Ontological approach to the relationship between logical and historical on the knowledge process

Abordagem ontológica da relação entre o lógico e o histórico acerca do processo de conhecimento

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ABSTRACT
Relationship between logical and historical is subject of different research within the education scope in Brazil, especially in mathematics education. According to Spacek (2023), a considerable number of these studies have as reference the contributions by Kopnin (1978). This research has theoretical nature and aims at presenting an alternative interpretation of the relationship between historical and logical that adopts a historical-social ontology as a reference. Thereunto, mature work by Lukács (2010, 2012, 2013) is constituted as a reference and main source of research, because the author places the object as multilateral at the center of the knowledge process. This viewpoint leads to the need for interpretations that are not reduced to homogenizing plans, as is the case with logic. Regarding the organization of teaching to the appropriation of theoretical thought, this interpretation contributes because it points to the need to specify the function of knowledge for the appropriation of reality, and its particularity as well. In turn, such aspects emerge from the analysis of this knowledge as historical production of humanity that has moments of permanence in change.

RESUMO
A relação entre o lógico e o histórico é tema de diferentes pesquisas no âmbito da educação no Brasil, em especial da educação matemática. Conforme Spacek (2023), uma quantidade considerável desses estudos tem como referência as contribuições de Kopnin (1978). A presente pesquisa, de natureza teórica, objetiva apresentar uma interpretação alternativa da relação entre o histórico e o lógico que adota como referência uma ontologia histórico-social. Para tanto, toma como referência e principal fonte de pesquisa a obra de maturidade de Lukács (2010, 2012, 2013). Isso porque o autor põe no centro do processo do conhecimento o objeto em sua multilateralidade. Tal postura condiz à necessidade de interpretações que não se reduzem a planos homogeneizadores, como é o caso da lógica. No que se refere à organização do ensino com vista à apropriação do pensamento teórico, essa interpretação contribui, pois aponta para a necessidade de especificação da função do conhecimento para a apropriação da realidade, assim como a sua particularidade. Por sua vez, tais aspectos emergem da análise desse conhecimento como produção histórica da humanidade que possui momentos de permanência na mudança.

1 This research is a synthesis of the second section of the item 2 of the fourth chapter of Spacek’s doctoral thesis (Spacek, 2023).
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1 Introduction

The relation between the logical and the historical has been the subject of various studies in the field of education in Brazil, particularly within the realm of mathematical education. This assertion is evidenced by the collection of articles comprising the current thematic issue. Like any phenomenon within the social realm, the formation of this body of research evolves historically and can be traced back to the dissertation by Newton Duarte, titled "The Relationship between the Logical and the Historical in Elementary Mathematics Education" (DUARTE, 1987). Among the works we have encountered, this dissertation stands out as one of the earliest Brazilian publications that explicitly addresses this relationship as one of its determining elements. In it, the author begins with the hypothesis which the relationship between the logical and the historical is fundamental and decisive for the intentional organization of a teaching sequence aimed at students' appropriation of knowledge. According to the author's premise, this relationship serves as a selection criterion that seeks, within the history of knowledge, essential stages in the evolution of mathematical content, which, in turn, must be replicated in the appropriation of its logic. The sequence developed by the researcher aimed to "not disconnect the logic of mathematical content from its historical development, without, however, falling into historicism, which considers the simple reproduction of history in the classroom as a solution to all teaching problems" (DUARTE, 1987, p. 5). The author argues that this stance enables the determination of the essential aspects in the historical evolution of concepts to be taught, with a focus on the

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5 The relationship is referenced in different ways in the Portuguese-language bibliographies consulted: "logical and historical", "logical-historical" (DUARTE, 1987), and "historical-logical" (PANOSSIAN, 2014). We opt for "logical and historical", as it appears in Kopnin (1978) and Davídov (1988). This choice is justified because, in our interpretation, the debate revolves around the issue of legitimacy and the rhetorical role of the relationship between the two categories in the process of development and apprehension of reality.
process, understood as the "essence of historical evolution" (DUARTE, 1987, p. 13). For the identification of these essential stages, it is imperative that the logical becomes the reference point. In these terms, the aim is to establish the relationship between the logical and the historical, wherein

 [...]the logical reflects the historical due to the fact that logic has evolved throughout the historical process. Consequently, logic serves as the starting point for the study of history, as it mirrors the essential stages of the historical process. Nevertheless, this reflection is neither direct nor immediate; otherwise, historical study would be rendered unnecessary (DUARTE, 1987, p. 13).

Thus, the relationship between the logical and the historical is considered fundamental for the organization of a teaching sequence that does not reduce itself to a mere logical sequence devoid of historical content, nor merely to a historical exposition, but rather constitutes a logical-historical sequence. The peculiarity of this sequence lies in the intention that the "stages of learning" succeed each other "in a logical manner, reproducing the essential stages of the historical process." According to the author, this would enable the student to "learn mathematics as a process" (DUARTE, 1987, p. 14).

These theoretical foundations support a sequence of teaching the numbering system and the four elementary arithmetic operations – addition, subtraction, multiplication and division – crafted by the author in alignment with the outlined stages and theoretical foundations.\(^6\)

Since the publication of the aforementioned work, there has been a considerable increase in the number of research studies in mathematics education in Brazil, which claim to be sustained, in some way, by the relationship between the logical and historical aspects of knowledge, more specifically, concepts. Spacek (2023) highlights that a good portion of these studies grounds their interpretations of such relationship in Kopnin's (1978)

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\(^6\) It is worth noting that Duarte (2013) engages in self-criticism regarding certain elements of the position advocated in Duarte (1987). It is not our intention to enumerate the counterpoints made by the author, as the focus here is solely on contextualizing the issue.
work. According to Spacek (2023), the Kopninian interpretation follows the tradition inaugurated by Engels (1982, 2017).

Alternatively, by adopting a historical-social ontology as a starting point (TONET, 2018), the objective of this present article is to advocate for the existence of an interpretation of the aforementioned categorical relation, regarding the educational organization process, grounded in a materialist-dialectical ontological conception. Such interpretation has been found, notably, in the mature works of Lukács (2010, 2012, 2013, 2017), which serve as our primary source of inquiry. In this regard, this article, stemming from a theoretical investigation, aims to contribute to the debate concerning the theoretical principles underlying the interpretation and organization of education, particularly in mathematics education, based on a materialist-dialectical ontology.

The exposition of the research findings commences with an exploration of the pivotal points of the debate, drawing primarily, though not exclusively, from the works of Lukács (2012, 2013). In the sequence, we will offer considerations regarding the potential contributions of such an approach to education. Lastly, in the concluding remarks, we will recapitulate the main points raised and indicate possibilities for educational organization engendered by the present study.

2 An interpretation of the relationship between the logical and the historical grounded in a materialist-dialectical ontology

One of the guiding principles of the ontological reading of social being present in Lukács (2010, 2012, 2013, 2017) is that of the central reference of the object itself in the process of human appropriation of reality. According to the author, such a process does not rely on any pre-established form or norm that would guide and shape it. Lukács (2012) seeks to highlight this centrality for a correct apprehension of reality, indicating a priority of the ontological over the epistemological. When commenting on Lenin's contribution, the Hungarian philosopher observes that in all concrete cases
analyzed from an epistemological perspective in *Materialism and Empiriocriticism*, in practice – by employing the theory of the reflection of a material reality that exists independently of consciousness – ontological considerations are implicitly present. According to the Magyar philosopher, Lenin's merit lies in being the only one, in his time, to reject the philosophical supremacy of modern logic and epistemology. To this end, he returns, with his forms of overcoming, to the original Hegelian conception of the unity of logic, theory of knowledge, and dialectics, but this time based on materialism. However, for Lukács (2012), such a return is not legitimized by the reading of Marx's work. According to the author, Marx's work does not support the unity between ontology and epistemology. At this point, according to Lukács (2012), one of the sources identified by Marx of Hegel's idealistic illusions is found. Therefore, it becomes important, as a first step, to analyze the excessive approximation between Marx and Hegel, as it influenced some Marxist currents, especially so-called Marxism-Leninism.

According to Chasin (2009), this theoretical link between Marx and Hegel enters into the heart of Marxism through the door of the thesis of the logical connection or inheritance of Hegel by Marx. Nonetheless, for the author, "the thesis of the logical connection between Marx and Hegel is not an issue authorized by Marx's work or intellectual convictions, but an unfounded formulation that has traces in Engels, footprints in Lenin, and which, later, was expanded, as in the case of Lukács" (CHASIN, 2009, p. 190)."  

From this trend, there are developments regarding methodological approaches that Marx would never have addressed. Lukács (2010) brings to light some of these notions when analyzing Hegelian philosophy. As stated by the author, there is an inherent duplicity in Hegel's philosophy, expressed on one hand by considering processuality as predominant in the world of objectivity – a notion brought about by the irreversibility of movement from less developed forms to more developed ones – and on the other hand by

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7 Here the author refers primarily to the Lukacsian writings preceding those with ontological orientations, especially Lukács (2018).
conceiving genesis as a logical derivation from the concrete based on the abstract – which results in the distancing from authentic categories of processual development and a reversal of the deduction process, conceived as the genuine form of development.

According to Lukács (2010, 2012), the driving force of development used by Hegel is the *negation of negation*. In this context, it is puzzling to the Hungarian philosopher that Engels was content with only materialistically conceiving the idealist construction of the *negation of negation*. As we know, Engels establishes the *negation of negation* as one of the *laws of dialectics*. As a law, it would be applicable to the entire field of processual, historical reality. In Lukács's view (2010, 2012), this external field of application is problematic because it becomes superfluous by not contributing to the apprehension of the specificity of the development process in what is essential, as Engels had recognized (2015).

To grasp the core of Lukács's critique, it is pertinent to transcribe Engels's words (2015, p. 165) which provide analytical support for the Hungarian philosopher:

> Let us consider a barley grain. Billions of barley grains are ground, boiled, fermented, and then consumed. However, if one of these barley grains encounters the conditions that are normal for it, when it falls on suitable soil, a very specific change occurs: under the influence of heat and moisture, it germinates; the grain disappears as such, it is *negated*, and its place is taken by the plant that has emerged from it, which is the negation of the grain.

According to Lukács (2012), in these lines lies the description of a process that follows the natural movement of development. In cases where the barley grain is destroyed, it can be asserted, as a legitimate ontological expression, that there has been a logically determined negation, but one that is of little significance for apprehending being in its determinations. Conversely, in the concrete case where the grain is in suitable conditions to sprout, there is a transformation of the grain into its normal other-being, the plant. The abstract
consideration of the negations of the grain leads to the loss of concrete determinations of this other-being. At the same time, it obscures the dialectical-real process by approaching and analogizing with cases that have merely formal relation to the process under study.

Lukács (2017, p. 134), in attempting to explain the formal character of the negation process, considers that

[...] "omnis determinatio est negation" I could also say that the lion is not shaving cream. On the logical plane, this proposition is faultless because the lion truly is not shaving cream. However, propositions of this sort could be uttered by the millions, and none of them would have an effective sense, as negation in other-being is a subordinate moment. This negation arises from comparison, and even in this case, it is a subordinate moment because the reciprocal other-being of the chair and the table, for example, comes from entirely positive things, and the circumstance that the table is not a chair is an exceedingly subordinate factor and has no importance in practical thinking. In contrast, negation, regardless of whether its true sense is already attenuated, assumes great importance as soon as we want to understand reality on purely logical bases. It is an effective negation, for example, when I say: "Two times two are not five." When I say: "There are no dragons," this is also a justified negation. The vast majority of negations, meanwhile, are not actually real negations. When I say that the lion is not shaving cream, this is not absolutely a real negation, but only a purely logical consequence of an assertion in which logic has secondary importance.

Regarding ontological considerations in which negation would be an important feature in development, for Lukács (2012), it is inconceivable to find negation in the sphere of inorganic being, since its characteristic, its way of being, is summarized by a chain of transformations from being-so to being-other. Conversely, in the sphere of organic being, negation can be identified in the process of death of living beings, as it is in this event that biological reproduction ceases and its laws are surpassed, leading the matter existing in organisms to be governed by chemical-physical laws. In general, "Only in cases where becoming

8 In a free translation, it means "every determination is a negation."
other objectively signifies a passage that radically subverts forms of objectivity or processes can it be understood as negation also on the objective ontological plane" (LUKÁCS, 2012, p. 217).

Another instance used by Engels (2015) is that of the negation of negation as the driving determination of mathematical operations. To comprehend this illustration, it is necessary to highlight that the author, initially, establishes a relationship between metaphysics/dialectics and formal logic/dialectics. Subsequently, he repeats the comparison but employs mathematical concepts: "the mathematics of variable magnitudes is to that of constant magnitudes as dialectical thought is, in general, to metaphysical thought" (ENGELS, 2015, p. 153). At another point, he draws a similar parallel when discussing formal logic as a method for seeking new knowledge based on the known, and dialectics as that which provides elements for surpassing formal logic and harbors the embryo of a more comprehensive worldview.

Elementary mathematics, the mathematics of constant magnitudes, operates within the bounds of formal logic, at least in general terms; whereas the mathematics of variable magnitudes, whose most significant part is constituted by infinitesimal calculus, essentially amounts to nothing but the application of dialectics to mathematical relations. The mere act of proving is decidedly secondary here, compared to the manifold applications of the method to new fields of investigation. However, from the standpoint of elementary mathematics, nearly all proofs in higher mathematics, starting from those of differential calculus, are strictly speaking false. This cannot be otherwise when attempting, as is the case here, to prove, through formal logic, the results obtained in the dialectical field (ENGELS, 2015, p. 165, emphasis added).

The assertion that the "mathematics of variable magnitudes," largely represented by infinitesimal calculus, corresponds to an "application of dialectics to mathematical relations" is evident in these lines. It is important to emphasize that Engels did not conceive of variables yet in terms of post-
Cauchy, mathematical analysis, that is to say, he had not engaged with the more elaborate discussions of his time. In these terms, Engels's thesis that higher mathematics contradicts elementary mathematics becomes fragile. In this context, it is valid to present how Engels (2015) interprets the negation of negation in elementary mathematics and in the mathematics of variable magnitudes before delving into the heart of the proposed discussion.

Let us take any algebraic magnitude, let's say \(a\). If we negate it, we get \(-a\). If we negate this negation by multiplying \(-a\) by \(-a\), we get \(+a^2\), that is, the original positive magnitude, but at a higher level, namely, squared. In this case, it does not matter if we can obtain the same result by multiplying the positive \(a\) by itself and thus also getting \(a^2\), because the negated negation is so deeply rooted in \(a^2\) that, in any circumstance, it has two square roots, namely, \(a\) and \(-a\). However, getting rid of this impossibility, of the negation of negation, of the negative root contained in the square, already acquires a quite concrete importance in the case of second-degree equations (Engels, 2015, p. 166-167).

It is not our intention here to dwell on the detailed analysis of the underlying conceptions of the mathematical process described, as well as its realization. However, it is important to emphasize certain aspects regarding the explicitation of the negation of negation applied to mathematics as found in Engels' lines. In this example, the German thinker employs "processes" of "changes" concerning certain conditions. The negation of the algebraic magnitude \(a\) is performed by assigning a sign \((-\) to its being-precisely-so, turning it into its opposite, or as Engels prefers, its negation. In turn, in the process of negating this negation, a multiplication of the negated entity \((-a)\) by another equal to itself \((-a)\) is performed, in order to obtain as a product the initial element, but in a new degree, \(a^2\). It is explicit that the negation in both cases is distinct. Firstly, he performs a multiplication by a negative number \((-1)\) and, in the second case, he performs a multiplication by the entity itself \((-a)\) which was negated. Let us agree that this is a maneuver by the author to

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According to the note in the edition used, Engels relied on the work of Heinrich Suter entitled *Geschichte der mathematischen Wissenschaften.*
demonstrate that the product is the same element to a greater degree, when, in fact, the logical negation would lead back to the initial \( a \). It is possible that we may be accused of making a metaphysical interpretation of the "dialectical process" shown by Engels, but we fail to see the dialectic embedded in the successive multiplications of different relative numbers.

Lukács (2010) highlights two additional issues regarding this example. The first concerns the fact that relative numbers present other meanings, such as in the coordinate system where \(-a\) and \(+a\) are arbitrary numbers that share something in common but do not inherently represent any negativity towards each other. The second pertains to the reason why multiplication, and not addition, expresses the negation of \(-a\) in relation to \( a \). According to Lukács (2010, p. 168), "Multiplication, moreover, shows an apparently useful analogy only in a purely formal manner, and exclusively from there does it acquire its privileged place," but "it contains no shadow of any ontological question."

What matters in the analysis of this example, as carried out by Lukács (2010), is, firstly, to demonstrate that the process of abstraction, within the movement of knowledge, does not stem from \textit{a priori} abstract forms to be applied to different branches of reality. Secondly, it is important to note that there are distinctions between the logical and ontological planes, and it is neither prudent nor, at the very least, unnecessary to engage in a strictly logical interpretation of ontological matters. Afterward, Lukács' critique (2010) aims to assert that the process of abstraction based on reality is fruitful only if it seeks what Marx (2011) terms "reasonable abstractions," meaning generalizations of reality grounded in actual processes, in order to obtain certain abstractions where the exclusion of particulars from exposition "never turns the latter into grotesque absurdity" (LUKÁCS, 2010, p. 169). Simply announcing the development of barley in the same manner as the multiplication of relative numbers as similar processes of negation of negation would expose absurd sides. This does not occur when
generalizations originate from being itself, resulting in abstractions that possess ontological determinations justifying them, such as affirming historicity as a general trait of forms of being.

[...] without even touching the realm of the absurd, I can affirm: geology demonstrates the irreversibility of natural processes as clearly as the history of France shows the irreversibility of historical processes. Here as well, the specific concrete moments of the two groups of phenomena have nothing to do with each other; however, the irreversibility of the process itself forms, here as there, the real basis of their respective particularities (LUKÁCS, 2010, p. 169-170).

This implies that the negation of negation is not a general law derived from the development of being, but rather a specific sphere arbitrarily applied across all of reality. It is in this sense that Lukács' critique is drawn in order to draw attention to the possibility, intrinsic to the logicization of ontological problems, of the mischaracterization of being itself, or in the less problematic case, of not aiding in the unveiling of its essence.

The question remains regarding the validity status of the law of negation of negation. For Lukács, the privileged locus of negation is the sphere of social being, as it arises with the existence of the subject. It emerges from choice and decision-making in praxis, between alternatives that are established for the individual as a bifurcation and opposition in the world, determining what aids or hinders the individual in achieving their objective. Here, a homogenization of reality is engendered in thought, where the negation or affirmation of certain properties is actualized through the process of labor. Along these lines, Lukács (2012) asserts that negation is the intellectual instrument of praxis and its inseparable reflection.

Let us return to the central aspect of Lukács' critique concerning the ontological concreteness of the object taking precedence over general epistemological abstractions. In connection with this aspect, Lukács (2010, p. 365) remarks on the tendency within Marxism to separate dialectical materialism as a philosophically
general and comprehensive doctrine, which, when applied to society, gives rise to historical materialism. In this sense, he asserts:

This stance contradicts Marxism on two decisive points. Firstly, by adopting a philosophical doctrine of abstract-general categories, whose findings are supposed to apply uniformly to all beings, and secondly, to the extent that the moment of historicity is reduced to a mere singular problem of being, which could only receive its authentic objective content, and consequently its intellectual formulation possibility, through the application of the general supra-historical universal principles of dialectical materialism in this "special sector". This codification of the essence of dialectical materialism appears as a precise univocity of its principles—compared to Marx's comments, which always refer to the historical process, unlike Lenin's hesitant attempts to approximately grasp, from various angles, the essential features of its procedural movement, thus, as an attempt to definitively fix univocal determinations of the categories.

The Hungarian philosopher argues against the tendency to identify and separate the existence of universal characteristics from certain determinations, converting them, through the realm of logic, into forms of chaining and hierarchizing reality under the pretext of a certain organizing principle. According to Lukács (2012), this process tends to homogenize the framework of reality on a certain aspect, not capturing it in its multiplicity of determinations. It is in this sense that logic can subvert the real process, as the author considers to be the case in Hegel. For Lukács (2012, p. 220), "Logic is one of the most important homogeneous means created by the praxis and mental work of man.” The problem that emerges from interpretations that elevate logic as the reference or director of the process of knowledge is that the starting points, the generative activities, tend to extinguish themselves in the homogeneous milieu of logic, condensed into a closed system that presupposes universality. The history of humanity is illustrative regarding the illusion, born there, of various thinkers being able to "through a well-finished system of the universe of thought homogenized in logical terms, give answers, from it, to all questions arising from the relations of human beings with reality" (LUKÁCS, 2012, p. 221).
The question that arises is how logic interacts with reality, given its essentially homogeneous nature which fundamentally differs from it. According to Lukács (2012, p. 221-222),

"...logic creates a homogeneous medium of thought, whose structure must be qualitatively different from reality, which is inherently heterogeneous; and this diversity must manifest itself, if for no other reason, by the fact that relationships in a homogeneous medium must be constituted differently from what they would be in the presence of objects, forces, etc., that are truly heterogeneous and acting upon each other." We have already referred to the intellectual operations that this fact makes necessary, such as the need for a physical interpretation, etc., of real phenomena that have been expressed in mathematical formulas; in this case, it is necessary for what has been mathematically homogenized to be re-approximated to objective reality through intellectual highlighting and clarification of the heterogeneous character of its components. [...] If the homogeneous medium that serves as the foundation for cognitive connection has a logical character, then the contrast between the homogeneous cognitive medium and the heterogeneous reality acquires a particular trait, whereby the complex - infinite - of phenomena heterogeneously related to each other and, therefore, not immediately systematizable and hierarchizable as such, will reproduce itself in thought as a homogeneously finished hierarchical system.

The homogenization of the process, according to Lukács (2012), is not necessarily an obliterating trait of the possibility of knowledge of reality and its apprehension in the sense of capturing its essence. This is explained by the fact that, in its constant return and calibration in relation to it, as well as in the denial of its inherently hermetic systemic character as an aspect of reality, the ability to assist in the concrete appropriation of a given object is established. Therefore, "through satisfactory, consciously critical handling of cognitive means, it can always be brought back to the parameters of a correct approach to real objects" (LUKÁCS, 2012, p. 222). However, this is not a possibility when creating and fixing a hierarchy, as it tends to distort the process of constituting reality through the conceptual chaining that composes the system. For Lukács (2012),
while the emergence of a formation is a problem of genesis - in which the real characteristics of the particular being are consequences of the legalities of its appearance and disappearance - the concept, on the other hand, has its emergence through the logical deduction of another concept, regardless of whether this path is from top to bottom or vice versa. Therefore, focusing on the logical aspect becomes a problem when logic becomes the model and foundation of ontology; thus, it implies that logical deductions transform into proper forms of ontological genesis.

In this context is situated the issue of the order of categories in the exposition of the object. According to Spacek (2023), among the authors advocating for an explicit unity of the logical with the historical and their consequent methods of apprehending reality, there is a tendency to affirm that the order of categories and the process of reality formation are correlated aspects. Due to this particularity, logic gains a privileged place and forms a dialectical pair with history (KOPNIN, 1972). However, such an attitude, according to Chasin (2009), transforms expository forms into stages of the object's own development.

According to Chasin (2009), this alleged correlation between the concrete development of reality and its logical exposition arises from the tradition, initiated by Engels (1982), of interpreting Marxian procedures as entailing both a logical and a historical stance, rather than analyzing the relationship between reality (history) and thought. It is under the pen of the German thinker and revolutionary, in a review of Marx's work "Contribution to the Critique of Political Economy", that the problematic of the relationship between categories, which drives the present article, gains relevance. In the extensive excerpt that follows, one finds the way in which Engels (1982) introduces it, as two modes of approaching the object - in the case analyzed, Political Economy. According to the author (1982), Marx would have been the only one who, starting from the "shell" of Hegelian logic, managed to extract the core containing real elaborations, thus rescuing the "dialectical method". This became possible only because the author of "Capital" would have
focused on the simple form of dialectics, without the idealistic embellishments, which was decisive for capturing the development of capitalism. Subsequently, Engels (1982, p. 541, emphasis ours), affirms that

[...] after acquiring the method, Economics could be approached in two ways: historically or logically. Just as in history, as well as in its literary reflection, development, in broad strokes, progresses from simpler to more complex relationships, the historical-literary development of Political Economy provided a natural thread to which criticism could attach itself, and, in broad strokes, economic categories would appear in the same order as logical development. This form apparently has the advantage of greater clarity, for thus, it follows the real development; in fact, however, it would at most become merely more popular. History often proceeds by leaps and zigzags, and if one were to follow it everywhere at the same time, not only would much material of little importance have to be gathered, but also the course of thought would often have to be interrupted; moreover, the history of economics could not be written without that of bourgeois society, and thus the work would become endless, since the preparatory works are lacking. Therefore, the logical mode of treatment was the only one that was in its place. This [mode], however, is in fact nothing but the historical, stripped only of its historical form and disturbing contingencies. Where this history begins, there must begin too the course of thought, and its further advance will be nothing more than the reflection, in an abstract and theoretically consistent form, of the historical course; a corrected reflection, but corrected according to laws that the actual historical course itself provides, so far as each moment can be considered in the point of development of its full maturity, of its classical form.

According to Chasin (2009), the correlation between concrete development and its logical exposition can become a trap, as historicity can be equated with logical realization, leading to the interpretation of logical forms as expressions of concrete historical development. According to the author, this is found in Lukács (2018), who, in his conceptions prior to the ontological ones, understands the forms of judgment as historical realizations in which the historicity of formation and succession always has a historical
substrate that constitutes them as form and content. In this sense, in Chasin's interpretation (2009), global historical processuality can be understood as syllogistic. According to the author, this transfiguration of the Marxian problem arises from an indistinction between the expository process and the analytic process of reality in Marx's works, in which:

> [...] the initial Marxian determination - the simple form of value, essentially expository, which is not discovered in act, but only the act that begins to show the discovered, that is, determination whose emphasis falls neither on the reproduction of the analytical complexity of the discovery, nor on the discovered as a concrete historical complex, whether immediately taken or forced to be understood as a theoretical reproduction of a specific stage of actual existence (CHASIN, 2009, pp. 181-182).

The exposition of categories as stages or historical forms of existence of the object has a broad repercussion in Hegel's thesis of the identity between the elements of objectivity and the subjectivity unfolded from totality. In these terms, Hegel (quoted in LUKÁCS, 2012, pp. 203-204) asserts that:

> "The order and connection of ideas" (of the subjective) are identical "to the order and connection of things" (of the objective). Everything exists in a single totality; the objective totality and the subjective totality, the system of nature and the system of intelligence are one and the same thing; to a subjective determinacy corresponds the same objective determinacy.

If the established order between ideas and reality is the same—or, in the case of some Marxist interpretations, if they are inverted orders—it is convenient to admit a relationship and hierarchization of development, insofar as, for Hegel, the path from being to concept is established from the simple to the complex, in which being, simple and abstract, is a property of everything that is new.

On the contrary, Lukács (2012) warns that it is not possible to conclusively advocate for this abstract and simple character of the new. This doubt cast upon
the Hegelian thesis does not signify a denial of the process of reality's complexification, which finds expression in the unfolding of the spheres of being—the emergence of organic being based on inorganic being and of social being based on these other two forms of being. Furthermore, it is possible to identify that in the sphere of social being there is a progressive tendency towards the universalization and socialization of relations. However, there is no form of categorization that can be deduced, by homogenizing heterogeneous processes, reduces the exposition of the logic of the object to a realization of the concept in its most finished form based on simpler ones. This line of argumentation resonates with Marx's elaborations, which can be found in the following words of Marx (2011, pp. 55-56):

But do these simple categories they do not equally have an independent, historical, or natural existence before the more concrete categories? It depends. Hegel, for example, correctly begins the philosophy of law with possession as the simplest legal relationship of the subject. But there is no possession before the family or relations of domination and servitude, which are much more concrete relations. On the contrary, it would be correct to say that there are families, tribes, which only possess but do not have property. Regarding property, therefore, the simplest category appears as a relation of simpler associations of families or tribes. In the most advanced society, property appears as a simpler relation of a developed organization. But the most concrete substrate, from which possession is a relation, is always presupposed. [...] the simplest category can express dominant relations of a still undeveloped whole, or subordinate relations of a developed whole that already had historical existence before the whole developed in the sense expressed in a more concrete category. In this case, the course of abstract thought, which rises from the simplest to the combined, would correspond to the actual historical process.

The relationship between simple and complex categories is not given, therefore, in the expository form, by inherent chaining and deduction within the concepts themselves, but by the development of reality that possesses regularities, but not teleology or blind necessity. The existence of complex
categories is not necessarily an unfolding of simple categories, but of a complexifying and acting together of reality. The existence of simple categories only has substance in dependence on the complexity of concrete forms of their expression. In general terms, this corroborates the assertion of a preponderance of the object in the process of investigation, and that the expository order of categories is related only to their intrinsic logic and historicity. Chasin (2009, p. 244), in the analysis of the categorical exposition developed by Marx in "Capital", supports this state.

Thus, the order of entry of materials into the discursive scene and the places they occupy therein are not stipulated by some kind of autonomous expository legality, but by the statute of ideal reproduction, forged in subservience to the ontological compound of the complex under study. This does not bear identity with the order and manner of their real engenderings, for it suffices to consider that the form of the commodity as commodity, i.e., the mode of existence of the product of labor in the particular sociability of commodity production, like any entity, comprehends the simultaneity of all its characteristics as an integrated presence sculpted by its categories, whereas, obviously, in analysis and discourse, this immediate unity is impossible, giving way to a framework structured by the sequentiality of the categorical approach.

Concerning these aspects that Lukács mention (2010, 2012, 2013), while attributing an important role to logic, but not necessarily a priority, in the process of knowledge, aims to rescue the development of the object in its own processuality, that is, in its own logic of development. In this sense, the logic of the object and its historicity are a single movement (CHASIN, 2009), but when represented through conceptual means, they are not a copy of reality by subjectivity. As moments captured by subjectivity in the process of seeking the means for teleological realization, the reflection of reality is realized in a way that captures the necessary moments for this process, that is, it constitutes a homogenization that, for its effective realization, is imperative to materialize under the governance of the object's own logic, and not by a supra-historical methodological principle.
We have already discussed the issue of the law of the *negation of the negation* principle and narrowed down the scope of its operation, but it is still valid to question the role and legitimacy of other supposed general laws. For instance, historicity, processuality, and the universal contradictoriness of the real are ontological properties of reality. Marx (1984, p. 17) acknowledges, for example, that Hegel, despite the mystification of dialectics, was "the first to present its general forms of movement in a broad and conscious manner." In this sense, Chasin (2009) asserts that in the movements of thought towards the reproduction of the concrete as a synthesis of multiple determinations, especially in the moment of determination, there is a contribution from these general forms, which, however, are always immanent to the object and not attributed to it by thought. In the object, it is always possible to identify the moments of universality, particularity, and singularity, given that the concretion of analysis also operates at these levels of generalization, which represent, through their dialectical interrelation, moments of the determinative process, despite their remote and abstract status.

Under this condition, a logic or dialectic of the universal, particular, and singular will be the bundle [...] of the most abstract of reasonable abstractions, which as such does not determine any concrete object. Given the maximum generality of this most abstract of reasonable abstractions, it is sayable of any object, it is the faintest abstract voice, a generality so possible, and in this sense, it can serve as a distant guide for the formulation of reasonable abstractions, and likewise for the steps of concretion (CHASIN, 2009, p. 216).

These ontological precepts, once derived from the utmost generality of reality, indicate some presuppositions, but they are not crutches or secure guides in the apprehension of knowledge of the object, just as supposed logical and historical modes of approaching reality are not. This overlap of approaches to the object is, according to Chasin (2009), a confusion inherent in the investigative process and in relation to reality itself, as it homogenizes and simplifies its movement for expository purposes.
It's clear that the criticism presented by Lukács (2010, 2012, 2013), as well as Chasin's interpretation of the Marxist method (2009), is structured around the rejection of the supposed existence of a general orientation towards objects, as well as a subordination of the ontological-historical to logical precepts. This can be seen in the analysis of societal development, where the ontological needs of the main trends of totality's development cannot be homogenized as purely logical needs. For example, it would be illegitimate to claim that Classical Antiquity arises from an ontological necessity, just as it is replaced by feudalism in a similar manner. On the other hand, even post festum, deducing serfdom from the slave economy in logical-rational terms is an absurdity (LUKÁCS, 2012). It is in this sense that Lukács (2012) establishes criticism of Engels regarding his distinction between the logical and historical modes of reality.

The decisive opposition with Marx's conception lies in the primacy of the "logical mode of treatment," which is posited here as identical to the historical, "only stripped of the historical form and unwelcome causalities." History stripped of its historical form: therein lies primarily Engels' recourse to Hegel. In Hegelian philosophy, this was possible: since history, like all reality, presented itself only as the realization of logic, the system could strip historical events of their historical form and reduce them to their own essence, namely, logic. But for Marx—and also for Engels—historicity is an ontological characteristic not further reducible of the movement of matter, particularly marked when, as is the case here, it concerns only social being. The most general laws of this being can also be formulated in logical terms, but it is not possible to derive them from logic or reduce them to it. In the cited passage, Engels does this, which is already evident by the use of the expression "unwelcome causalities"; on the ontological level, something casual may well bear an essential tendency, regardless of whether, from the perspective of pure logic, chance is understood as "unwelcome" (LUKÁCS, 2012, p. 373, emphasis ours).

The foundation of approaching an object of knowledge based on the considerations made points towards, in the specific case of organizing school education, the necessity of considering it, in the process of its objectification
as an object of teaching, as a form of reflection of reality. However, as a consequence of the intensive infinitude of objects and the structured nature of reality as totality—which presupposes its internal differentiation and the simultaneous multiplicity of factors acting in its uninterrupted unfolding—depending on the purposes of the activity in which reflection is one of its moments, other knowledge is required for its objectification, its progressive process of concretion. It becomes imperative, for the appropriation of its essence, that homogeneous forms become moments of the analysis of the object, understood as a "synthesis of multiple determinations" (MARX, 2011, p. 54). This runs counter to the tendency that often establishes formal and consequently homogeneous parameters for complex, heterogeneous problems. Such an attitude distorts them and transports them to a formal, supposedly rational plane that aims to mitigate the need for taking a position by transferring the responsibility of choosing among alternatives to a plane that is supposedly free from the influence of other social spheres. An example of this tendency can be found in the case of Bertrand Russell, who, according to Mészáros (2009), advocated for a preemptive attack by Western capitalist powers against the Union of Soviet Socialist Republics (USSR) when there was no imminent risk of this act resulting in mutual destruction.10

Unfortunately, this kind of reasoning, which makes sense on a purely formal and homogenized level – "better millions dead than all of humanity" – was revived during the pandemic period as it dragged complex and vital problems into the formal plane. According to Mészáros (2009, p. 31),

10 According to Mészáros (2009), the philosopher would later regret this, condemning his own stance.
deductions, depart from its own material assumptions — necessary though not explicit — or that claims to be able to transfer them to a separate "sphere of emotions," can only lead to total arbitrariness in matters of such importance — literally, matters of vital importance.

The discussion presented, by highlighting the incompleteness of the isolated reflection of objects of knowledge for analyzing the totality of reality, underscores its particularity — that is, the moments and aspects of reality of which it constitutes a reflection — which may be essential for capturing the essence of the objects of reality based on the apprehension of their own legalities. At the same time, in the context of human activity, in the form of a concept, reflection can support generalized forms of action that enable, by being embedded as moments of the realization of this activity, human beings to act beyond the constraints of the given situation. Consequently, beyond the inseparability of reality and its knowledge, despite ontologically asserting their distinct nature (LUKÁCS, 2013), the relationship between the particularity and generality of knowledge presents itself as an indissociable aspect of human activity.

The fact that reality always exceeds knowledge, which generally operates on a homogeneous plane, reinforces the specificity of knowledge in relation to it, indicating both its weakness — the delimitation of its field of action — and its strength — the spheres of reality that play a more effective role — as well as the degree of generalization of the subject’s action in this sphere. In this framework, the study of the process of its development becomes important, especially for the teacher, because it is through this study that it becomes possible to grasp what constitutes the trait of permanence in change (LUKÁCS, 2012). In other words, such a study can reveal what makes it emerge and makes it its own, or in the terms of Davídov (1988, p. 143), its "substantial abstraction."

In summary, the ontological interpretation carried out here regarding the relationship between the historical and the logical, mainly based on Lukács (2010, 2012, 2013, 2017), helps us sustain a reading of the teaching
and learning process as a historically delimited phenomenon that must be interpreted based on its own legalities. In this context, the materialist-dialectical ontological reading of the relationship between the logical and the historical aspects of knowledge becomes important because, by emphasizing the object as the guiding pole of the knowledge process, it indicates its potentiality and limitation by eliminating any claim to establish a priori categorical foundations to guide it. This fact demands from the teacher a constant questioning about the essence of knowledge as historical development, as well as the role it plays in the current mode of production, in order to grasp its contributions to students' formation regarding their emancipatory possibilities.

3 Final considerations

The relationship between the categories of the historical and the logical as one of the determinants for the organization of teaching aimed at the formation of students' theoretical thinking has been explored by different research studies, with special emphasis on research in mathematics education. Our objective was to advocate for the existence of an interpretation of the aforementioned relationship grounded in a materialist-dialectical ontological conception. We sought to present a non-hegemonic interpretation among research studies in mathematics education that are based on historical and dialectical materialism in Brazil, as these have largely been formulated under the influence of the works of Kopnin (1978), as shown by Spacek (2023).

An ontological interpretation grounded in the works of Lukács (2010, 2012, 2013, 2017) highlights the need to position the object at the center of the process of knowledge as its reference point. This, in turn, demands the adoption of a multilateral and consequently heterogeneous approach. It is imperative, for the adoption of such a stance, that it not be restricted to the unilateral and homogeneous plane of logic as a reflection of the knowledge of the object or of its own being.
Another highlighted aspect is the ineliminable historicity of the object, a characteristic that synthesizes its process of transformation as an uninterrupted movement of change, as well as the continuity of what constitutes its essence. Such historicity of the object also prevails in the process of knowledge of reality, which presents its own legalities.

Such considerations can be taken as guiding principles for the organization of teaching that aims to develop students' theoretical thinking, as they can assist in the analysis of the object of knowledge in its genesis and development in the pursuit of what is specific to it, that is, what characterizes permanence in change (LUKÁCS, 2012), as well as indicating the need to highlight its contribution to the formation of the student.

Enfoque ontológico de la relación entre lo lógico y lo histórico acerca de conocimiento

RESUMEN
La relación entre lo lógico y lo histórico es tema de diferentes investigaciones en el ámbito de la educación en Brasil, en especial de la educación matemática. Según Spacek (2023), una cantidad considerable de esos estudios tiene como referencia las contribuciones de Kopnin (1978). La presente investigación, de naturaleza teórica, tiene como objetivo presentar una interpretación alternativa de la relación entre lo histórico y lo lógico que adopta como referencia una ontología histórico-social. Para ello, se constituye como referencia y principal fuente de investigación la obra de madurez de Lukács (2010, 2012, 2013). Eso es porque el autor pone en el centro del proceso del conocimiento el objeto en su multilateralidad. Tal postura conduce a la necesidad de interpretaciones que no se reducen a planes homogeneizadores como es el caso de la lógica. En lo que se refiere a la organización de la enseñanza con vistas a la apropiación del pensamiento teórico, esa interpretación contribuye, pues apunta a la necesidad de especificación de la función del conocimiento para la apropiación de la realidad, así como a su particularidad. A su vez, tales aspectos emergen del análisis de ese conocimiento como producción histórica de la humanidad que posee momentos de permanencia en el cambio.

Palabras clave: Ontología; Lógico e histórico; Conocimiento.

4 References


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