

The organization of fraction teaching in Basic Education from the logical-historical movement

A organização do ensino de fração na Educação Básica a partir do
movimento lógico-histórico

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ABSTRACT

The article aims to discuss the need to reflect, together with teachers of Mathematics of Basic Education, both in undergraduate courses and in schools, the possibilities of organizing the teaching of mathematical concepts, among them fraction from learning triggering situations that are based on the logical-historical movement of the concept. In this context, “the fraction can be understood in at least two ways: as an operative technique or as language, thought, creativity and reading of the world” (LIMA, 1998, s/p) when one understands its conceptual concepts, quantities and measures, which are historically constituted by the various social and cultural groups. It is assumed that the logical-historical movement of the concept can be understood as a didactic perspective for the teaching of Mathematics. It is from this movement that the teachers of Basic Education, when dealing with mathematical concepts in their classrooms, can prioritize the theoretical thinking of the fraction, so that it is able to provide the development of students, according to the assumptions of historical-cultural theory.

Keywords: Learning triggering situations. Theoretical thinking. Historical-cultural theory.

RESUMO

O artigo² tem como objetivo discutir a necessidade de refletir, juntamente com professores de Matemática da Educação Básica, tanto nos cursos de licenciaturas, quanto nas escolas, as possibilidades de organizar o ensino de conceitos matemáticos, dentre eles o de fração a partir de situações desencadeadoras de aprendizagem que se fundamentem no movimento lógico-histórico do conceito. Nesse contexto, “a fração pode ser entendida pelo menos de duas formas: como técnica operatória ou como linguagem, pensamento, criatividade e leitura do mundo” (LIMA, 1998, s/p), quando se compreendem seus nexos conceituais, grandezas e medidas, os quais são constituídos historicamente pelos diversos grupos sociais e culturais. Tem-se como pressuposto que o movimento lógico-histórico do conceito pode ser entendido como perspectiva didática para o ensino de Matemática. É a partir desse movimento que os professores da Educação Básica, ao tratarem dos conceitos matemáticos em suas salas de aula, podem priorizar o pensamento teórico da fração, de modo que esse seja capaz de proporcionar o desenvolvimento dos educandos, conforme os pressupostos da teoria histórico-cultural.

Palavras-chave: Situações desencadeadoras de aprendizagem. Pensamento teórico. Teoria histórico-cultural.

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1 Introduction

In the 21st century, more precisely in the year 2022, despite the discovery of the vaccine against Covid-19³, we are still experiencing a global pandemic that was announced in March 2020. In this context, Brazilian schools, especially public schools—which remained closed for two years in terms of in-person teaching but which, since April 2020, have been regulated by the Ministry of Education and Culture (MEC) through actions called non-face-to-face⁴ educational activities—are being challenged to convince children and young people to return to studying concepts in various areas of knowledge, including mathematics. According to Honorato and Nery (2020, p. 2)

Specifically, school education—from early childhood education to higher education—was the first to have its activities suspended and will likely be one of the last sectors to fully reopen. And this will not mean a return to “normality,” not least because many impacts will be felt. COVID-19 has already caused the closure of schools, affecting approximately 87% of the world's student population, around 1.5 billion students in 165 countries, according to records from the United Nations Educational, Scientific and Cultural Organization [UNESCO] (2020). In fact, the solutions adopted for the “new normal,” especially in the form of distance learning, have been received by school stakeholders as controversial measures.

In other words, among the various challenges faced by mathematics teachers in basic education, there is one in particular: thinking about how to organize the teaching of mathematics in basic education to break with what Skovsmose (2000) called the exercise paradigm.

³ COVID-19 is an acute respiratory infection caused by the SARS-CoV-2 coronavirus, which is potentially severe, highly transmissible, and globally distributed (FEDERAL, 2022, n/p).

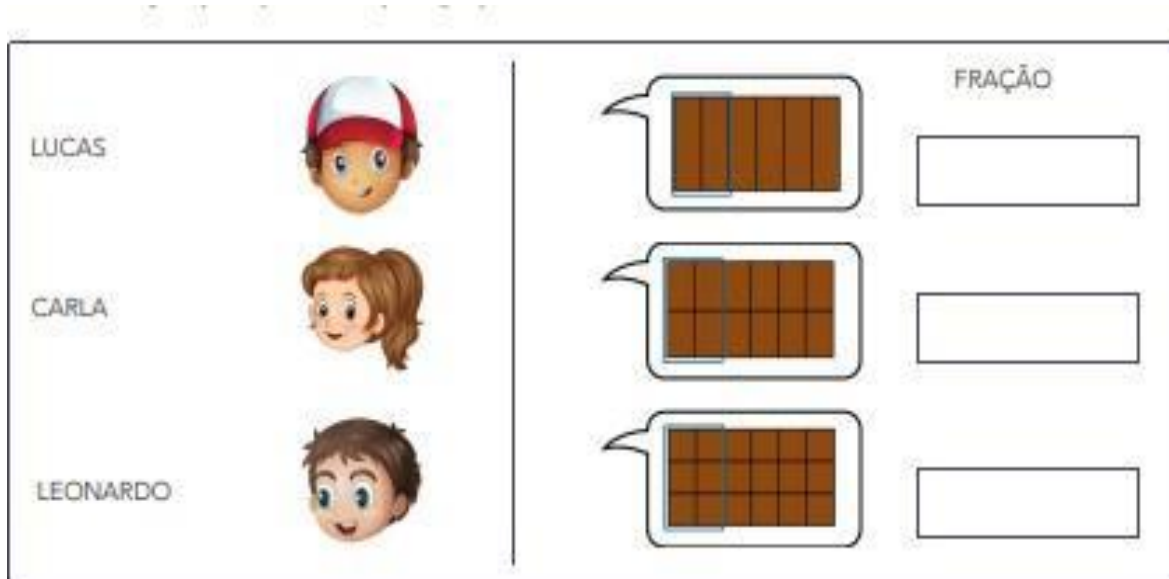
⁴ **Elementary school (final years) and high school** – Adult supervision of activities can be provided through guidance and monitoring, supported by plans, goals, and schedules for in-person or *online* study, since students have more autonomy at this stage. In this case, the recommendation is to give more space to non-face-to-face educational activities. Among the suggested activities is the distribution of educational videos (MEC, 04/28/2020, n/p).

We might ask ourselves, would it be possible to break with the exercise paradigm so that both mathematics teachers and students in basic education can be humanized by mathematics, according to the assumptions of historical-cultural theory? How could the logical-historical movement (KOPNIN, 1978) of concepts contribute to such an organization, considering that, according to Stamato (2003), the history of mathematics was implemented as a discipline in mathematics degree courses almost twenty years ago?

To begin answering these questions from the perspective of historical-cultural theory, which are part of the research we are developing, entitled: “History of Mathematics in the Training of Basic Education Teachers” (CNPq), it is necessary to revisit the paradigm of the exercise, since it manifests itself when teachers organize teaching based on the assumptions of Skovsmose (2000), in which what is emphasized in the classroom are the perceptible elements of concepts (DAVYDOV, 1982). There is a predominance of the use of formulas, memorization, exercise training, and assessments. A single answer is required for problem solving. From this perspective, Basic Education teachers make no mention of the logical-historical movement of concepts, since the focus of classes is on solving exercises. This does not mean that teachers are solely to blame for teaching.

In the specific case of teaching fractions, when analyzing the proposal of the State of São Paulo, entitled “Currículo Paulista” (2022), which is based on the assumptions of the National Common Core Curriculum (Base Nacional Comum Curricular - BNCC), published in 2017, we find that the materials available to students and teachers for the 5th grade of Basic Education, at least initially, prioritize the perceptible elements of the concept and representations, which come from figures whose starting point are everyday examples involving the division, into equal parts, of pizzas, chocolates, etc., as shown in lessons 7 and 8, called by the authors “Rational numbers” and whose objective is “to solve problems with rational numbers in fractional and decimal representation” (SEE/SP, 2022, p. 249).

After solving the challenge set by his father, Lucas called his siblings to eat the chocolate bar his mother had bought for them. Paula bought a chocolate bar of the same size for each of them, but they didn't eat the whole bar. They saved the rest for after dinner. Write the fraction that represents the piece each of them ate from their chocolate bar:



Source: Always Learning Notebook (SEE/SP, 2022)

The authors then present the following definition: “a rational number has two representations: fractional and decimal” (SEE/SP, 2022, p. 250).

In this proposal for organizing fraction teaching, teachers are invited to focus on the representation of fractions in the form of a/b , where a is called the numerator and b is the denominator, which is not equal to zero. The starting point for the lesson is an example from everyday life, and the focus is on representations.

It is no coincidence that the studies by Alves (2018, p. 216), conducted in the north of the country on the teaching of fractions, point out that “lessons on fractions in the sixth grade (...) show that 84% of students indicated that teachers began their lessons with the definition followed by examples and exercises, therefore, in the traditional way.” What changes in the proposal approved by the São Paulo State Department?

Only the order. Everyday examples precede the definition, but the focus is still on representation, therefore, on the operational aspect of fractions. In the third

stage, the authors' guidelines are directly related to solving exercises, which are sometimes called problems and sometimes activities (SEE/SP, 2022). They require only a single answer, as pointed out in the studies by Skovsmose (2000).

Finally, the authors of the São Paulo Curriculum suggest that teachers evaluate students through self-assessments. It should also be noted that the 5th grade mathematics workbook is integrated with the Portuguese language workbook and seeks to contextualize what the authors call didactic sequences, which are composed of exercises that have come to be called problems and activities.

This way of organizing teaching has been around for a long time. It is designated as traditional by Alves (2018) and has proven to be inefficient, since students do not understand this concept, as pointed out by the studies of Dionizio et al (2019). For this reason, it is necessary to ask: why teach the concept of fractions? What is this concept for? Is there another way to organize the teaching of the concept of fractions in basic education?

We can answer these questions from two points of view: the exercise paradigm and the logical-historical movement of the concept.

If the starting point continues to focus only on representation derived from everyday examples, as indicated by the BNCC (2017) and, consequently, some curricula, such as the São Paulo Curriculum (2020), we can agree with those who argue that the concept of fractions is unimportant because it becomes merely operational (LIMA, 1998) and, for this reason, together with operations, can be abolished from curricula. Thus, mathematics students and teachers do not need to study it to teach it.

If the starting point is the logical-historical movement of the concept, it is necessary for students and teachers to be aware of the conceptual links (internal and external) that are present in the concept of fractions.

From this perspective, that of the logical-historical movement of the concept, the main objective of teaching fractions is for students in basic education to acquire theoretical thinking in relation to the concept of fractions. In this sense, fractions can help children and young people analyze the reality that surrounds them, based

on fundamental elements, which are called conceptual links (internal and external) that involve measurement and quantities (continuous and discrete), for example, beyond representation and operations.

Here, there is the possibility of breaking with the exercise paradigm. Or, even, with the teaching of fractions that prioritizes only the operational aspect, harming the conceptual. To this end, it is necessary for mathematics teachers and teacher trainees to become familiar with what Moura (2010), inspired by Leontiev's studies, calls teaching activity (TA) and teaching guidance activity (TGA), since

The teacher's teaching activity should generate and promote student activity. It should create in students a special reason for their activity: to study and learn theoretically about reality. It is with this intention that teachers plan their activities and their guidance, organization, and evaluation actions. (MOURA, 2010, p. 90).

Here, the definition given by Moura (2010) differs from what the authors of the São Paulo Curriculum call an activity. The same author defends Teaching-Guiding Activity (TGA) as a possibility for mediation to organize the teaching of mathematics, considering the humanization of those who teach and those who learn, since the teaching and learning processes are not dissociated from the historical and cultural experiences of the subjects, because teachers and students are active. In addition, the concepts dealt with in the classroom have a history and are therefore in flux, since they are not ready and finished.

This way of organizing teaching, which has Educational Activity (EA) and TGA as its theoretical and methodological assumptions, presupposes the need for mathematics teachers and teacher trainees to study how to learn to design and develop learning trigger situations (LTS) that consider the logical-historical movement of the concept as a didactic perspective. Such movement presupposes the study and analysis of conceptual links (internal and external). Therefore, SLS contains them.

It is about the need to reflect, together with mathematics teachers in basic education, both in teacher training courses and in schools, on the possibilities of

organizing the teaching of mathematical concepts, including fractions, based on SLS, which are grounded in the logical-historical movement of the concept discussed in this article.

In the next section, we will present the theoretical and methodological foundations of the research, then discuss the organization of fraction teaching in Basic Education from two perspectives: BNCC (2017) and the logical-historical movement of the concept. Finally, we will make our concluding remarks.

2 Theoretical and methodological foundations

When defending the logical-historical movement of concepts in classrooms, it should be considered that, despite technological advances—especially those related to non-face-to-face⁵ activities and the new demands presented to today's society— we cannot fail to draw attention and insist, once again, on the fact that when we analyze the teaching of mathematics in basic education, we find that very few changes have been made in terms of its organization.

In this sense, even though Davidov (1988) made this observation more than thirty years ago in the Russian context, based on studies related to “the didactic principles of the traditional school and possible teaching principles in the near future” (p. 143), we must recognize, as the author does, that

Indeed, over the course of hundreds of years, the main social purpose of mass education has been to instill in most working-class children only those knowledge and skills without which it is impossible to obtain a more or less meaningful profession in industrial production and social life (knowing how to write, count, and read; having elementary ideas about the surrounding world). Elementary school achieved these goals and served as the first and only stage in the education of most of the population, a stage that directly prepared children for work as more or less skilled labor or for vocational training in relatively simple specialties. The solution

⁵ The transition, in just a few weeks, from the conventional school model to the online model, or to mixed formulas combining telematic interaction and personalized tutoring, led us to believe that we were facing a revolution in the strategies, curricula, and methods of formal education, with very rapid and positive achievements, such as the universalization of digital innovations, the effectiveness of remote interaction, the collaboration of families, openness to new scenarios, and others. Many of the aspirations that we have been formulating for decades as goals have been partially realized or have been seen to be plausible expectations for the future.

to this social task corresponded entirely to the utilitarian-empirical content provided by traditional primary school and the teaching methods that had been developed there over many years. (DAVYDOV, 1988, p. 144).

As we are now in the year 2022, in the Brazilian context, after reading the quote, we could question the author about its validity, arguing that, as more than three decades have passed since this analysis was written, compulsory schooling has expanded here in Brazil. Currently, children spend at least 12 years learning more than just reading, writing, and counting, and since the publication of the BNCC (2017), the focus of teaching and learning processes has been on skills development⁶.

If we agree with Davidov (1987) and consider his arguments that schools—in their social practice, through the didactic principles they list—have prioritized empirical reasoning to ensure that children and young people can perform strictly everyday tasks; if we analyze how mathematics is taught, including the teaching of fractions, we will find that the BNCC (2017) itself and the curricula based on this document – as is the case with the São Paulo Curriculum (2022) – can lead teachers to organize the teaching of the concept of fractions in a way that prioritizes empirical-discursive mathematical thinking, through proposals that consider four distinct moments, studied by Lima (1998): 1) showing the concept; 2) demonstrating how the concept works; 3) training the concept; and 4) evaluating the concept.

This way of teaching fractions is based on the argument that students need to relate the mathematical concepts taught in schools to everyday life and, at the same time, pass internal and external assessments, including the National High School Exam (Enem) and university entrance exams.

We understand that this way of organizing teaching, which day by day becomes rote learning for students, is based on what Davydov (1982) called

⁶ In the BNCC, competence is defined as the mobilization of knowledge (concepts and procedures), skills (practical, cognitive, and socio-emotional), attitudes, and values to solve complex demands of everyday life, the full exercise of citizenship, and the world of work (BRAZIL, 2017, p. 8).

traditional didactics, which disregards the conceptual links (internal and external) present in the concepts that are taught in school. Here, priority is given to empirical thinking, the study of the perceptible elements of concepts.

In the specific case of the concept of fractions, the focus of teaching is on the symbolic aspect: a/b , where a is defined as the numerator, b is the denominator and must necessarily be different from zero, as well as on the operations and rules for solving the exercises. It is for this reason that we agree with Libâneo (2004) when he states that

(...) teaching methods need to incorporate the latest research on ways of learning and teaching and on the mediating role of teachers in preparing students for thinking. More precisely, it will be essential to understand that knowledge presupposes the development of thinking and that developing thinking presupposes methodology and systematic thinking procedures. (LIBÂNEO, 2004, p. 6).

In order for didactics to review and expand the principles that govern traditional schools to principles that are more in line with what Davidov (1988) called “teaching in the near future,” which, in our view, is no longer *near* because it has already arrived, as we have entered the 21st century and have a new global reality after a long period of the COVID-19 pandemic, it is necessary to consider

(...) the new principles of the school. Logically, all teaching should preserve the link and “succession” of knowledge, but it should be a link between qualitatively different stages of teaching, different both in content and in the procedures used to convey this content to children. (DAVIDOV, 1988, p. 150).

In this sense, the teaching of concepts must consider the principles of activity and object character as a way of abolishing a teaching structure that has sought, for centuries, to preserve only “the traditional interpretation of the principle of conscious character” (DAYDOV, 1982, p. 150). Otherwise, methodological dogmatism may be directly related to the paradigm of exercise, the antithesis of the logical-historical movement of the concept that is rejected by those who develop curricula, including the São Paulo Curriculum.

It is as if concepts had to be learned to be used immediately; that is, empirical-discursive thinking (DAVYDOV, 1982) is prioritized over theoretical thinking, which requires an understanding of the conceptual links (internal and external) between concepts.

In this sense, from the perspective of historical-cultural theory,

We must therefore consider two fundamental interrelationships in teaching: first, the human aspect, consisting of the teacher, their students, and the group in constant and fruitful interaction; second, the cultural aspect, consisting of objectives, content, methods of organization, and evaluation. It will always be a serious distortion of the teaching perspective to attribute exaggerated importance or emphasis to content or method, as if they were unique or decisive in the situation; they will only be valuable insofar as they contribute to the development and formation of the students' personalities (...). It should be noted that teaching also encompasses the general principles, criteria, and practical norms that regulate all teaching activities, framing them within a rational set of broad meaning and direction. (FERNÁNDEZ, 2007, p. 5-6).

In other words, the logical-historical movement of concepts can be considered didactic for mathematics because it encompasses the two interrelationships proposed by Fernández (2007, pp. 5–6): the human and the cultural.

By assuming that both new didactic principles and dialectical theoretical thinking should be part of didactics and, consequently, guide the organization of mathematics teaching, whether in basic education schools or in higher education, more precisely in mathematics degree courses, we are arguing that

Students must learn to contextualize their professional activity historically and socially, questioning the social significance of the content they convey and the methods and forms they use within the set of current social relations. This means situating phenomena from the perspective of their historical-classist development. (FERNÁNDEZ, 2007, p. 4).

In this context, teachers who already teach mathematics and teacher trainees, and consequently elementary school students, are now exposed to a different type of mathematics teaching organization, very different from the one that promotes training and fragmentation of mathematical concepts.

But how can mathematics teaching be organized in Brazilian schools, considering the particularities of each school, teacher, and student, as well as new teaching principles and dialectical theoretical thinking, to break with traditional teaching methods?

To foster theoretical thinking, the teaching of scientific concepts must be based on didactic procedures aimed at the appropriation of the concept as a mental activity, which differs greatly from the conceptual teaching model typical of the school tradition and materialized in textbooks and handouts. Organizing teaching from this perspective is therefore a major challenge, as it involves treading paths that are still little known. (SFORNI, 2015, p. 377).

This means that, when prioritizing theoretical thinking in elementary school classrooms, we need to challenge ourselves, together with mathematics teachers and teacher trainees, to develop and organize proposals that consider the historical aspects of concepts, which need to be appropriated by children and young people.

Thus, in these new times of curriculum implementation in schools, based on the BNCC (2017), it must be strongly considered that

In advocating teaching focused on the development of theoretical thinking by students, our goal is not to create a teaching method based on historical-cultural theory, but rather to identify some principles that can guide teachers in the planning, execution, and evaluation of teaching. The studies by Moura (1996; Moura, 2001; Moura, 2010), Núñez (2009), and, more recently, Nascimento (2010) also point to principles or teaching actions based on this theoretical perspective and were therefore taken as a reference in our work. (SFORNI, 2015, p. 381).

Based on research involving educational experiments grounded in a historical-cultural perspective, the aforementioned author presents five principles that can guide teachers' actions regarding the organization of teaching:

- "1. Principle of teaching that develops:
2. Principle of the active nature of learning:
3. Principle of conscious nature;

4. Principle of unity between the material (or materialized) and verbal planes;
5. Principle of concept-mediated action” (SFORNI, 2015, p. 382).

We understand that these principles are similar to those developed by Davidov (1988) and are not present in the BNCC (2017).

Based on the studies by Davydov (1982), Davidov (1987), and Fernández (2007), which deal with didactics, as well as the studies by Sforni (2015), which present us with the didactic principles that would be appropriate for Brazilian schools, we find that the three researchers suggest that the logical-historical movement of concepts should be frequent in classrooms. Thus, we propose that this movement be understood as a didactic perspective for teaching mathematics. In this context, LTS are organized by teachers

[...] based on their teaching objectives, which, as we have said, translate into content to be appropriated by students in the learning space. The teacher's actions will initially be organized with a view to setting in motion the construction of a solution to the situation that triggers learning. These actions, in turn, when triggered, will consider the objective conditions for the development of the activity: the material conditions that allow for the choice of methodological resources, the knowledgeable subjects, the complexity of the content under study, and the cultural context that frames the subjects and allows for socio-affective interactions in the development of actions aimed at the objective of the activity—the appropriation of certain content and the general mode of learning action. In other words, the subjects, mobilized from the triggering situation, interact with others according to their potentialities and aim to reach another level of understanding of the concept in motion. In addition, the way of approaching the concept also endows it with a new quality when having to solve problems, because, in addition to having learned new content, they have also acquired a way of appropriating content in general. The triggering situation for learning must contemplate the genesis of the concept, that is, its essence; it must explain the need that led humanity to construct the concept in question, how problems and human needs arose in a given activity, and how humans developed solutions or syntheses in their logical-historical movement. (MOURA, 2010, p. 222-3).

We argue that EA and, consequently, LTS should consider: a) the historical development of the concept, that is, the logical-historical movement of the concept being studied; b) the dialectical moments of its formation; and c) the experience of the subjects' participation linked to a reflective-active-explanatory process, measured by the individual-group-class relational dynamic.

2.1 Materials and methods

The research is qualitative, theoretical in nature, and characterized, according to Fiorentini and Lorenzato (2007, pp. 102-103), as “bibliographic or historical-bibliographic” (...). This type of research is also called a documentary study, with an emphasis on “typically historical studies,” since “primary sources” are used, such as theses, dissertations, articles, and pedagogical policy projects as a way of collecting information.

The study methodology consists of the following stages and strategies:

1) The logical analysis of content, which consists of a theoretical study of the logical-historical movement of concepts addressed in Basic Education. The study necessarily refers to bibliographic research involving both the historiography of mathematics and the relationships between the historiography of mathematics and the logical-historical movement.

2) The proposal of LTS for Mathematics, which consists of a theoretical study on the logical-historical movement and the analysis and elaboration of LTS for Mathematics that deal with Basic Education content. The elaboration of the situations counts on the participation of both students from the Mathematics course at UFSCar and Basic Education teachers who develop research in graduate programs in Education (Academic and Professional), at the Master's and Doctorate levels, who are part of the "Shared Teacher Training Research Group – School and University (GPEFCom)," which is under our coordination.

3) Theoretical deepening of how the History of Mathematics has been incorporated into Mathematics degree courses at Brazilian federal public universities. It consists of a study of pedagogical projects in Mathematics courses

at Brazilian federal public universities, as well as a survey and analysis of theses, dissertations, and articles published in journals dealing with the same theme.

Thus, this article is directly related to the second moment, since it deals with the LTS analysis of the concept of fractions based on the logical-historical movement of concepts carried out with mathematics teacher training students and teachers of basic education in training activities.

It also considers the fact that, although Brazilian mathematics degree courses have been including the subject of the history of mathematics in their curricula for almost twenty years, according to studies by Stamato (2003), the bibliographic survey we have conducted so far (SOUSA, 2021) shows that the focus of the subjects is not on the analysis of historiographies whose main objective is for undergraduates and teachers of mathematics in basic education to learn to analyze conceptual links (internal and external) present in historiographies, to develop, with a certain degree of autonomy, LTS in their classrooms, based on the didactic principles indicated by Sforini (2015). In other words, it seems that in most subjects, the logical-historical movement of the concept is not the object of study.

We highlight two types of instruments to be used in the research: those that are contributing to the construction of facts: the theoretical texts already produced, and the mathematics LTS based on historical-cultural theory, which are being developed by researchers, elementary school teachers, and teacher trainees. These instruments make it possible to consider the more general movement of the research.

The LTS encompasses two essential characteristics to fulfill the research objectives: 1) They are a teaching and research tool, that is, they are planned by the researcher with or without the participation of mathematics education students from UFSCar and elementary school mathematics teachers, with the goal of obtaining data that reveal the relationship that may involve the organization of teaching in the classroom. 2) They are a tool for teacher training, especially in degree programs, by teaching students how to develop LTS based on logical-historical movement.

The analysis of the data follows an interpretive line, whose characteristic is particularization rather than generalization of results. The search is not for

abstract universals, which, according to Moreira (1990), are reached through statistical generalizations, but for concrete universals, which are reached through the detailed study of a specific, culturally localized case. To this end, we are constructing categories that represent how mathematics teachers and students in basic education can create LTS of mathematical concepts, including fractions, that consider the logical-historical movement.

3 The organization of fraction teaching in Basic Education based on the BNCC

Before presenting LTS on the concept of fractions that considers the logical-historical movement, let us refer to the BNCC, a document that has been the basis for Brazilian state curricula since 2017, so that we can analyze the guidelines that Basic Education teachers receive for organizing the teaching of the concept of fractions in mathematics classes.

According to the recommendations of the authors who drafted it,

Based on recent Brazilian curriculum documents, the BNCC considers that the different fields that make up mathematics bring together a set of fundamental ideas that produce connections between them: equivalence, order, proportionality, interdependence, representation, variation, and approximation. These fundamental ideas are important for the development of students' mathematical thinking and should be converted into objects of knowledge in school. Proportionality, for example, should be present in the study of: operations with natural numbers; fractional representation of rational numbers; areas; functions; probability, etc. In addition, this notion is also evident in many everyday actions and other areas of knowledge, such as sales and commercial exchanges, chemical balances, graphical representations, etc. (BRAZIL, 2017, p. 268).

In other words, in this proposal, fractions are understood as objects of knowledge, are included in the thematic unit Numbers, and emphasize proportionality in their fractional representation of rational numbers. It should be noted that there is no mention of the logical-historical movement of this. It is implied that fractional representation is sufficient to indicate, for example, the notion of proportionality. It fails to indicate that fraction, ratio, and proportion are

different concepts that were historically configured at different times. They have different objectives. What they have in common is only their representation in the form of a/b , where b is different from zero.

The authors express concerns about fractional representation, operations, and relationships involving the numerical fields of natural and rational numbers. They suggest that the following objects of knowledge be addressed from 5th to 8th grade:

(5th year, p. 294): Fractional representation of rational numbers: recognition, meanings, reading, and representation on the number line; Fractional representation of rational numbers: recognition, meanings, reading, and representation on the number line; Comparison and ordering of rational numbers in decimal and fractional representation using the notion of equivalence; Calculation of percentages and fractional representation; Problems: addition and subtraction of natural numbers and rational numbers whose decimal representation is finite; Problems: multiplication and division of rational numbers whose decimal representation is finite by natural numbers; (6th grade, p. 300): Decimal number system: characteristics, reading, writing, and comparison of natural numbers and rational numbers represented in decimal form; Fractions: meanings (part/whole, quotient), equivalence, comparison, addition, and subtraction; calculation of the fraction of a natural number; Operations (addition, subtraction, multiplication, division, and exponentiation) with rational numbers; (7th grade, p. 306): Fractions and their meanings: as part of integers, result of division, ratio, and operator; Rational numbers in fractional and decimal representation: uses, ordering, and association with points on the number line and operations; (8th grade, p. 312): Repeating decimals: generating fraction (BRAZIL, 2017, 294-312).

Although the authors of the BNCC (2017) suggest that elementary school teachers can organize the teaching of fractions based on relationships involving natural numbers, operations, and fractional representations, these relationships are very weak and only operational, making it difficult for students to relate the topics. There is little conceptual depth, which explains the difficulties students face, according to studies by Dionizio et al (2019).

Despite these studies, when analyzing the São Paulo Curriculum

(2022), we found that this way of organizing teaching, which is based on skills, was strictly followed by the authors who wrote the notebooks referring to the organization of teaching the concept of fractions, insofar as they prioritize the operational aspect of the concept of fractions, as we presented in previous paragraphs.

4 The organization of fraction teaching in Basic Education based on the logical-historical movement

Based on the assumptions presented in previous paragraphs, we have suggested that mathematics teachers and teacher trainees design and develop LTS on concepts, including fractions, that have as their starting point, in Basic Education classes, conceptual links, with an emphasis on internal links, breaking with the current “contemporary mathematical formulation of content” (RADFORD, 2011, p. 16).

In the specific case of the concept of fractions, teaching has been based on the perceptible elements of the concept and on fractional representation, a/b .

We believe that it is necessary for teacher trainees and teachers to understand that fractions represent a “new extension of the realm of numbers” (KARLSON, 1961, p. 50), because

It is truly curious that humanity sensed very early on the existence of other numbers besides whole numbers. Due to circumstances – never on his initiative – the hunter was sometimes forced to share a fish or game, when he had only one left. They would then divide it into two halves, four quarters, or even more fractions, depending on the number of mouths to feed. Hunger, as we can see, was not only the best of cooks but also an excellent mathematician. It taught humanity fractions, whose name still recalls, in many languages, the act of breaking things into pieces. (KARLSON, 1961, p. 50).

Karlson (1961, p. 51) also points out that, apart from hunters, “all ancient civilizations, from the Egyptians and Abilonians to the Hindus and Chinese, knew fractions without exception, and even knew how to use them with relative skill.” It is no wonder that “that practical factor of coercion, already mentioned, led them to

see fractions as the most natural thing in the world” (p. 51). Here, there is clear evidence of the “existence of proper fractions, such as $1/2$; $3/4$; $5/7$; $17/89$. It is only a small step from here to improper fractions, such as $17/13$; $5\ 2/8$ ” (p. 51). Already

Calculation with simple fractions developed even before the definitive regulation of division, which was not really an easy task, as there was no adequate numerical notation. (KARLSON, 1961, p. 51).

In the case of Caraça (1998), when dealing with the concept of fractions, he presents us with the problem of measurement as a starting point. In analyzing it, he constructs both the rational field and its properties. For the author, when measuring, we need

1) Establish a single standard of comparison for all quantities of the same type; this standard is called the unit of measurement for the quantity in question—for example, the centimeter for lengths, the gram for weights, the second for times, etc. 2) Answer the question – how many times? – (...), which is done by giving a number that expresses the result of the comparison with the unit (...). There are, therefore, three distinct phases and three distinct aspects to the problem of measurement – choice of unit; comparison with the unit; expression of the result of this comparison by a number. (CARAÇA, 1998, p. 30).

The same author also states that the measurement operation has direct links to private property and the State, since,

In all these relationships, which encompass, so to speak, all economic activity of landowners, it is necessary to carefully determine areas, which depend, according to rules taught by geometry, on the measurement of certain dimensions (CARAÇA, 1998, p. 31).

The historiographies of Karlson (1961) and Caraça (1998) suggest that we can organize the teaching of fractions in basic education in a way that considers conceptual links: quantities (discrete and continuous) and measurement, based on issues that involve the reality of different social groups. These connections are internal, while fractional representations are external connections.

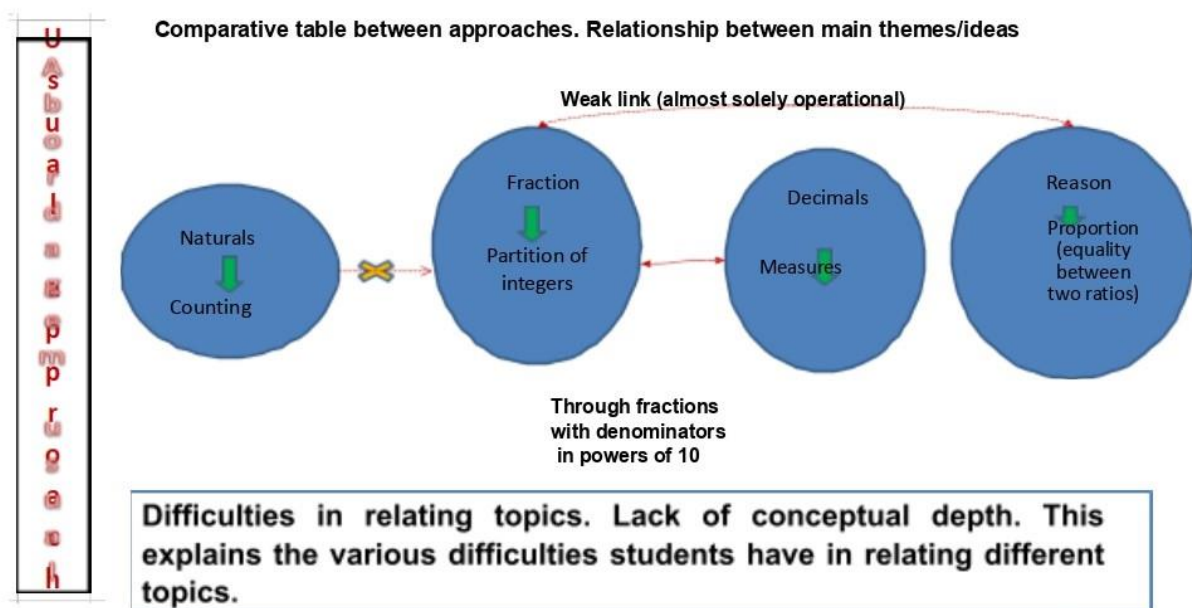
Thus, to analyze the conceptual connections that arise in this way of thinking about fractions and with the aim of making our teaching practice an experience of the reciprocal movement between theory and practice, we decided to offer, in the second semester of 2021, the extension activity: “Teaching fractions, measurements, and decimal numbers from a logical-historical perspective” at the Federal University of São Carlos (UFSCar), in a remote format due to the COVID-19 pandemic.

We intended to analyze, based on the studies by Karlson (1961) and Caraça (1998), together with mathematics teachers and students, the Basic Education LTS involving the concepts of fractions, measurements, and decimal numbers that are based on the logical-historical movement of the concept, in an articulated way emphasizing the expansion of natural numbers, based on historically studied problems that deal with the measurement and definability of fractions, ratios, and proportions.

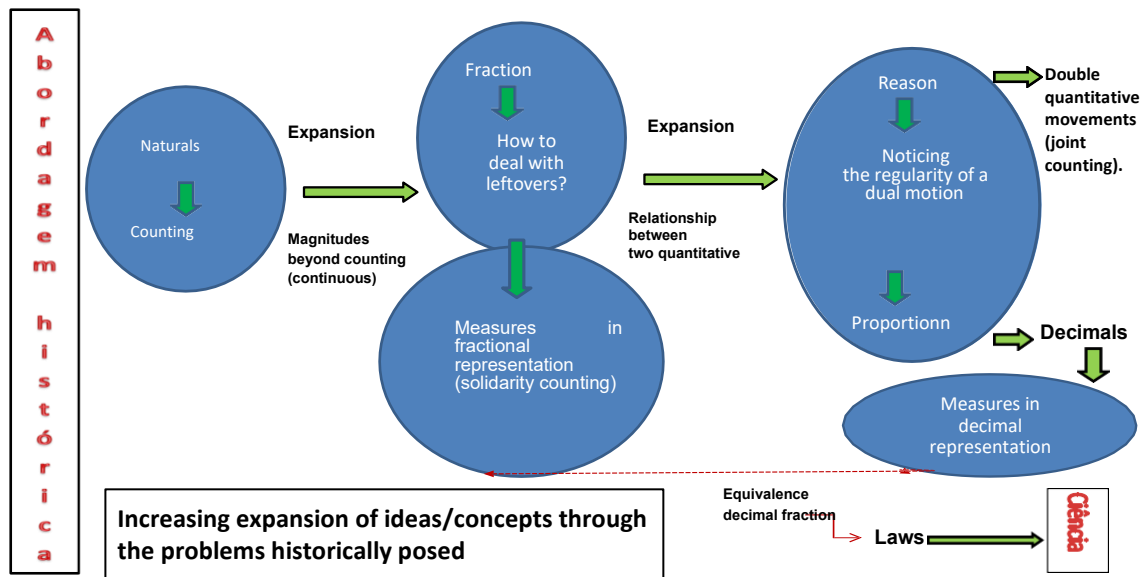
Table 1 compares the ideas defended by the two authors with those presented in various textbooks and in the BNCC (2017).

Chart 1: Comparison between the traditional approach and the logical-historical movement of the concept

Comparative table between approaches. Relationship between main themes/ideas



Textbooks



Source: Caraça Group (2000)

As Table 1 shows, the logical-historical movement allows us to understand that the concept of fraction has continuous and discrete quantities, as well as quantitative variations, as internal conceptual links. The field of rational numbers is formed by the concepts of fraction, ratio, and proportion, which, regarding the counting of movements in life, involve double movements. For this reason, rational numbers allow us to perform joint counts, while natural numbers allow us to perform solitary counts. In the case of natural numbers, counting is done based on one-to-one correspondence and groupings. Joint counting, on the other hand, requires an internal conceptual link: measurement.

In this sense, we chose Lima's (1998) proposal as a possibility for organizing the teaching of the concept of fractions in elementary school mathematics classes because it is in line with the studies by Karlson (1961) and Caraça (1998), as it considers both the operational and conceptual aspects of fractions. For the author,

The fraction can be understood in at least two ways: as an operative technique, or as language, thought, creativity, and reading of the world. It encompasses multiple historical, geographical, geometric, philosophical, cultural, physical, chemical, literary, artistic, and other connections. This is what makes fractions the best part of the whole. (LIMA, 1998, n/p).

From this perspective, when measuring lengths of land, for example, we may have to answer the question: *what will we do with the leftovers?* The answer to this question may lead us to solid counting, since natural numbers allow for solitary counting. This question does not make sense when counting discrete objects. Based on this understanding, the same author asks us the following question: “How can we understand fractional thinking?”

To answer it, he invites us to organize our learning process of the concept of fractions by studying four moments that arise in the construction of this concept: 1) the opposition between the part of nature that is organized into natural units and the part that is presented in continuity; 2) the practice of geometrizing the Earth; 3) the idea of measurement; and 4) the history of a working tool (LIMA, 1998). In other words, he prioritizes the conceptual links between quantities (discrete and continuous) and measurements.

At the same time, attention must be drawn to the fact that, according to the author, the identification of rational numbers with fractional notation is so great that the two are confused: a rational number is a fraction, and a fraction is a rational number. This mechanical identification leads to the algorithmic mechanization of the operative technique with fractions, which we call fractional specialization (LIMA, 1998).

That is why, during 2021, we studied, with the help of mathematics students and elementary school teachers in training, three LTS entitled: 1) Magnitude; 2) The illusion of unity; and 3) Working with “Mother Earth.”

Chart 2: LTS involving conceptual links: quantities and measurements**LTS: The greatness**

Let's answer the following questions as a group:

- 1) Let's return to the counting of a flock of sheep by a shepherd:
 - a) What is the quality of the flock that allows the shepherd to count its size?
 - b) How big is the flock?
- 2) Choose any five classmates. Then:
 - a) Choose any quality they have in common;Write their names in order of magnitude according to that quality.

LTS 2: The illusion of unity

Let's answer the following questions as a group:

- 1) Discuss the following cases, explaining how “the illusion of unity” appears in them:
 - a) Mrs. Lurdes went to the supermarket and bought three one-pound bags of sugar;
 - b) Mrs. Rubenita went to the bakery and bought two liters of milk;
 - c) Juvenal paid for his bus ticket, which cost four reais;
 - d) Robério bought six chairs for his house;
 - e) Whenever I go to the gas station, I fill up my car with the same amount: ten reais.
- 1) Give any example of a situation in which “The illusion of unity” appears.
- 2) Explain what “The illusion of unity” is.
- 3) Analyze all the activities you participated in and answer: do you have “The illusion of unity” or not? Explain your answer.

LTS 3: The Measure – Working with “Mother Earth”

Since the Earth is not divided into lots or portions, that is, it is not naturally organized into units, man had to invent a way to divide it into private family properties. This is what the ancient Egyptians did. The shape they adopted to divide their land was rectangular.

However, an unexpected problem soon arose: every year, the Nile floods cover the divided land, erasing the division marks. In addition, the river flooded many portions, reducing the size of the plots.

- 1) Imagine that you are an employee of the pharaoh in charge of overseeing the distribution of land. After a flood of the Nile, you receive a visit from families who want to discuss the resumption of their land.
 - a) What is the problem presented to you by the families? (Write the problem in the form of a simple and direct question – language of the historical context).
 - b) And for you, the modern student living in the 21st century, what is the problem? (Write the problem in the form of a simple and direct question – language of the modern context).
 - c) Now rewrite the two problems into one, using only mathematical language, that is, using only mathematical words.
 - d) How to count an amount of land?
 - e) How to get rid of the illusion of unity?
 - f) How to enumerate quantities that are not presented in natural units?
 - g) How to “count” the amount of land each family has?
 - h) What is the first procedure we should take when we want to enumerate quantities that are not organized into natural units?

Source: Lanner de Moura et al. (2000, p. 03-04)

Chart 2 shows us that, when studying LTS mathematics, undergraduates and elementary school teachers can reflect on the relationships between the qualitative and quantitative aspects of quantities, noting that these aspects only make sense when we are faced with situations that need to be measured, i.e., when dealing with continuous quantities. However, sometimes we are not clear whether we are dealing with a unit or whether we are deceiving ourselves. This is the case with the question to be analyzed: *Whenever I go to the gas station, I fill up my car with the same amount: ten reais*. Where would the illusion of unity be? Is it in the quantity of *ten*? Is it in the quality of the gasoline? Or is it in the relationship between quality and quantity?

In other words, this question does not admit a single answer. Therefore, the answer cannot simply be memorized. The focus is not on the result of an operation. Thinking about this situation is part of all our lives, as is measuring the land under our responsibility.

Thus, we argue that learning to plan LTS that can mobilize the conceptual links (internal and external) of the concept of fractions should be part of mathematics degree courses and teacher training activities in basic education.

5 Final considerations

The analysis and discussion of the studies we have conducted on the organization of teaching, based on the assumptions of historical-cultural theory, have shown us that the logical-historical movement of the concept, which is configured as a dialectical unity between the logical and the historical, can constitute a didactic perspective for teaching mathematics, as well as a training activity for basic education teachers and mathematics students, because it prioritizes the theoretical thinking of concepts, among them that of fractions.

This means that without an understanding of the conceptual links (internal and external) between concepts present in historiographies that present us with different versions of the logical-historical movement of concepts taught in basic education, teachers in basic education and teacher training students do not have the necessary understanding to defend the teaching of fractions today.

Using only the operationality of fractions as the starting and ending point for organizing the teaching of fractions does not guarantee that children and young people will understand that the field of rational numbers is composed of fractions, ratios, and proportions, which provide us with an understanding of much of the reality in which we live.

Each of these concepts has internal and external connections, and therefore conceptual connections, which are linked, while at the same time differing in content when we analyze, from a logical-historical perspective, the issues that led to them being studied by various socially and culturally distinct groups.

In other words, it is necessary that, during mathematics classes, the form, in this case the fractional representation a/b , does not take the place of the content, which presupposes the *counting of quantities*, which contain magnitudes (discrete and continuous) and measurements present in our reality.

Children and young people of the 21st century have the right to appropriate these logical and historical connections, which stem from different syntheses of social and cultural practices developed by our ancestors and are still valid today in the counting practices they perform practically every day.

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