to familiarize teachers with algebraic conceptual nexus such as variables and unknown quantities. This adaptation highlighted the capacity to manage and articulate the fluctuation of magnitudes through various languages (written or symbolic) while also introducing the concept of systems of equations in general, emphasizing the significance of establishing relationships between the variable quantities at hand.

As usual, the teachers were split into smaller groups and presented with a Jackstraws game devoid of standard instructions. The researcher established that the rules should be culturally familiar, but collectively, participants would determine the score value for each stick color retrieved, except for yellow and red. The values for these colors would be independently set by each smaller group in confidence. These values were to fall within the set of natural numbers, ranging from 0 to 60. At the end of the game, each group was tasked with recording the number of sticks of each color acquired by each participant, along with their respective final scores.

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Table 1 — Jackstraws game

Jackstraws game

Directions:

- *Form a group of 4 people.*
- *Read the game rules carefully.*
- *The score value for 3 of the 5 stick colors that make up the game should be set collectively.*
- *Each smaller group will secretly assign scores for the other 2 stick colors in the game.*
- *At the end of the game, each group is required to document the total scores of all four players and enumerate the quantity of sticks of each color that each player collected, refraining from recording the values assigned to the two specified colors.*
- *Subsequently, the recording sheets are exchanged among the groups, and each group is tasked with discerning the values assigned by the other group to the two specified colors.*

Rules:

- *One of the players drops the sticks onto the table.*
- *Each player, in turn, must extract the sticks from the pile one by one, either by using their hands or an auxiliary stick known as "emperor," "general," "mikado," etc., without disturbing or touching the other sticks.*
- *Each player's round is concluded If a player moves or touches any of the stick.*
- *The scores assigned to each stick color will be considered.*
- *The objective is achieving the highest score possible and win.*

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Source: Santos, 2020, p.88.

Teacher Bia's group is progressing toward grasping algebraic conceptual nexus and moving closer to a broader relationship that entails revealing the two unknown values.

> *Bia:* But now, how do we calculate it? Could it be any value between 1 and 60?
> *Jonas:* Yes, but it isn't 60, as it is black. It's neither 30 or 10 too. So it's any number between 1 and 60, minus these three… Here, 160 was left for red and yellow.
> *Bia:* Let me see… We add up the ones we already know the value of and see how much is left for red and yellow.
> *Jonas:* Sure. But everyone's score is different…
> *Olga:* Yes… everyone took an amount of each color.
> *Bia:* But the value of yellow and red is the same for all.
> *Jonas:* So we must find a value that would work for all scores. That's the hard part (laughs).

Teacher Bia reinforces, in the form of a question, the variance range of magnitudes by asking, "Could it be anything from 1 to 60?" Furthermore, they recognize that the scoring values set for the yellow and red sticks should be consistent among all members of the group analyzed. This realization stems from their utilization of the score from just one player of the opposing group at that moment. This assumption, supported by Teacher Jonas' remarks, aligns with one of the fundamental conditions of the concept of a system of equations, wherein the values of the unknown quantities must fulfill all related equations.

> *Jonas:* Then we have to find the values, or think of a formula that helps finding these values? There should be one. We know it's a comparison, but how do we solve it?
> *Bia:* Here, I think we really have to find the values… we gotta know how much yellow and red are worth to their group. But there must be something that works for any score.
> *Jonas:* I think so too… but, look here (writes down)… if I think that I have 60 for yellow, then each stick is worth 20. That leaves 100 for the red ones… as there are 4, then each would be worth 25. Am I right?

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> *Bia:* Yes, but we still need to confirm this works for the other scores.
> *Jonas:* I wanna learn this formula soon (laughs).

In this scenario within the game, they come to the realization that they need to determine values for the yellow and red sticks that align with the scores of all players in the analyzed group, ensuring consistency with their respective final scores. This calls for a practical application of the concept of equations systems, transitioning from the abstract to the concrete, even though the primary focus at that moment was not for the teachers to master system resolution algorithms. Teacher Bia also emphasizes the importance of seeking a general rule that addresses the requirement to formulate the concept from the abstract to the specific, stating, "there must be something that works for any score."

Upon gathering all the groups' findings, the researcher presents the scores of two teachers from the same group: the first represented as $1A + 1V = 20$ and the second as $3A + 5V = 90$. Here, A is the value assigned to each yellow stick, while V corresponds to each red stick. Now, the teachers were tasked with determining the value of the red sticks for the first and second teachers, given that the yellow stick was worth 1.

> *Jonas:* If we consider balance, three plus 5V equals 90. So 5V would have to be worth 87, right?
> *Bia:* But then five times something would be 87… it's impossible….
> *Alice:* Guys, this result would be in the decimals.
> *Researcher:* What numerical set have we established to be used?
> *Jonas:* Natural…meaning this result is useless…A can't be worth 1.
> *Researcher:* Why?
> *Jonas:* 'Cause the result can't be a decimal…
> *Bia:* Also, because our result didn't match Olga's [outcome]. If they belong in the same group, I believe the result should be consistent.

Bia reiterates her understanding of the variance field and that the values assigned to the yellow and red sticks must align with the scores of all members of the corresponding group.

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The researcher creates a table with the potential responses for each teacher's score and inquires what values would satisfy this criterion based on the scores provided.

> Jonas: It will be 5 and 15.
> Researcher: Why?
> Jonas: Those are the two identical values that appear in both cases.
> Researcher: What if the value of a color changed?
> Jonas: The other color would also change…
> Researcher: Why?
> Bia: Because they are within the person's score. So, if the value of one color changes, the other would change too. That would affect the whole score.
> Jonas: We knew that we had to compare, but kept throwing in random values from 1 to 60. We didn't pay attention to the fact that it maybe wouldn't reach 60.
> Edna: But here's the thing… we were working with values up to 60… if it had been a higher number there would have been many more possibilities… until we found a value that worked for both…
> Bia: I believe this comparison is meant to help us see where it comes from… how it works. Our group speculated that there might be a simpler solution.

Similar to Jonas, Bia demonstrates an understanding of the fluctuation of quantities and how, at a given moment, variables take on specific values (unknown quantities); grasps at the extent of variability in these quantities and acknowledges that, by comprehending the general resolution framework, rules can be formulated to be applied across diverse situations, potentially through algorithms. Bia also hints at the quantities variability and the interconnectedness between such variable quantities by stating that "if the value of one color changes, the other would change too, affecting the whole score," giving evidence of comprehension of algebraic concepts intrinsic to algebra and its conceptual interrelations.

Various methods of notation were deliberated upon across the collective synthesis, with some groups opting to write the color names followed by the number of sticks for each participant, while others employed letters for symbolism. Consequently, the researcher posed questions to the teachers, fostering discussions on topics such as the correlation between variables and unknowns, how variables can convey the fluctuation of quantities, and ways of representing these variables.

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At the end of the second module of the experiment, the researcher prompts the teachers once more to articulate their understanding of algebra both in writing and speech.

> *Bia (writing):* Algebra is the relationship between quantities. It means working with unknown numbers that may vary.
>
> *Bia (speaking):* I wrote that algebra involves things that may vary. Myself… I've learned that it's also about working without the numbers. And I began to grasp at the idea, because to me, it was basically mechanical, just crunching numbers for x and that was the end of it. So much so, I didn't even have a clue what algebra was (laughs).

Teacher Bia exhibits significant progress in her thought process, articulating both verbally and in writing aspects pertaining to the unknown value, quantity variance, and the potential for establishing relationships among them. Such indications suggest a movement towards the core of algebra and its conceptual nexus, a shift that epitomizes the evolution of algebraic thinking.

5 Final considerations

This article provides a snippet of research conducted with elementary school teachers who specialize in teaching mathematics at the early stages of Elementary Education, delving into the exploration of their development of algebraic thinking within the framework of ongoing professional development. The research was grounded in the principles of the Historical-Cultural Perspective, drawing heavily on the contributions of Vigotski, Leontiev, and Davidov. Algebraic thinking was conceptualized as a form of theoretical reasoning facilitated by algebraic concepts.

The data collection process revolved around a formative experiment guided by Teaching Orientation Activities, incorporating Learning Triggering Situations designed to familiarize teachers with the essence of algebra and its conceptual nexus. The data analysis segment focused on Teacher Bia's developmental trajectory, spotlighting key moments to provide evidence of her progression in comprehending algebraic elements, particularly concerning conceptual linkages and the process of generalizing from the abstract to the specific.

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It is evident that structured training initiatives for early years educators, aimed at immersing them in the essence of algebra and its conceptual nexus through collective and meaningful activities facilitated by Teaching Orientation Activities, can bolster the cultivation of theoretical reasoning mediated by algebraic concepts, i.e., algebraic thinking.

Given that algebraic conceptual nexus are pivotal components within the logical-historical evolution of concepts and the fostering of algebraic thinking, comprehending these interconnections, namely fluency, variable, and variance field, remained a central focus throughout the formative experiment conducted by the research. Moreover, this article's primary focus of analysis lies in Teacher Bia's approach to these connections throughout her training journey.

As she collaboratively engaged in resolving Learning Triggering Situations, Teacher Bia experienced shifts in her perception of algebra as a domain of knowledge intertwined with the dynamics of quantity variance and its resultant relationships. Furthermore, she seemed to gravitate towards the concepts of fluency, variance field, and unknown value (variable and unknown quantity), i.e., algebraic conceptual nexus, a progression indicative of the development of algebraic thinking.

Desarrollo del pensamiento algebraico en docentes desde los primeros años en el contexto de la educación continua.

RESUMEN
Las discusiones sobre la enseñanza del álgebra en los primeros años de la educación primaria han sido objeto de investigación en las últimas décadas y se han intensificado en los últimos años, especialmente con la nueva Base Nacional Común Curricular. Las investigaciones relacionadas con la formación de profesores que enseñan matemáticas en los primeros años se vuelven relevantes ya que el profesor es responsable de organizar la enseñanza, estando directamente relacionado con el movimiento de aprendizaje de los estudiantes. En este artículo, presentamos un recorte de una investigación, fundamentada en la perspectiva Histórico-Cultural, que tuvo como objetivo investigar el desarrollo del pensamiento algebraico en profesores de los primeros años, considerando el pensamiento algebraico como el pensamiento teórico mediado por conceptos algebraicos. Los aspectos metodológicos se basaron en el materialismo histórico-dialéctico y la producción de datos se realizó a través de un experimento formativo organizado en base a la Actividad Orientadora de Enseñanza, con Situaciones Desencadenantes de Aprendizaje que consideraran el movimiento lógico-histórico del álgebra, con el objetivo de acercarse a la esencia del álgebra y a los vínculos conceptuales algebraicos. El recorte de análisis presenta escenas del movimiento formativo de una de las profesoras participantes en el experimento, permitiendo la identificación de posibles superaciones en la forma en que la profesora define qué es el álgebra, además de su comprensión sobre la fluidez, variación de cantidades, variable y campo de variación, en un movimiento de lo

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general a lo particular, es decir, de la búsqueda de reglas generales que resuelvan situaciones específicas. Tales movimientos de aproximación a los vínculos conceptuales algebraicos y la esencia del álgebra por parte de la profesora se refieren al proceso de desarrollo del pensamiento algebraico.

Palabras clave: Pensamiento algebraico; Pensamiento teórico; Vínculos conceptuales algebraicos; Formación continua de profesores de los primeros años; Actividad Orientadora de Enseñanza.

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