Obutchénie
Revista de Didática e Psicologia Pedagógica

DOI: https://doi.org/10.14393/OBv7n3.a2023-69584

Vadim Andreevich Krutetskii's contributions to the field of high abilities/giftedness in mathematics[1] [2]

As contribuições de Vadim Andreevich Krutetskii para o campo das altas habilidades/superdotação em matemática

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ABSTRACT
With the affirmation of Mathematics Education as a research field and, more recently, with the emergence of Inclusive Mathematics Education, research on mathematical giftedness raises discussions about its definition. Although different authors refer to Krutetskii's theory as the most complete attempt to explain the phenomenon, his work is often cited for being considered a pioneer, but rarely highlight the potential for analysis that his conception of mathematical giftedness presents. This article is a theoretical and bibliographical study and aims to discuss the relevance of Krutetskii's theory for studies on mathematical giftedness nowadays. It was possible to identify convergence with the contemporary theories and that, from the point of view of scientific research, Krutetskiian theory

RESUMO
Com a afirmação da Educação Matemática como área de investigação e, mais recentemente, com a emergência da Educação Matemática Inclusiva, as pesquisas sobre a superdotação matemática suscitam discussões sobre a sua definição. Ainda que diferentes autores se refiram à teoria de Krutetskii como a mais completa tentativa de explicação do fenômeno, sua obra é frequentemente citada pelo fato de ser considerada pioneira, mas, raramente, destacam as potencialidades de análise que sua concepção de superdotação matemática apresenta. Este artigo é um estudo teórico, bibliográfico e tem como objetivo discutir a relevância da teoria krutetskiiana para os estudos sobre a superdotação matemática na atualidade. Foi possível identificar convergência com teorias de superdotação contemporâneas e que, do ponto de vista da pesquisa científica, a teoria krutetskiiana

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DOI: https://doi.org/10.14393/OBv7n3.a2023-69584

presents us with rich elements that can help us reflect on the theoretical-methodological aspects of our investigations.

Keywords: Krutetskii; Giftedness; Mathematical abilities; Research; Theory.

nos apresenta ricos elementos que podem nos ajudar a refletir sobre os aspectos teórico-metodológicos de nossas investigações.

Palavras-chave: Krutetskii; Superdotação; Habilidades matemáticas; Pesquisa; Teoria.

1 Introduction

At least since the early 1990s, we have seen a series of government initiatives intensify in Brazil, in line with a worldwide movement for educational inclusion, in which students with different specific educational needs (SEN) have been protected by legal provisions, reaffirming everyone's right to education and the role of the school as a privileged space for developing the potential of all students and, in particular, of those who are the target audience for Special Education, i.e. students with disabilities (intellectual, physical, hearing or visual), global development disorders and high abilities/giftedness (BRASIL, 2008).

In this sense, different areas of knowledge, such as Psychology and Education, have advanced in the production of research related to the development and learning processes of students with SEN, based on different theoretical frameworks. On the other hand, when we focus specifically on students with high abilities/giftedness, we see more modest progress, both in terms of the implementation of government initiatives (PÉREZ, 2018) and in terms of the development of research (MORI, 2021).

In this vein, Mathematics Education, as an autonomous area of research and knowledge production with its own problems and theories and which, at the same time, moves between different scientific fields, including Psychology and Education (MOREIRA *et al.*, 2016), has dedicated itself to investigating the processes of teaching and learning Mathematics in relation to students with SEN, but not limited to them, giving rise to a new branch of research referred to in the literature as Inclusive Mathematics Education.

With regard to students with high abilities/giftedness, academic

DOI: https://doi.org/10.14393/OBv7n3.a2023-69584

research in the field of Mathematics Education is recent and scarce (FERREIRA, 2020; FEREIRA; MOREIRA, 2021), which leads us to reflect on the theories that have underpinned such research in order to understand perspectives, trends and possibilities for analyzing the topics researched, especially since it is a construct for which different definitions are presented.

From a general domain perspective, Renzulli (2018) sees giftedness as a result of the interaction between three sets of characteristics: above-average ability, creativity and commitment to the task. In contrast, Gagné (2005) distinguishes between the terms giftedness and talent, reserving giftedness for naturally high aptitudes beyond the standard expected for the child's age, while talent is understood as the exceptional mastery of systematically developed skills and knowledge in a specific area of knowledge. In turn, Mönks and Katzko (2005) define giftedness as an individual's potential for remarkable or extraordinary achievements in one or more domains.

The different theoretical currents and the various definitions of high abilities/giftedness imply multiple criteria to be considered when identifying the gifted, so that the process requires a multi-referential assessment, diversifying the intervention proposals, agents, resources, procedures and assessment instruments (POCINHO, 2009).

Alencar (2007) highlights some of the factors that have driven the growing attention to gifted students. These include the development of the varied skills of these students, who have the potential to contribute significantly to scientific and technological progress, promoting the future well-being of the population and consolidating leadership in various areas of knowledge. Another relevant aspect is associated with the transition to a new paradigm in the conception of wealth in the information society. This new approach focuses on high-tech products, which in turn require the formation of high-level intellectual capital.

From a domain-specific perspective, as is the case with mathematical giftedness, the multiplicity of definitions also applies. Furthermore, studies in

DOI: https://doi.org/10.14393/OBv7n3.a2023-69584

the fields of mathematics education and gifted education have developed independently, without connections (LEIKIN, 2011).

Consistently, researchers from different countries point to the theory of mathematical giftedness proposed by Vadim Andreevich Krutetskii [5]in the mid-twentieth century as a pioneer (LEIKIN, 2011; PARISH, 2014; SCHINDLER; ROTT, 2017) and that this remains, to this day, the most complete attempt to explain this construct given the richness and breadth of the study carried out (LEIKIN, 2021).

Considering that the work in which V. A. Krutetskii presents his theory was launched more than five decades ago, the changes that have taken place in the fields of gifted education and mathematics education in this period, and the fact that research into mathematical giftedness is still recent on the national academic scene, we raised the following questions: What are the influences of the socio-historical context on the formulation of V. A. Krutetskii's theory? What are the main points of V. A. Krutetskii's theory? How does Krutetskii's conception of mathematical giftedness converge with contemporary conceptions of giftedness? What possibilities for analysis does this theory still offer us?

Thus, this article is a theoretical and bibliographical study. Its main objective is to discuss the relevance of Krutetskii's theory for studies on mathematical giftedness today. The text is divided into five sections, of which this introduction is the first. Next, we discuss the life and work of V. A. Krutetskii and highlight some aspects of the research that resulted in the formulation of his theory and convergences with contemporary theories of giftedness. Finally, we make some concluding remarks in which we point out some of the potential of this work in the current research movement into mathematical giftedness.

[5] Throughout the text we will use the abbreviated form V. A. Krutetskii

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2 Historical-biographical considerations: context, life and work of V. A. Krutetskii

Vadim Andreevich Krutetskii (Russian: Вадим Андреевич Крутецкий), sometimes spelled Krutetsky, was born in Moscow in 1917, the year the Russian Revolution began, and died in the same city, coincidentally in 1991, the year that marked the end of the Union of Soviet Socialist Republics (USSR).

Between November and December 2017, in celebration of the centenary of V. A. Krutetskii's birth, the International Conference "Psychological practices in Russian education: an innovative perspective" was held in the city of Armavir (Russia). The purpose of this initiative was to explore the scientific heritage of this and other Russian researchers who contributed to the development of the scientific foundations of psychology and outlined ways of implementing them in pedagogical practice. It is worth noting that in 2007, on the occasion of his 90th birth anniversary, the community of Russian psychologists had already dedicated a series of events to this illustrious researcher, highlighting the lasting resonance of his ideas to this day (NEDBAEV; NEDBAEVA, 2017).

According to Dubrovina (2017), V.A. Krutetskii was one of the most brilliant psychologists of the second half of the 20th century. He graduated in 1941 from the Department of Economic Geography at Moscow State University and, after working as a teacher and director of a technical school in Astrakhan, began his postgraduate studies in 1947 at the Institute of Psychology of the Academy of Pedagogical Sciences (IPACP) of the then Russian Soviet Federative Socialist Republic (RSFSR) (now the Institute of Psychology of the Russian Academy of Education).

In 1950, after defending his doctoral thesis at the IPACP, V. A. Krutetskii remained at that institution, where he worked for more than 30 years, 20 of which as director of the Abilities Laboratory and, for 16 years, as deputy director of the Institute of Basic Research of the Russian Academy of Medical Sciences (SLASTENIN, 2017).

V. A. Krutetskii had a wide range of interests, represented in more than 130 publications. He conducted research in the field of educational and

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developmental psychology (the psychology of instruction and education of schoolchildren) and personality psychology. He also studied individual psychological characteristics and psychological qualities of the personality of children and adolescents, among others. His research was aimed not only at solving scientific problems, but also focused on using his results in pedagogical practice (DUBROVINA, 2017).

Despite so many topics of interest, V. A. Krutetskii devoted himself more extensively to the problem of abilities (literary, artistic and pedagogical) (CHUPRIKOVA, 2017), assigning them a central place among individual psychological characteristics and stating that the development of a child without the development of their abilities is impossible, since abilities are characteristics of a person as a subject of life and activity and that their development determines the formation of the child's personality (DUBROVINA, 2017).

However, among the many abilities investigated, the scientific work on mathematical abilities was the one that made V. A. Krutetskii's ideas cross the borders of the former USSR and be recognized and incorporated by researchers from different countries.

Working with his team of around 50 people, V. A. Krutetskii conducted an extensive investigation into the mathematical abilities elicited in different problem-solving situations between 1955 and 1966, reported in his best-known work Психология математических способностей школьников (Psikhologiya matematicheskikh sposobnostey shkolnikov) published in 1968.

The work had an international resonance beyond the barriers imposed by the Cold War and, in the United States of America (USA), the book was translated into English by Joan Teller and edited in 1976 by professors Jeremy Kilpatrick (1935-2022) (University of Georgia) and Izaak Wirszup (1915-2008) (University of Chicago) under the title “The psychology of mathematical abilities in schoolchildren” and there is no record of official translations of this work into Portuguese or even Spanish.

It is noteworthy that Professor Izaak Wirszup, a Polish Holocaust survivor who emigrated to the USA after World War II, played a key role in

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bringing to the fore discussions about the large gap between American and Soviet expectations regarding the mathematical performance of schoolchildren and the urgency of improving mathematics teaching in American schools.

In view of this, Wielewski (2005) highlights the importance of understanding the socio-political context during the production of V. A. Krutetskii's main work, which substantiates the arguments of the editors of the English version. Research approaches in psychology differed between the capitalist and socialist regimes. While North American and British researchers adopted a behaviorist approach, emphasizing intelligence tests, Soviet researchers, including V. A. Krutetskii, following a Marxist perspective, prioritized the observation and description of cognitive development in various activities, such as problem solving, rejecting the use of tests.

In this way, the context of the development of scientific research in the former USSR followed different paths when compared to the USA and Europe, so the work of V. A. Krutetskii has nuances that deserve our attention and can help us reflect on the future of research and education of mathematically gifted people.

In the field of gifted education research, there is a distinction between so-called general domain theories, which conceptualize giftedness across domains, and specific domain theories, which conceptualize giftedness in a particular domain. In this way, V. A. Krutetskii's theory is considered domain-specific (SCHINDLER; ROTT, 2017).

Recognizing the growing interest in different countries, including the Soviet Union, in the problems faced in Mathematics Education and mathematics as an essential area for the development of other sciences, Krutetskii (1976) started from the principles that the problem of abilities is a problem of individual differences and that abilities are not innate, but are developed through experience and work, i.e. they depend on social factors. In addition, he considered the notion that special attention to the development of gifted children conflicts with the objective of the comprehensive development of children's abilities in general to be mistaken.

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According to Chuprikova (2017), V. A. Krutetskii's interest in mathematically gifted students was strongly influenced by the works of Boris Mikhailovich Teplov on musical ability and Nathan Semionovich Léytes on giftedness, as well as his own interest and inclination for mathematics.

In his last years of work, V. A. Krutetskii was a professor at the Psychology Department of Moscow State Pedagogical University. Under his leadership, methods of studying teachers' abilities were successfully developed. The essence and structure of pedagogical skills and the conditions for their formation and development among students of pedagogical colleges and universities were studied, and methods for diagnosing pedagogical abilities were developed (SLASTENIN, 2017).

This gives us an idea of how important V.A. Krutetskii was to the development and advancement of Soviet psychology, especially during a period of great social and cultural change, which helped some of his ideas to transcend the borders of the former USSR. Next, we'll discuss his work, which has had the greatest repercussions in international academic circles.

3 From mathematical abilities to high abilities in mathematics

In The psychology of mathematical abilities in schoolchildren, V. A. Krutetskii devoted himself to studying the mathematical abilities of schoolchildren and his research was not intended to reveal the nature of such abilities, but rather a study of the ability to master mathematics, as he himself observed (DUBROVINA, 2017).

Its aim was therefore to identify which individual psychological characteristics influence the successful mastery of mathematics, i.e. what makes a person mathematically capable. This study paid special attention to the differences in mathematical performance exhibited by students classified as capable, average and incapable[6].

[6] Although we recognize that this term is inappropriate for today, we must consider that it is a study developed in the middle of the last century, and therefore considered appropriate in that context.

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After carrying out an extensive review of the General Theory of Abilities, resulting from a combination of efforts by several Soviet psychologists such as Boris Mikhailovich Teplov, Lev Semionovich Vigotskii and Alexei Nikolaevich Leontiev, as well as studies carried out by other Russian and foreign psychologists and mathematicians, by ability to learn mathematics Krutetskii (1976, p. 74-75,) refers to the,

> [...] individual psychological characteristics (primarily characteristics of mental activity) that answer the requirements of school mathematical activity and that influence, all other conditions being equal, success in the creative mastery of mathematics as a school subject – in particular, a relatively rapid, easy, and thorough mastery of knowledge, skills, and habits in mathematics.

In his view, "mathematical ability is an internal, complex phenomenon, resulting from the interaction of various components which, in order to be studied, need to be observed during the execution of the activity" (WIELEWSKI, 2005, p. 32). Success in mastering an activity derives from a well-defined combination of characteristics, and is conditioned by the presence of an active/positive relationship with the activity (interests and inclinations), character traits (persistence, self-discipline and diligence) and certain characteristics of mental processes (sensation, perception, thinking, memory and imagination) which, in turn, constitute the source from which students' mathematical abilities originate (KRUTETSKII, 1966).

V. A. Krutetskii was in the habit of writing in diaries her observations on the development of her daughter and then her grandchildren. Their words, actions, games, small and big achievements were all recorded in detail. He firmly believed that, sooner or later, psychology would achieve its place of prominence in the USSR (LEVCHENKO-KRUTETSKAYA, 2017).

Krutetskii (2017) stated that the method of observation is one of the most effective methods of psychological research into a child's personality and that the preschool period is the most intense and most important period of psychological

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development, since the characteristics of this stage are manifested in progressive changes in all spheres, from the improvement of psychophysical functions to the emergence of complex personality transformations.

Due to the ban on mental ability tests in Soviet education, educational psychologists adopted alternative research methods. Prominent among these was the practice of having children recite aloud how they solved a problem, providing a clearer understanding of the mental processes involved. This method included researchers presenting hints or introducing new problems. Interviews with students could be repeated as necessary to ensure comfort. In some instances, teachers took on the role of interviewers, collaborating in the design of lesson sequences that combined individual performance tests and interviews to map the learning process (KILPATRICK; WIRSZUP, 1976).

With around 200 Russian students aged between six and 17 over a period of about a decade (1955-1966), V. A. Krutetskii and his team carried out a wide-ranging and lengthy investigation into mathematical abilities using both statistical techniques and observations of the students while solving different problems.

In one part of the research, V. A. Krutetskii and his team worked from 1958 to 1966 with a group of 34 students considered mathematically capable whose mathematical abilities, as a rule, manifested themselves at an early age. During problem-solving situations, typological differences were identified regarding the structure of mathematical talent, the existence of which is due to the logical-verbal and pictorial-visual components and the relative role of these components in the students' mental activity. The three cognitive styles identified were named: analytical style, geometric style and harmonic style (abstract-harmonic and pictorial-harmonic) and were considered not to be mutually exclusive (KRUTETSKII, 1976).

Students with an analytical style stand out for their superior development of the logical-verbal component compared to the pictorial-visual component, preferring to use abstract schemes when solving problems, even when the situation requires the use of visual concepts. In contrast, students

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with a geometric style show a predominance of the pictorial-visual component, always opting for visual resources when solving problems and facing challenges when dealing with abstract schemes. Students with a harmonic style, on the other hand, display a relative balance between the logical-verbal and pictorial-visual components, with variations that include a preference for abstract mental operations (abstract-harmonic) or the use of visual resources (pictorial-harmonic).

Although this classification into three cognitive styles predates the work of V. A. Krutetskii, his contribution is due to his distinction between the level of mathematical skills (largely determined by the logical-verbal component) and the type of mathematical skills (largely determined by the pictorial-visual component) (PRESMEG, 1991).

The research by V.A. Krutetskii and his team also included a study of the differences in the age of students in the development of the components of mathematical abilities. The analysis covered six parameters: 1) formalized perception of mathematical material; 2) generalization of mathematical material; 3) quality of abbreviated mathematical thinking; 4) flexibility of the mental process; 5) search for an economy of mental effort (rationally solving a problem) and 6) mathematical memory. The results showed regular qualitative and quantitative changes in the manifestation of the components according to age, with the ability to generalize being the first to be formed. In the last stages, abbreviated thinking and economy of mental effort are formed (KRUTETSKII, 1966; 1976).

Krutetskii (1976) pointed out that in some of the children investigated, mathematical abilities were developed in a context of general giftedness. In other cases, mathematically gifted children were not marked by general giftedness, i.e. in all other respects they were children who did not differ from their peers. In this respect, Wielewski (2005) points out that the correlation between the general and specific domains was not explored by him because he considered it to be a complex issue that had not yet been fully resolved by Soviet psychology.

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Then, from the gathering of experimental and non-experimental material, he presented a general outline of the structure of mathematical abilities (Table 1), significant for mathematical talent during school age, which include: obtaining mathematical information, processing mathematical information, retaining mathematical information and the general synthetic component (mathematical mind).

Table 1 – General outline of the structure of mathematical abilities

1. Obtaining mathematical information
(a) The ability to perceive formalized mathematical material, to understand the formal structure of a problem
2. Processing mathematical information
(a) The ability for logical thinking in the sphere of quantitative and spatial relationships, numerical and letter symbols, the ability to think by means of mathematical symbols; (b) The ability to generalize quickly and widely about mathematical objects, relationships and operations; (c) The ability to reduce the mathematical reasoning process and the corresponding system of operations; the ability to think in reduced structures; (d) Flexibility of mental processes in mathematical activity; (e) The search for clarity, simplicity, economy and rationality of solutions; and (f) The ability to quickly and freely reconstruct the direction of the mental process, switching from a direct line of thought to a reverse one (reversibility of the mental process in mathematical reasoning).
3. Retaining mathematical information
(a) Mathematical memory (generalized memory for mathematical relations, characteristic types, schemes of arguments and proofs, problem-solving methods and principles of approach)
4. General synthetic component
(a) Mathematical cast of mind.

Source: Adapted from Krutetskii (1976).

For Krutetskii (1976), the general synthetic component is expressed in the effort to make environmental phenomena mathematical, a constant need to pay attention to the mathematical aspect of phenomena, in other words, to see the world through a mathematical lens. Leikin (2021) understands that the general synthetic component is linked to mathematical curiosity and fascination with new ideas.

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Thus, "mathematical giftedness is characterized by generalized, curtailed and flexible thinking in the realm of mathematical relationships and number and letter symbols and by a mathematical cast of mind". These abilities are expressed to varying degrees in able, average and unable students (Krutetskii, 1976, p. 352).

However, the author emphasizes that the outline presented refers to the structure of the mathematical abilities of school-age children and it is therefore not possible to say, unless studies are carried out to this end, to what extent it can be considered an outline of mathematical abilities in general.

> Krutetskii's research was important because it provided a differentiated picture of the variety of talents and cognitive styles, as well as exploring various aspects of mathematical thinking present in the problem-solving process. It was a wide-ranging and in-depth exploratory study, which is why the results of the study of his work cannot be summed up by listing a few concepts and terms (WIELEWSKI, 2005, p. 2).

For his part, Leikin (2018) points out that systematic studies on mathematical giftedness have been neglected for decades and that V. A. Krutetskii's work remains unique given its focus on the mathematical reasoning characteristics of gifted students and the involvement of different actors (teachers, children and their parents), as well as the combination of qualitative and quantitative research tools. Another highlight was the series of problems that integrated different components that characterize mathematical reasoning.

4 Krutetski's theory: dialog with other theories of giftedness and possibilities for contemporary research

Bearing in mind that mathematical giftedness is still an under-researched topic, reflecting on the theories of giftedness that underpin our research becomes a necessary exercise, especially with a view to broadening the dialog between gifted education and Mathematics Education.

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Leikin (2018) validates this perspective by noting a significant increase in the attention paid to research into mathematical giftedness over the last decade. This area is becoming increasingly interdisciplinary, with researchers establishing connections between mathematics education research and giftedness research more broadly. On the other hand, the growing number of publications on the subject also reveals a lack of consensus among researchers regarding the definition of mathematical giftedness.

As a subject of growing interest and for which there is no unanimous definition, theorizing about mathematical giftedness and developing empirical investigations are actions that should occur in parallel and we believe that (re)visiting the work of V. A. Krutetskii represents a possibility to expand our knowledge base about the phenomenon, since his domain-specific theory of giftedness remains relevant today.

Theories of giftedness that have been revised, implemented in school practice and reaffirmed through empirical research are those that have shown the greatest potential for dissemination and adoption by both researchers and educational policymakers. In this sense, the most fruitful example is undoubtedly Joseph Renzulli's theory of giftedness.

It is a general domain theory of giftedness, made up of four sub-theories, namely the Three-Ring Concept of Giftedness, the Triadic Enrichment Model, Operation Houndstooth and Executive Functions, which has been examined and re-examined for more than four decades. In the Three-Ring Concept of Giftedness, giftedness is characterized by the interaction of three sets of traits: above-average ability, creativity and commitment to the task (motivation) (RENZULLI, 2018).

Schindler and Rott (2017) point out that the Three Ring Concept of Giftedness is often applied in research in the field of Mathematics Education. In this context, Leikin (2018) clarifies that, within the field of gifted education, mathematical giftedness is commonly recognized as a specific form of giftedness. It is distinct from general giftedness, yet most of the models designed to address

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general giftedness can be applied to mathematical giftedness, considering its intrinsic associations with mathematical abilities and competencies.

In Brazil, we can consider it natural that Renzulli's theory underpins most research on mathematical giftedness (FERREIRA, 2020), since it underpins current legislation (CALLEGARI; RONDINI, 2021) and most Specialized Educational Assistance programs aimed at students with high abilities/giftedness (CARNEIRO, 2015). In this way, we highlight below some points of convergence between V. A. Krutetskii's theory and theories of general giftedness, more specifically Joseph Renzulli's theory.

Contemporary models of giftedness consider the role of non-cognitive personality traits in the manifestation of general giftedness (GAGNÉ, 2005; RENZULLI, 2018). These traits, although not explicit in the structure of mathematical abilities (Table 1), are considered by Krutetskii (1976) when he mentions the general psychological conditions necessary for successful performance in an activity, which includes characteristic traits such as diligence, self-discipline, independence and clarity of purpose.

The above-average ability ring includes both general and specific performance areas and any student's performance in this ring is more stable, as it is more related to more traditional cognitive/intellectual traits (RENZULLI, 2018). Schindler and Rott (2017) identified in their study that the mathematical skills investigated by V. A. Krutetskii find greater expression in this ring, demonstrating that this is a trait of greater importance in his theory.

In both conceptions, the definition of a reference group based on peer comparison is more important than a certain percentage obtained in intelligence tests, which are criticized by both.

Motivation (or commitment to the task) is another component of giftedness present in different theories (GAGNÉ, 2005; RENZULLI, 2018). According to Renzulli (2018, p. 27), this set of traits includes perseverance, determination and willpower and that "the meaning of this set of traits in any definition of giftedness is derived from a myriad of research and studies, as well as

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autobiographical analyses of creative and productive individuals". By this, the author is referring to the capacity for immersion and dedication and for facing obstacles demonstrated by individuals who have made important contributions to their respective areas of knowledge.

In general, researchers consider the division of motivation into intrinsic and extrinsic, the former being related to individual engagement or personal satisfaction in carrying out an activity and the latter being related to carrying out an activity due to external demands.

In this direction, Subotnik, Pillmeier and Jarvin (2009) highlight the importance of motivation (intrinsic and extrinsic) as a condition for the development of mathematical ability. Leikin (2010) explains that, in the case of mathematics, intrinsic motivation refers to enthusiasm and pleasure in the process of solving problems or making mathematical discoveries (in the sense that they are new to the student). Extrinsic motivation, on the other hand, is related to some outcome such as getting a high mark in a test/exam or even pleasing parents and teachers.

Krutetskii (1976) explored the intrinsic dimension of motivation and considers essential, among a combination of qualities that lead the student to success in the implementation of mathematical activity, an active and positive attitude towards mathematics and an interest and inclination to study it. In addition, he states that perseverance, capacity for work and diligence were characteristics manifested in the mathematical activity of the gifted students observed in his study.

Another component of giftedness present in different theories is creativity (GAGNÉ, 2005; RENZULLI, 2018), which includes inventiveness, imagination and originality. In the field of Mathematics Education, this has been a topic of growing interest, particularly because of its connection with mathematical giftedness. Although there is no precise definition of what mathematical creativity is and no consensus on the extent to which mathematical giftedness and mathematical creativity are related, much progress has been made in recent decades on this topic (SRIRAMAN, 2005; LEIKIN, SRIRAMAN, 2017).

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Although mathematical creativity is mentioned by Krutetskii (1976), it is neither defined in his theory nor made explicit in the structure of mathematical abilities (Table 1). When describing the methods used in his experimental research, he highlights the use of problems with elements of mathematical creativity and defines them as non-standard problems. Is the ability to come up with answers to non-standard problems therefore one of the elements of mathematical creativity?

In the preface to the Russian edition, the author sets out proposals for the future continuation of his work, including investigating, in cooperation with mathematicians, the more developed structure of mathematical creativity. When presenting the problem and objectives of his study, he emphasizes that we should treat the concept of mathematical ability on two levels: 1) as a creative (scientific) ability, of mathematical professionals, and 2) as a school ability, of schoolchildren studying mathematics.

For Krutetskii (1976), the difference between the two levels is not absolute and they are connected in the sense that the thorough, independent and creative study of mathematics is a prerequisite for the development of a skill for creative mathematical activity and would be present at both levels. Therefore, the mathematical "discoveries" made by a student in math classes, such as a method of solving an already known problem, but which has been "discovered" by the student, would be similar in nature to those made by professional mathematicians.

In view of the above, we can observe convergences between Krutetskii's theory and other contemporary theories of giftedness, in particular with the Renzullian theory. However, our perception of this convergence is limited and we are convinced that detailed research can improve our understanding of the convergence between domain-general and domain-specific theories, since this relationship has not yet been fully clarified.

V. A. Krutetskii reveals that the work carried out with his team was strictly psychological in nature and there was no intention of analyzing math

DOI: https://doi.org/10.14393/OBv7n3.a2023-69584

teaching methods or creating any new method, as they were aware that this should be done by researchers, mathematicians and educators [mathematicians].

Thus, his theory and, in a restricted way, his structure of mathematical abilities, have immense potential for investigation, not only in relation to mathematically gifted students, but students at all levels of performance, as well as problem-solving skills, given the wealth of the 79 problems used in his study.

5 By way of conclusion: the relevance of V. A. Krutetskii's theory

A number of misconceptions persist in the Brazilian educational scene about high abilities/giftedness, which is often perceived as a rare phenomenon. This misconception contributes to the difficulty in implementing effective strategies that provide adequate development for students with this characteristic. In addition, this misinformation impacts the expansion of research on the subject, especially in areas other than Psychology, such as Mathematics Education.

Thus, knowing the genesis of studies on mathematical abilities seems essential to give greater robustness to discussions and research on mathematical giftedness, hence the importance of (re)visiting the most prominent work to date in this area, presented in the main work of V. A. Krutetskii, whose contributions boosted the development of Soviet psychology.

Since this is an extensive work and a more in-depth analysis would be beyond our scope and the limitations of this text, the points highlighted here give us an idea of the richness of the research that resulted in Krutetski's theory of mathematical abilities.

It is crucial to note that V.A. Krutetskii's work not only reflected the influence of the socio-historical context in which it was conceived, but was largely shaped by it. The political-ideological conflict characteristic of the Cold War, marked by the dispute between the USA and the USSR over world hegemony, had significant ramifications, expanding attention to the processes of teaching and learning mathematics. Recognized by both countries as a fundamental area

DOI: https://doi.org/10.14393/OBv7n3.a2023-69584

for scientific and technological progress, mathematics emerged as an object of research, incorporating the identification and development of talented individuals in this field.

In addition to identifying the structure of mathematical abilities made up of the abilities to obtain, process and retain mathematical information, as well as the presence of a "mathematical mindset" and making progress in relation to cognitive styles, V. A. Krutetskii also identified non-obligatory components in the structure of mathematical giftedness whose presence or absence and level of development determine the type of mathematical mindset. To this end, he used a research method and style that differed from the research carried out in the same period.

Although V. A. Krutetskii's theory has been recognized by American and European researchers, it has not escaped criticism, especially in relation to the way he used the statistical technique of Factor Analysis in his studies. On the other hand, Krutetskii's theory reveals points of convergence with contemporary theories of giftedness. In particular, its affinity with Joseph Renzulli's Three Ring Concept of Giftedness stands out. However, the demonstration of these connections lacks empirical investigations to more solidly substantiate these theoretical relationships.

With data collected between the 1950s and 1960s, it is pertinent to question the relevance of the framework presented by V. A. Krutetskii in research on mathematical giftedness today and what we can say is that it has consistently influenced research in the area over the last few decades as can be seen in the works of Greenes (1981), Presmeg (1991), Sriraman (2005), Szabo; Andrews (2017), Leikin; Sriraman (2017) and Leikin (2021).

From a practical point of view, mathematics educators can use the mathematical problems developed and used in the research, which cover arithmetic, algebra and geometry, and, with the necessary adaptations, explore different mathematics learning objectives, which are not restricted to gifted students. They can also be used in other research contexts, such as problem-solving research, which is considered one of the trends in mathematics education.

DOI: https://doi.org/10.14393/OBv7n3.a2023-69584

We would like to thank the Dzeta Research Group Investigations in Mathematics Education (DIEM); the Federal District State Department of Education (SEEDF), the Federal District Research Support Foundation (FAPDF, Notice 12/2022, FAPDF Learning Program) and DPG/UnB Notice 11/2023.

Aportaciones de Vadim Andreevich Krutetskii al campo de las altas capacidades/superdotación en matemáticas

RESUMEN
Con la afirmación de la Educación Matemática como área de investigación y, más recientemente, con la aparición de la Educación Matemática Inclusiva, la investigación sobre la superdotación matemática ha dado lugar a discusiones sobre su definición. Aunque diferentes autores se refieren a la teoría de Krutetskii como el intento más completo de explicar el fenómeno, su trabajo es citado a menudo por ser considerado pionero, pero pocas veces se destaca el potencial de análisis que presenta su concepción de la superdotación matemática. Este artículo es un estudio teórico y bibliográfico y pretende discutir la relevancia de la teoría de Krutetskii para los estudios actuales sobre la superdotación matemática. Fue posible identificar convergencias con las teorías contemporáneas de la superdotación y que, desde el punto de vista de la investigación científica, la teoría de Krutetskii nos presenta elementos ricos que pueden ayudarnos a reflexionar sobre los aspectos teórico-metodológicos de nuestras investigaciones.

Palabras clave: Krutetskii; Superdotación; Habilidades matemáticas; Investigación; Teoría.

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Received in June 2023.
Approved in October 2023.