Obutchénie
Revista de Didática e Psicologia Pedagógica

DOI: http://doi.org/10.14393/OBv8.e2024-21

# The nature of mathematical knowledge and the historical dimension of the concept in the organization of teaching for meaningful learning[1]

## A natureza do conhecimento matemático e a dimensão histórica do conceito na organização do ensino para uma aprendizagem significativa

*Ana Paula Gladcheff*[2]
*Neusa Maria Marques de Souza*[3]
*(In Memoriam)*

**ABSTRACT**
In this article, based on the foundations of Cultural-Historical Theory and our research findings, we aim to demonstrate how a teaching situation organized based on the logical-historical movement of the concept enables meaningful learning processes. We consider mathematical knowledge to be a humanly constructed product, the result of social life practices, and that the child, by attributing meaning to the ideas shared in such practices, appropriates historically constructed social meanings, learns, and develops by

**RESUMO**
Procuramos evidenciar neste artigo, a partir dos fundamentos da Teoria Histórico-Cultural e de resultados de nossas pesquisas, como uma situação de ensino organizada com base no movimento lógico-histórico do conceito viabiliza processos de aprendizagem significativa. Consideramos o conhecimento matemático como produto humanamente construído, resultado de práticas da vida social e que a criança, ao atribuir sentido às ideias compartilhadas em tais práticas, vai se apropriando de significados sociais construídos historicamente, aprende e se

[1] English version by Felipe de Carvalho Menezes. E-mail: Menezes-felipe@outlook.com

[2] Post-doctoral degree in Education by CECH/UFSCar, São Carlos, with the line of research "Education in Sciences and Mathematics". Doctoral degree in Education by FE/USP, São Paulo, with the line of research "Sciences and Mathematics Teaching". Member of Grupo de Estudos e Pesquisas sobre Atividade Pedagógica - GEPAPe and Grupo de Estudos e Pesquisa do Ensino e Aprendizagem da Matemática na Infância – GEPEAMI. Orcid: https://orcid.org/0000-0002-8281-4651. E-mail: anapaula.glad@alumni.usp.br.

[3] Post-doctoral degree in Mathematical Education by FE/USP, São Paulo. Doctoral degree in Education (Curriculum) by PUC/SP. Member of Grupo de Estudos e Pesquisas sobre Atividade Pedagógica – GEPAPe and Coletivo de estudos e pesquisas sobre psicologia escolar e atividade pedagógica-TECER. Collaborating Professor at the Graduation PRogram in Mathematical Education (Master's and Doctoral degrees) at UFMS-Universidade Federal de Mato Grosso do Sul. Orcid: https://orcid.org/0000-0003-4775-7548. E-mail: neusammsouza@gmail.com.

We regret to inform our readers that Neusa Maria Marques de Sousa, a co-author of this paper, passed away on 2024 June 14th. Sousa made significant contributions to this research and was instrumental in Mathematics Education with studies in the area of initial and continuing training of teachers . Her expertise, dedication, and passion for Mathematics Education will be greatly missed by colleagues and peers.

DOI: http://doi.org/10.14393/OBv8.e2024-21

establishing relations and creating new ideas. As a result of our studies on the conceptual nexus of the concept of measurement, which is defined as a link between concepts and which is fixed by the logical-historical movement of the concept, we have developed a brief analysis based on a triggering learning situation developed with early childhood education students to work with this concept and we point out how human relations mediated by historically produced cultural instruments are essential for meaningful learning to take place from the beginning of schooling.

**Keywords:** Logical-historical movement of the concept; Teaching organization; Mathematics teaching activity; Conceptual nexus of measurement; Problem situation.

desenvolve estabelecendo relações e criando novas ideias. Decorrente de nossos estudos sobre os nexos conceituais do conceito de medida, os quais são definidos como elo entre os conceitos e que se encontram fixados pelo movimento lógico-histórico do conceito, desenvolvemos uma breve análise a partir de uma situação desencadeadora de aprendizagem desenvolvida com alunos da educação infantil para o trabalho com este conceito e pontuamos como as relações humanas mediadas pelos instrumentos culturais historicamente produzidos são imprescindíveis para que aprendizagens significativas ocorram desde o início da escolarização.

**Palavras-chave**: Movimento lógico-histórico do conceito; Organização do ensino; Atividade de ensino de matemática; Nexos conceituais de medida; Situação-problema.

## 1 Introduction

In this article we aim to highlight the historical dimension of the concept as a necessary condition for organizing mathematics teaching in a meaningful way, as well as the relevance of the teacher's role as mentor of processes intentionally organized and mediated by culturally produced instruments, based on our research ground on Cultural-Historical Theory. Our narrative includes the investigation of the process of signification of the activity of teaching mathematics, intrinsically linked and with possible derivations from actions that make up a teacher training activity[4]. We will address studies on conceptual nexuses, understood as links between concepts established by the logical-historical movement of the concept of fraction that originates in the relation between quantities, present in human relations and, therefore, in the movement of formation of the concept of measurement.

To this end, we will first describe our view of mathematics as a human product and our understanding of the need for children to appropriate mathematical knowledge and its processes of signification from the beginning of

[4] The research received funding from CAPES.

DOI: http://doi.org/10.14393/OBv8.e2024-21

schooling (Leontiev, 1978). Otherwise, they end up being deprived of a life full of possibilities for coexistence in the community in which they live and, above all, for them to become aware that the human social experience is the source of their experience. The notes and syntheses that we will make arise from this understanding when we propose the historical dimension of the concept of measurement, evidenced through its logical-historical movement, as a necessary condition for the movement of organizing mathematics teaching since the beginning of schooling. We show a brief analysis of how human relations mediated by historically produced cultural instruments are essential for learning to occur. This allows the concept to be placed in the movement of appropriation by children, involving them in a social and ludic learning activity.

## 2 The nature of mathematical knowledge and the role of schooling from the early years

Mathematical knowledge in its theoretical form is conceived, from a historical perspective, as the result of practical needs of social life. Alexandrov (2016) states that, like other sciences, mathematics, produced through a process of abstraction, "reflects the laws of the world around us and serves as a powerful instrument for knowledge and mastery of nature" (p. 11, our translation). And this is presented as a characteristic of all mental activity in general. The numerical symbol (numeral) is a good example of an abstraction that arises from the human need to record increasingly larger quantities accompanied by the numbering system, which allows us to represent the name of any possible number by a symbol, without having to resort to concreteness of what symbolizes such quantity. This means operating with abstract numbers, through the production of mental models, without having to resort to its concreteness (Lanner de Moura; Lima; Moura; Moisés, 2016).

What happens, however, is that such abstractions have reached such a high degree of generalization that they end up apparently losing the connection they have with practical life, making mathematical knowledge often incomprehensible if we do not appropriate its concepts in a meaningful way.

In this context, when we conceive mathematics as the result of practical

DOI: http://doi.org/10.14393/OBv8.e2024-21

needs of social life, we mean that all mathematical knowledge has as its foundation a human activity practiced to satisfy the needs of social life (in collectivity) (Caraça, 2010). For that reason, we understand it as a human product and, therefore, a social object, knowledge related to real life, both in its origin and in its applications.

Through this vision, we conceive that mathematical knowledge in its theoretical form, as a cultural product, is considered a human wealth and, therefore, everyone has the right to appropriate it in such a way that it, in a symbolic tool dimension, becomes an instrument of thought that allows the individual to transform reality and not just adapt to it (Munhoz, Moura, 2019). With this understanding, we can give school education an important role in the humanization of individuals through this knowledge. This means that, understood as a path to human development, school education has the function of intentionally promoting the socialization of knowledge that was historically produced and systematized by humanity up to the present moment, given by scientific concepts (Saviani, 2011).

In the case of children, we are aware that they have had contact with mathematics since birth, as they interact daily with human productions that represent mathematical knowledge (Serrão *et al.*, 2012). However, when you enter school, whether in Early Childhood Education or in the early years of Elementary School, they come to know a way of learning that is different from the way they were used to in daily life. It happens because "the entire system of their vital relations is reorganized" and they come into contact with systematized and elaborated knowledge, that is, scientific knowledge (Leontiev, 1978, p. 289). From that moment on, their obligations cease to be solely towards their parents and the educator, and become related to society. Therefore, from the beginning of schooling, where children's first relations with mathematical knowledge in its theoretical form occur, it must have meaning for them. Their knowledge begins to expand without abandoning what they carry in their life story.

By understanding, for example, the meaning of one-to-one correspondence as a human product, as one of the fundamental relations of the concept of number,

DOI: http://doi.org/10.14393/OBv8.e2024-21

through other actions of grouping, inclusion, ordering, frequency. and position, children are able to understand the numbering system we use in a way that is meaningful to them: as a product of social life practices for the control of discrete quantities. Mathematical operations, from this perspective, also take on a meaning that makes it possible to develop the relation with controlling the variation of continuous quantities, in a geometric dimension, and with variables and movement, in an algebraic dimension, for example. In this way, children appropriate elementary and basic mathematical knowledge, as a human production, from the beginning of schooling. And this is important because all the more complex mathematics is based on the elementary knowledge that is developed throughout Elementary School.

In this sense, as Moura (2013) states, the function of mathematics from the beginning of schooling happens through mathematical literacy with the aim of satisfying the needs of human integration. This means understanding it not just as algebraic, arithmetic, geometric, and stochastic decoding, but in the relations that exist in controlling the variation of quantities, whether discrete or continuous, of shapes and spaces, of variables and movement, of data collection, organization, and analysis. In this way, mathematical knowledge may become, as already mentioned, an instrument of thought.

## 3 The logical-historical movement of the concept as a possibility for the organization of mathematics teaching from the early years of schooling

In the context of Cultural-Historical Theory, we can say that children learn by attributing meaning to ideas, by appropriating historically constructed social meanings, which enables them to establish relations and create new ideas. Thus, the school can be understood as the privileged space where the contents developed are intentionally constituted as objects of an activity whose purpose is to make the subjects who participate in it appropriate both these objects and the way to deal with them, providing meaning to the life that takes place in community (Moura, 2013).

DOI: http://doi.org/10.14393/OBv8.e2024-21

The system of meanings, although in eternal transformation, is *ready* when the individual is born and it is up to the previous generation to create conditions for this new generation to appropriate this system. Gladcheff (2015) reiterates that "meaning has different amplitudes as individuals appropriate it. Through the process of appropriating meanings with ever greater depths, new meanings are formed" (p. 64, our translation). That is why meaning not only allows communication between individuals, but also reflects existing social relations and the way in which such appropriation occurs depends on the meaning attributed to it. In this sense, Leontiev (1978) states that conscious meaning is created by the objective relation between what incites action in the subject (mean) and what their action is oriented towards as an immediate result (end). Each individual's action acquires meaning in the whole of social activity, that is, in the social relations between the individual and the rest of the group (Duarte, 2004).

However, appropriating knowledge in such a way as to turn it into belonging and participation in social practices is not a simple task and we can understand the need to develop ways of teaching and ways of approaching concepts that make this process possible in a meaningful way. Libâneo (2004) states that it is "fundamental to understand that knowledge presupposes the development of thought and that developing thought presupposes methodology and systematic procedures for thinking" (p. 6, our translation).

To this end, we believe in learning through educational situations that are challenging and ludic, while "make children aware of the need of the concept to be taught" (Moretti; Souza, 2015, p. 28, our translation). In this case, ludic activities, such as games or play, as they are the children's main way of relating to reality, are what enhances their possibility of learning and appropriating new knowledge. Therefore, they can be important methodological resources in the process of teaching mathematics, as they encourage the learning of "mathematical structures that are often difficult to assimilate", developing the "ability to think, reflect, analyze, understand mathematical concepts, raise hypotheses, test and evaluate them" (Leontiev, 2010; Grando, 2008, p. 26, our translation).

DOI: http://doi.org/10.14393/OBv8.e2024-21

When bowling, for example, each child participates above all aiming to score points. The score must have a meaning for them. This process takes place through the one-to-one correspondence that the child apprehends in the relation between the collection of points and the collection of bottles knocked over (Lanner de Moura; Rocha; Moura; Silva, 2023). In this way, we are reproducing, in a particular way (through bowling), the generic form of human activity embodied in the concept of number, which at the moment represents the number of points the children score in each of their moves. By comparing their score with that of each of their classmates, the children begin to understand the relation of inclusion between the collections now given by the scores of the whole class. Thus, they begin to understand the meaning of one quantity being higher than, lower than, or equal to another.

The act of playing, in this case, is oriented towards the construction of the concept of number, which initially takes place in the one-to-one correspondence between the number of points and the number of bottles knocked over. This may therefore be seen as an initial moment in the appropriation of the concept of number by the children who, when needing to record their score, begin to give meaning to the numerical symbol that represents it. In this way, the concept of number is placed in the movement of appropriation by the children, involving them in a social and ludic activity. What is always important to remember is that the content to be learned is not in the game itself, but in the act of playing with social mediations (Moura, 2013).

What has been described allows us to understand the possibility of approaching a concept as a result of the practical needs of social life. In the same direction, in order to conceive of a concept as a social object, knowledge related to real life, both in its origin and in its applications (Caraça, 2010), we must know what Davydov (1982) names as *conceptual nexus of a concept*. For the author, the scientific concepts that make up mathematical knowledge in its theoretical form contain external and internal nexuses. The external nexuses are associated with the formal language of the concept because, as Sousa & Moura (2016, p. 2) express, "they are clean, stripped of the human work that

DOI: http://doi.org/10.14393/OBv8.e2024-21

generated them" (our translation). The internal nexuses (or conceptual nexuses), on the other hand, are associated with the logical-historical movement of the studied object, which represents the essential aspect of the concept and, in this case, "contains the logic, the history, the abstractions, the formalizations of human thinking in the process of becoming human through knowledge" (Sousa; Panossian; Cedro, 2014, p. 96, our translation).

In order to understand a concept in a meaningful way, therefore, it is necessary to apprehend the development process of the human activity that is embodied in it, that is, its logical-historical process. In a historical dimension, we show the essence of the human needs that motivated the production of the concept and, in a logical dimension, we show the systematization that reveals the responses created by humans to meet these needs. These responses, in their theoretical form, when proposed as the object of teaching and learning, are translated into the symbolic or intellectual tools that must be appropriated by the students.

However, we emphasize that when we approach the role of the history of mathematics in teaching, we do so by highlighting the logical-historical development of concepts. This does not mean that we should teach mathematics through history, nor repeat the historical path in the formation of a mathematical concept, but to search the historical process for the movement of thought in the context of the formation of this concept and its symbolic representation.

For this reason, the ways in which teaching is organized play an essential role in defining the quality of the teaching situations that teachers will provide for their students to develop in activities. This is because it is known that the appropriation of knowledge does not occur through simple natural contact with the surrounding physical and social phenomena, but through mediation that is established under educational conditions and in an intentional way. In this process, in which "school learning guides and stimulates development processes" (Vigotski, 2007, p. 116, our translation), human relations mediated by historically produced cultural instruments are essential for human learning to take place.

DOI: http://doi.org/10.14393/OBv8.e2024-21

One possibility proposed by Moura (2012) is teaching organized in such a way as to privilege the logical-historical movement of the concept, which works methodologically with the proposition of problem situations organized by *triggering learning situations*, which may be materialized as: a *game* with a pedagogical purpose, which preserves the character of a problem; a *problematization of everyday situations*, which gives children the opportunity to experience the solution of problems that are significant to them; or, a *virtual history of the concept*, which puts children in front of a problem situation similar to that experienced by humanity (in the generic sense). In this case, the history that surrounds the triggering situation "is not the factual history, but that which is impregnated in the concept, considering that this concept objectifies a historically placed human need" (Moretti; Moura, 2011, p. 443, our translation).

With the *learning triggering situation*, teaching actions are planned to guide students towards the collective solution of a *triggering problem* that carries in itself the genesis of the concept. The genesis given by the reproduction of the human need that historically generated that concept, setting it in motion so that it can be appropriated by the students.

We emphasize that this way of organization of teaching is anchored in the concept of Teaching-Orienteering Activity developed by Moura (1996, 2000, 2017) and collaborators when they focus on pedagogical practice based on the elements that make up the psychological concept of activity formulated by Leontiev (2010). The Teaching-Orienteering Activity, understood as a theoretical-methodological basis, is governed by principles that make it explicit as a unity between the teaching activity (by the teacher) and the learning activity (by the student) in the pedagogical activity context. It is, therefore, a way of thinking about the organization of teaching, which considers the unity between teaching and learning, as indicated by the theoretical and methodological assumptions of Cultural-historical theory

To corroborate the considerations we have made so far, we are going to keep explaining what we understand by the conceptual nexuses of measurement. Then, we are going to present a teacher training episode in which an Early Childhood

DOI: http://doi.org/10.14393/OBv8.e2024-21

Education teacher reports on what she and her students experienced during the development of a teaching activity triggered by a *learning triggering situation*, materialized in a virtual story entitled *Verdim e seus amigo*s. The virtual story presented to the students poses a problem to be solved collectively. Guided by the actions planned by the teacher, while they search for a solution to the problem, the concept of measurement is put into motion of appropriation. As we will be able to see, the way in which the concept is being approached makes it possible for its meaning to be put into a learning motion and, with this, the subject now has a new tool to act with. This symbolic tool, given in this case by the concept of measurement, may be used to solve other problems that may arise.

## 4 The conceptual nexuses established by the logical-historical movement of the concept of measurement

The conceptual nexuses of measurement originate from the relationship between magnitudes. Magnitude, in turn, may be defined as a quality of an object or phenomenon that may be quantified. And, we refer to quality as the "set of relationships in which a given being encounters other beings in an aggregate" (Caraça, 2010, p. 93, our translation). Therefore, we can say that quality, as an attribute of an object or phenomenon, is always relative to something, based on comparison and identification.

The moment the considered quality may be expressed in numbers, we affirm that it may be measured, that is, through measurement we may numerically express the quality of an object or phenomenon (Caraça, 2010). However, the quantities involved in measuring a magnitude relative to an object or phenomenon are not presented in *discrete* form (as elements that are organized in natural units), but in *continuous* form (which theoretically admit infinite divisibility).

Alexandrov (2016) expresses his understanding on the conflict of opposites, the discrete and the continuous, reporting that:

> [...] Each object separately [discrete] is indivisible in the sense that, when it is divided, it almost always ceases to be what it

DOI: http://doi.org/10.14393/OBv8.e2024-21

> was, as is clear from the examples of "a third of a man" or "a third of an arrow". On the other hand, continuous and homogeneous magnitudes and objects are capable of being divided and grouped together again without losing their essential character. Mashed potatoes offer an excellent example of a homogeneous object that in itself is not made up of parts, but may nevertheless be separated into parts as small as desired. Lengths, areas. and volumes have the same property: although they are essentially continuous and are not really divided into parts, they undoubtedly offer the possibility of unlimited division (p. 52-53, our translation).

When we compare two magnitudes, therefore, we are beginning the process of measurement and we do so for the practical needs of social life. Lanner de Moura (1995, p. 67, our translation) tells us that, "[...] just as the need to control the variation of quantities led humans to create numbers, the need to control variations in the dimensions of objects led them to measure space".

Caraça (2010) exemplifies the comparison stage by performing it between the lengths of two line segments. However, the author states that the process is not finished just by issuing a simple result of - *length greater or less than* -. In general, an answer to this question is requested:

> [...] - how many times does one length fit into another? But that is not all yet; if there is no single term of comparison for all magnitudes of the same kind, the exchange operations required by today's social life become, if not impossible, at least extremely complicated (Caraça, 2010, p. 29, our translation).

We conclude that there are, therefore, in the movement of forming the concept of measurement, three steps to be completed: identification of the magnitude of an object or phenomenon to be measured (what will be measured); comparison between magnitudes of the same type belonging to two (or more) objects or phenomena (with this we establish relations of greater, lesser or equal); establishment of a common unit of measurement that allows us to quantify magnitude and express the comparison using a number.

The logic described, as we can see below, comes from a human activity carried out to satisfy a need in social life. It represents the systematization

DOI: http://doi.org/10.14393/OBv8.e2024-21

that highlights the responses created by humans to meet this need. The synthesis made up of these steps forms the conceptual nexuses from which new syntheses will be structured.

According to Caraça (2010), Herodotus, a Greek historian from the 5th century b.C., when writing about the Egyptians, justifies the need for numerical expression of measurement by the individual's relations with the State based on property, approximately four thousand years ago. The historian refers to the origins of Geometry with the following narrative:

> *This king (Sesostris) divided up the land among the Egyptians, giving an equal square plot to each man; from this he derived his revenue, imposing a rent to be paid each year. But if the river (Nile) carried away a portion of any man's lot, he would come to him and report what had happened. And the king would send men to examine and to measure by how much the land had been diminished, in order that he might pay only a proportionate amount of rent. And it appears to me that geometry was discovered in this way, and that it afterwards came over into Greece* (Caraça, 2010, p. 32, author's highlights, our translation).

However, in the measurement process, it is generally impossible to place the selected unit an integer number of times in the magnitude to be measured. Therefore, the simple arithmetic calculation of the number of units is not sufficient. It is in this way, through this human practice, that the need arises to divide "the unit of measurement and express magnitude with greater accuracy in parts of the unit; that is, not through integer numbers, but through fractions". Fractions, as a numerical expression, arose "from the division and comparison of continuous magnitudes; in other words, from measurements" (Alexandrov, 2016, p. 43-44, our translation).

From the above, we can infer that the process of meaning of the concept of measurement comes from its historical-logical movement. This enables us to undertake pedagogical actions in order to explain the relationships, or conceptual nexuses, essential for the concept of measurement to be understood. In this way, "this element that arises from the relationships of

DOI: http://doi.org/10.14393/OBv8.e2024-21

men among themselves and with nature when constructing measurement" is considered a guiding principle for how to pose the measuring problem to children (Lanner de Moura, 1995, p. 67, our translation).

Referring to the elementary means of logical thinking, Talízina (2009, p. 74) highlights how promoting the development of logical thinking in students is in the teachers' hands and that, for this purpose, they need to know the general and specific actions that are included in the school learning activity. The author brings this domain as relevant in view of her research notes regarding the fact that from elementary to higher levels of schooling, "the majority of students do not master the initial means of logical thinking" (our translation).

Among such actions, the author mentions the following as indispensable means for studying mathematics from the initial levels: leading to the concept, deduction of consequences, and comparison. We are going to make some considerations about the means of comparison that will be explored in a triggering learning situation presented in the next topic of this article. According to Talízina (2009, p. 83-84),

> The comparison will only be concrete when it is used, firstly, during the comparison of homogeneous objects and phenomena of reality (plants, buildings, animals, etc.); secondly, when the comparison is carried out according to the essential characteristics. Comparison assumes the ability to perform the following actions: 1) identification of characteristics in objects; 2) establishment of general characteristics; 3) identification of the basis of comparison (one of the essential characteristics); and 4) comparison of objects according to the chosen basis (our translation).

It is important to highlight that comparisons are possible for both qualitative and quantitative aspects, as long as attention is paid to the selection of a basis for comparison. Quantitative comparison demands a unit of measurement with which the comparison is carried out, which, at first, may be established as an "*immediate* comparison" on the basis of which "the *mediated* comparison is formed", since this does not occur "immediately, but rather with the help of the measure, that is, in a *mediated* way" (Talízina, 2009, p. 84, our translation).

DOI: http://doi.org/10.14393/OBv8.e2024-21

Aware of these assumptions, we are going to develop analyzes of the episode presented in the following topic.

## 5 An episode in teacher training activity: Mediated processes in the development of the triggering learning situation "Verdim and his friends" by the teacher with the students

In one of our studies (Gladcheff, 2015), we experienced a training activity with teachers who teach mathematics in Early Childhood Education and in the early years of Elementary School. In this process, lasting four years, we were able to demonstrate and propose actions that enhanced the significance of the mathematics teaching activity of those who participated in it, triggered by the unity between mathematical theoretical knowledge and the planning of teaching actions for such knowledge, based on principles governed by the Historical-Cultural Theory. The scene we are going to present was taken from training meetings that took place weekly. It brings the presentation prepared by two teachers, identified by Teacher1 and Teacher2, about Teacher1's first experience with the development of a triggering learning situation in her classes: the virtual story of the concept *Verdim e seus amigos*.

Chart 1: The virtual story *Verdim and his friends*

| *Virtual story: Verdim and his friends* |
|---|
| *Once upon a time there was Verdim, an enchanted being who lived in a forest from another world. Verdim had many friends and they played together every day in the forest clearing. Almost all of them lived close to Verdim's house, except three of them: the giant called Tililim and two dwarves, Edim and Enim.* |
| *One day, Verdim invited everyone to play at his house. As Tililim, Edim, and Enim lived very far away, Verdim explained how to get to his house.* |
| *Leaving the clearing, on the side where the sun sets, they should take fifty steps forward, then thirty steps until the big tree, and then they should continue straight ahead and his house would be just ten steps away.* |
| *With Verdim's explanation, they wrote down everything they needed to do so as not to forget anything.* |
| *The next day, they headed in the right direction. However, they could not reach Verdim's house.* |

DOI: http://doi.org/10.14393/OBv8.e2024-21

*What could have happened? Why could they not arrive?*

*How can we help Verdim find out what happened so he can find another way to explain how to get to his house?*

Source: Virtual story elaborate by members of GEPAPe and GEEAMI, from Lanner de Moura (1995).

The virtual story *Verdim and his friends*, shown in Table 1, was originally created by a group of researchers members of Grupo de Estudos e Pesquisa sobre Atividade Pedagógica (GEPAPe) and Grupo de Estudos do Ensino e Aprendizagem de Matemática na Infância (GEEAMI), from Lanner de Moura (1995). It was presented to the teacher training group with the aim of carrying out an analysis of its potential to set the concept of length measurement into a movement of appropriation, through mediated, planned, and oriented actions to solve the problem proposed by the teacher.

The following dialogue excerpt refers to the presentation of an experience by Teacher1 developed at her school and which was stimulated by her participation in the training activity that had been taking place for nine months. In this meeting, Teacher1, with the collaboration of Teacher2[5], presented her experience to the group so that they could analyze and reflect on what was exposed.

***Teacher1**: Our presentation is: the need to measure as a problem.*

*So, is it possible to reflect on the need to measure with children in Early Childhood Education? And this work was done with the children I teach there in the East Zone.*

*We searched a little about what the official documents from the city hall, from Early Childhood Education (we have both), talk about measuring, in Early Childhood Education.*

*We observed that they place more emphasis on everyday life situations and what children bring in terms of prior knowledge, rather than what they know about measurements in everyday life. And the use of specific vocabulary related to measuring.*

[5] Teacher1 e Teacher2, teachers in Early Childhood Education, participated in the research project: "Educação matemática nos anos iniciais do Ensino Fundamental: princípio e práticas da organização do ensino". Observatório da Educação – Edital CAPES 2010. University of São Paulo (USP) - Faculdade de Educação (FE). Nucleus coordinator: Prof. Dr. Manoel Oriosvaldo de Moura – FEUSP.

DOI: http://doi.org/10.14393/OBv8.e2024-21

*They often mention this issue of measuring with a straw, with a string, even with steps. Well, they asked it to be done. And not the way we did it. So, the proposal is the need to measure as a problem.*

*The story of* Verdim and his friends. *Then I told them the story (*she tells at this moment that they had planned to use polystyrene foam to keep the characters in the model standing. However, it would not hold them so the teacher used gouache paint boxes to support them, as shown in Figure I*).*

*So, how was the discussion with the children? At first, they were just stuck in the context. When I asked the questions, "Oh! It's because the giant has his face in front", "Oh! But they did not arrive because there were too many leaves in the way", and another one said, "It's easy, just go straight".*

*And I was always drawing their attention, leading them, to see if they could get to my idea (*she played a video recording part of the activity in the classroom).

*So, at that first moment, they were very stuck in the context. There was a moment when I put Verdim together with the others and then a child said that Verdim could teach them, since they were together.*

*Then, leading the discussion, they managed to come up with the idea that the difference in the step size of each of the characters was the reason why they did not reach Verdim's house. The little girl, Julia, arrived at this idea.*

***Teacher2**: Because first they were going for foot size. Then she* (Teacher1) *said, "So how does the little dwarf walk?"*

***Teacher1**: Then she showed it. Then they showed Verdim's and the giant's steps (*The teacher then asks Julia to show the dwarf's step to the whole class, then Verdim's and then the giant's).

*So what do we conclude from this?*

*For children, there has to be a ludic context, because they get involved with the story. It was not just about telling. They position themselves as if they were the characters.*

***Teacher2**: It's interesting that even afterward, there are still some children who get stuck. She* (Teacher1) *asks, "What does the dwarf's foot look like?" "Shoeless".*

*So, there are children who have already understood the measurement and there are children who are still stuck in the figure.*

***Teacher2**: You promote circulation of several proximal zones.*

***Teacher1**: The role of the adult, right? The teacher articulates and mediates the children's ideas to maintain their focus on the problem situation.*

DOI: http://doi.org/10.14393/OBv8.e2024-21

> *They begin to imagine, to raise their hypotheses. And we have to always be calling, focusing attention on the problem to be solved. Otherwise, they will disperse.*
>
> *This situation, then, leads to thinking, mathematical reflection, analysis and synthesis on a certain aspect of reality that leaves common knowledge and has the possibility of leading thinking towards scientific knowledge.*
>
> *So, the paradigm shift has an impact. It causes insecurity. Because we say, "What now? I do not know how to teach anymore. Faced with a proposal like this, because we had a way of teaching, but now we see it like this. So, everything I have taught them so far is wrong, poor children (*making gestures showing discontent*)*"
>
> *I saw another way of teaching here. We learned one way and we end up reproducing that way. This here (*referring to the development of the class initiated by the triggering situation*) forms us. We start to see things differently, we become more selective.*
>
> *The central thing, what I think is so important, is to make the child think about the object of study.*

By using the virtual story to trigger collectivity and set the concept into motion in the classroom, Teacher1 is faced with situations different from those she has experienced during her teaching career. She demonstrates her concern when realizing the possibility of leading a five-year-old child to, in her words, "think, reflect mathematically" leaving common knowledge and guiding them to scientific knowledge.

To this end, Teacher1 understands the importance of mediation constituted by herself when saying "The role of the adult, right? The teacher articulates and mediates the children's ideas to maintain their focus on the problem situation"; by the material resource she is using – the model presented in Figure I, when saying "There was a moment when I put Verdim together with the others and then a child said that Verdim could teach them, since they were together"; and by the story, when she points out that "For children, there has to be a ludic context, because they get involved with the story. It was not just about telling. They position themselves as if they were the characters".

DOI: http://doi.org/10.14393/OBv8.e2024-21

Figure I - Part of the setting constructed by Teacher1 for the development of the activity..

Source: Photography provided by Teacher1 (Gladcheff, 2015, p. 161).

The students, as reported by the teacher, at first, were stuck in the context of the story itself and developed hypotheses directly related to what they were manipulating. But when a child exposes the idea that "the difference in the step size of each of the characters was the reason why they did not reach Verdim's house", Teacher1 starts to lead the discussion based on this idea, as the magnitude to be considered was explicit: the length.

Faced with the teacher's questions ("So how does the little dwarf walk?"), children were led to get involved in the activity as subjects in the discovery of knowledge. Guide questions, according to Vigotski (2004), should lead students to theoretical syntheses and, for this, it is necessary for the teacher to value the way in which they are proceeding to solve the proposed problem, as demonstrated by Teacher1 when asking the child shows the dwarf's step to the group, then Verdim's and then the giant's. At this moment, we are able to notice the action of comparison between magnitudes of the same type, that is, between the lengths of each character's steps. The teacher, therefore, begins to reflect on her pedagogical interventions and demonstrates a conflict between the way she worked before and the way she believes to be the most appropriate at that moment. She expresses her confidence in the training group by expressing her concerns, stating that she has learned a new way of teaching. This represents a great challenge, as commented by Sforni:

> [...] It is necessary that the teaching of scientific concepts is based on didactic procedures aimed at appropriating the concept as a mental activity, which is very different from the conceptual teaching model typical of school tradition and materialized in textbooks and

DOI: http://doi.org/10.14393/OBv8.e2024-21

> workbooks. Organizing teaching from this perspective is, therefore, a great challenge as it implies following paths that are still little known (Sforni, 2015, p. 377, our translation).

With this teaching activity developed by the teacher in the classroom, it is also possible to highlight the way in which theoretical knowledge may be consciously appropriated by students and teachers. The *triggering learning situation*, materialized in the virtual story *Verdim and his friends*, reconstitutes a human activity that embodies the genesis of the concept of measurement. In this way, *measurement* is treated as a problem to be solved by the children, and the teacher, through pretend play, involves them in a problem situation in mathematics. She does so consciously, remembering that playing is the main activity for this age group (Leontiev, 2010), and this allows children to participate in the story and appropriate a human social experience that took place in the production of worked knowledge. She works with the movement of thought carried out to solve a problem related to human social practice.

The problem situation inserted in the virtual story *Verdim and his friends* rescues the essence of the concept of measurement and makes use of the logical-historical movement of the concept as opposed to the idea that the concept is assimilated in a mechanized way, in its systematized form as a final product. As Leontyev (1981, p. 422) indicates, the child, contrary to a widely held opinion, "does not adapt itself to the world of human objects and phenomena around it, but makes it its own, i.e. appropriates it". Appropriation, according to the author, "is a process that has as its end result the individual's reproduction of historically formed human properties, capacities, and modes of behaviour".

We were able to verify that, during the attempt to solve the problem posed here, the children became involved in the activity, as mentioned by the teacher, as if they were the characters in the story themselves, which makes it possible to appropriate the concept as reported by Leontyev (1981). As these were five-year-old children, the numerical expression equivalent to the quantity for the length magnitude was not worked on. However, the children came up with a solution to

DOI: http://doi.org/10.14393/OBv8.e2024-21

the problem by establishing a common unit of measurement so that everyone could find *Verdim's* house.

Consider the teacher's awareness when she brings at the end of her speech that "*The central thing, what I think is so important, is to make the child think about the object of study".* She is, at this point, in line with what Ilyenkov (2007) argues when stating that:

> We have to organize the process of the mastery of knowledge, the process of the assimilation of intellectual culture in the same way as the best teacher—life—has organized it for thousands of years. Namely, in such a way that in the course of this process the child should be forced constantly to train not only (and even not so much) the "memory" but also the ability independently to solve tasks that require thinking in the proper and precise sense of the word [...] (Ilyenkov, 2007, p. 16-17).

Finally, we highlight the possibility that a teaching situation such as the one described must provide the educational environment with "attention to individual differences, the particularities of the problem put into action, and the various types of knowledge present in it, with the aim of forming subjects in the social direction of human formation that has the collective as a reference" (Munhoz, Moura, 2020, p. 358, our translation).

## 6 Final considerations

Throughout this text we sought to highlight the historical dimension of the concept, based on the study of its logical-historical movement, as a possibility for organizing mathematics teaching, since the beginning of schooling.

To this end, we discussed a view of mathematics as the result of practical needs of social life and based on the conceptual nexuses established by the logical-historical movement of the concept of measurement, as well as the need for knowledge, by the teacher, of actions for the development of logical thinking of students and their essential characteristics.

DOI: http://doi.org/10.14393/OBv8.e2024-21

We presented a *triggering learning situation* created by a group of researchers and developed by a teacher, in a classroom, with five-year-old children. This *triggering learning situation*, materialized in the virtual story *Verdim and his friends*, brings in a playful way an engaging problem, which contains the genesis of the concept of measurement, given by the reproduction of the human need that historically generated such concept. This means that, in this case, measuring became a problem to be solved by children through pretend play, involving them in a problem situation in mathematics.

The steps that define the relations, or conceptual nexuses, essential for the concept of measurement to be understood in a meaningful way, are present in the way in which children end up solving the problem proposed by the triggering situation presented.

Thus, in accordance with the theoretical assumptions of the authors mentioned here and the research results we developed, whose appropriate focus in this space was presented here, we defend the historical dimension of the concept as a necessary condition for the movement to organize teaching.

Depending on such organization and the practice of setting it into motion, an intentional training process takes place that goes beyond the appropriation of knowledge, in itself important, by educational subjects, to the condition of mastery of generalized forms of thought. Forms that allow their insertions as social subjects and the condition of access and appropriation of cultural means generated by the movement of human significance.

## La naturaleza del conocimiento matemático y la dimensión histórica del concepto en la organización de la enseñanza para un aprendizaje significativo

**RESUMEN**
En este artículo procuramos evidenciar, a partir de los fundamentos de la Teoría Histórico Cultural y de los resultados de nuestra investigación, cómo una situación de enseñanza organizada a partir del movimiento lógico-histórico del concepto posibilita procesos de aprendizaje significativos. Consideramos que el conocimiento matemático es un producto humanamente construido, resultado de prácticas de vida social y que el niño, al atribuir significado a las ideas compartidas en dichas prácticas, se apropia de significados sociales históricamente construidos, aprende y se desarrolla

estableciendo relaciones y creando nuevas ideas. Como resultado de nuestros estudios sobre el nexo conceptual del concepto de medida, que se define como un vínculo entre conceptos y que está fijado por el movimiento lógico-histórico del concepto, hemos desarrollado un breve análisis basado en una situación de aprendizaje desencadenante desarrollada con alumnos de educación infantil para trabajar con este concepto y señalamos cómo las relaciones humanas mediadas por instrumentos culturales producidos históricamente son esenciales para que se desarrolle un aprendizaje significativo desde el inicio de la escolarización.

**Palabras clave**: Movimiento lógico-histórico del concepto; Organización de la enseñanza; Actividad de enseñanza de las matemáticas; Nexo conceptual de la medida; Situación problemática.

## 7 References

ALEXANDROV, Alexander Danilovich. Visión general de la matemática. In: ALEXANDROV, Alexander Danilovich; KOLMOGOROV, Andrei Nikolaevich; LAURENTIEV, Mikhail. *La matemática*: su contenido, método y significados. Madrid: Alianza Editorial, 2016.

CARAÇA, Bento de Jesus. *Conceitos fundamentais da matemática.* Revisto por Paulo Almeida. Lisboa: Gradiva, 2010.

DAVYDOV, Vasili Vasiliévitch. *Tipos de generalización en la enseñanza.* Ciudad de La Habana: Editorial Pueblo y Educación, 1982.

DUARTE, Newton. A formação do indivíduo, consciência e alienação: o ser humano na psicologia de A. N. Leontiev. *Caderno Cedes,* Campinas, v. 24, n. 62, p.44-63, abril 2004. DOI: https://doi.org/10.1590/S0101-32622004000100004.

GLADCHEFF, Ana Paula. *Ações de estudo em atividade de formação de professores que ensinam matemática nos anos iniciais.* Tese (Doutorado em Educação) – Universidade de São Paulo, São Paulo, 2015. DOI: https://doi.org/10.11606/T.48.2016.tde-09032016-103554.

GRANDO, Regina Celia. *O jogo e a matemática no contexto da sala de aula.* São Paulo: Paulus, 2008.

ILYENKOV, Evald Vassilievich. Our schools must teach how to think! *Journal of Russian and East European Psychology,* vol. 45, no. 4, July–August 2007, p. 9–49. DOI: https://doi.org/10.2753/RPO1061-0405450402.

LANNER de MOURA, Anna Regina. *A medida e a criança pré-escolar.* Tese (Doutorado em Educação) – Universidade Estadual de Campinas, Campinas, 1995.

LANNER de MOURA, Anna Regina; LIMA, Luciano Castro; MOURA, Manoel Oriosvaldo de; MOISÉS, Roberto Pérides. *Educar com a matemática:* fundamentos. São Paulo: Cortez, 2016.

LANNER de MOURA, Anna Regina; ROCHA, Cristina Helena de Souza; MOURA, Manoel Oriosvaldo de; SILVA, Silem Santos. *Controle da variação das quantidades:* iniciação à linguagem numérica. São Paulo: FEUSP, 2023. DOI: https://doi.org/10.11606/9786587047447.

LEONTIEV, Alexis. *O desenvolvimento do psiquismo.* Tradução de Manuel Dias Duarte. Lisboa: Livros Horizonte, 1978.

LEONTIEV, Alexis. Uma contribuição à teoria do desenvolvimento da psique infantil. In: VIGOTSKII, Liev Semionovich; LURIA, Alexander Romanovich; LEONTIEV, Alexis. *Linguagem, desenvolvimento e aprendizagem*. Tradução de Maria da Pena Villalobos. São Paulo; Ícone, 2010.

LEONTYEV, Alexis. *Problems of the development of the mind***.** Moscow: Progress publishers, 1981.

LIBÂNEO, José Carlos. A didática e a aprendizagem do pensar e do aprender – a Teoria Histórico-Cultural da Atividade e a contribuição de Vasili Davydov. *Revista Brasileira de Educação*, n. 27, 2004.

MORETTI, Vanessa Dias; MOURA, Manoel Oriosvaldo de. Professores de matemática em atividade de ensino: contribuições da perspectiva histórico-cultural para a formação docente. *Ciência e Educação*, Bauru, v. 17, n. 2, p. 435-450, 2011. DOI: https://doi.org/10.1590/S1516-73132011000200012.

MORETTI, Vanessa Dias; SOUZA, Neusa Maria Marques. *Educação matemática nos anos iniciais do Ensino Fundamental:* princípios e práticas pedagógicas. São Paulo: Cortez, 2015.

MOURA, Manoel Oriosvaldo de. A atividade de ensino como ação formadora. In: CASTRO, Amelia Domingues de; CARVALHO, Ana Maria Pessoa (orgs.) *Ensinar a ensinar:* Didática para a Escola Fundamental e Média. São Paulo: Cengage, Learning, 2012.

MOURA, Manoel Oriosvaldo de. A dimensão da alfabetização na educação matemática infantil. In: KISHIMOTO, Tizuco Morchida; OLIVEIRA-FORMOSINHO, Júlia (orgs) *Em busca da pedagogia na infância:* pertencer e participar. Porto Alegre: Penso, 2013.

MUNHOZ, Ana Paula Gladcheff; MOURA, Manoel Oriosvaldo de. Ações formadoras em atividade de formação contínua com professores que ensinam matemática nos anos iniciais da escolarização: uma iniciativa na perspectiva da

teoria histórico-cultural. *Revista Paranaense de Educação Matemática,* v. 8, n. 15, p. 62–88, 2019. DOI: https://doi.org/10.33871/22385800.2019.8.15.62-88.

MUNHOZ, Ana Paula Gladcheff; MOURA, Manoel Oriosvaldo de. Atividade de formação de professores de matemática mediada pela Atividade Orientadora de Ensino. *Obutchénie: Revista de Didática e Psicologia Pedagógica*, *[S. l.]*, v. 4, n. 2, p. 355–381, 2020. DOI: https://doi.org/10.14393/OBv4n2.a2020-57487.

SAVIANI, Demerval. *Pedagogia histórico-crítica:* primeiras aproximações. Campinas: Autores Associados, 2011. (Coleção educação contemporânea)

SERRÃO, Maria Isabel; DAMAZIO, Ademir; SAMPAIO, Elaine Araujo; ASBAHR, Flavia da Silva Ferreira; ROSA, Joselia Euzébio; MOURA, Manoel Oriosvaldo de. Relações entre educação infantil e conhecimento matemático. In: *Encontro Nacional de Didática e Práticas de Ensino* (ENDIPE). 2012, Campinas.

SFORNI, Marta Sueli de Faria. Interação entre Didática e Teoria histórico-cultural. *Educação & Realidade,* Porto Alegre, v. 40, n. 2, p. 375-397, abr./jun. 2015. DOI: https://doi.org/10.1590/2175-623645965.

SOUSA, Maria do Carmo; MOURA, Manoel Oriosvaldo de. O movimento lógico-histórico em atividades de ensino de matemática: unidade dialética entre ensino e aprendizagem. In: *Encontro Nacional de Educação Matemática* (ENEM). 2016, São Paulo.

SOUSA, Maria do Carmo; PANOSSIAN, Maria Lucia; CEDRO, Wellington Lima. *Do movimento lógico e histórico à organização do ensino:* o percurso dos conceitos algébricos. Campinas, SP: Mercado das Letras, 2014. (Série Educação Matemática)

TALÍZINA, Nina Fiodorovna. *La teoría de la actividad aplicada a la enseñanza.* Tradução direta do russo para Castellano por Yulia Solovieva e Luis Quintanar Rojas. Puebla-MX: Universidade Autonóma de Puebla, 2009.

VIGOTSKI, Lev Semeonovich. *A formação social da mente:* o desenvolvimento dos processos psicológicos superiores. Tradução de José Cipolla Neto, Luís Silveira Menna Barreto, Solange Castro Afeche. São Paulo: Martins Fontes, 2007.

VIGOTSKI, Lev Semeonovich. *Psicologia pedagógica.* Tradução de Paulo Bezerra. São Paulo: Martins Fontes, 2004.

Received in March 2024
Approved in April 2024