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On a point kinetic with explicit presence of neutron poisons: reactivity decomposition according to long time scales

Cinética pontual com presença explícita de venenos de neutrons: decomposição da reatividade de acordo com escalas de tempo longas

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Abstract. The present work consists of deducing the point kinetics model considering neutron absorber poisons from the neutron diffusion equation. When considering poisons products, the non-linear system has different orders of magnitude in short and long time scales. The first one represents operational reactor control, whereas the second is due to the change of the chemical composition of the nuclear fuel as a result of burn-up. This represents a first step in a new direction, as it shows how we arrived at the extension of the point kinetics model considering Xenon-135 and Samarium-149 poisons. The non-linear system is solved using the Adomian decomposition method that expands the non-linear terms into an infinite series, obtaining a recursive system, where the recursion initialization is a homogeneous linear equation and the subsequent recursion steps consider the non-linear contributions as source terms constructed from previous recursion steps. The formulation of the source terms of the decomposition method is shown. In order to demonstrate the robustness of the current approach to this type of problem, case studies are presented where nuclear fuel uses reused elements up to third generation, taking a step in a new direction where poisons are already present in the reactor initially.

Keywords. Neutron Diffusion. Neutron Poisons. Adomian Decomposition Method. Short and Long Time Kinetics. Reused Nuclear Fuel.

Resumo. O presente trabalho consiste em deduzir o modelo de cinética pontual considerando venenos absorvedores de nêutrons a partir da equação de difusão de nêutrons. Ao considerar venenos produtos de fissão, o sistema não linear possui diferentes ordens de grandeza em escalas de tempo curtas e longas. A primeira representa o controle operacional do reator, enquanto a segunda é devido à mudança na composição química do combustível nuclear como resultado do *burn up*. É o primeiro passo em uma nova direção, pois mostra como chegamos à extensão do modelo de cinética pontual considerando os venenos Xenônio-135 e Samário-149. O sistema não linear é resolvido usando o método da decomposição de Adomian, o qual expande os termos não lineares em uma série infinita, obtendo um sistema recursivo, em que a inicialização da recursão é uma equação linear homogênea e os passos subsequentes da recursão consideram as contribuições não lineares como termos fonte construídos a partir dos passos anteriores da recursão. A formulação dos termos fonte do método de decomposição é apresentada. Para demonstrar a robustez da abordagem atual para esse tipo de problema, são apresentados estudos de caso em que o combustível nuclear utiliza elementos reutilizados até a terceira geração, dando um passo em uma nova direção, uma vez que os venenos já estão presentes no reator inicialmente.

Palavras-chave. Difusão de nêutrons. Venenos de nêutrons. Método da decomposição de Adomian. Cinética de curta e longa escala de tempo. Combustível nuclear reutilizado.

Mathematics Subject Classification (MSC): 82D75

1 Introduction

Generally, in nuclear reactor control, neutron point kinetics models are applicable. The traditional point kinetics model is suitable for short-term intervals, while burn-up, which refers to the effects of poisons from fission products, is convenient for long-term intervals. Short-term intervals are generally up to 10^1 seconds, while long-term intervals are of the order of 10^4 seconds.

Poisons produced from neutron fission fragments in nuclear reactors are an important aspect of their operation. These fission fragments generate fission products which, in turn, are parasitic neutron absorbers, resulting in long-term heat sources [6, 12]. The most important poisons for reactor operation are Xenon-135 and Samarium-149.

Nuclear fuel can be reused in subsequent cycles. In this case, older elements, usually up to the third generation, are added. However, it is important to remember that some poisons, such as stable Samarium-149, are always present in the fuel. Therefore, it is essential to know the chemical composition of old fuel before its reuse.

In the literature some works involving the effects of poisons in the nuclear reactor, such as [8, 9, 17]. We can mention that of [8] that works with the Neutron Point Kinetic equations taking into account the effects of Xenon-135 and Samarium-149 poisons, in which is developed an analytical solution through a decomposition method and where the case studies are presented from the neutron poisons starting nulls in the reactor, i.e, using as initial conditions new fuel elements. In the spirit of reference [17] are considered in the model the effects of fission product poisons in the final behavior of the neutron density. For this, the equations of the Neutron Point Kinetics are solved considering the temperature effects, by the Rosenbrock's method.

We point out that simulations with the coupling of the effects of the main neutron absorbers poisons to the point kinetic model are not very common in the literature. Only a small number of studies use simulations to test the accuracy of methodologies and the significance of their findings. A recent study by [10], however, tackled the NPKEs by incorporating the impacts of not only the primary neutron poisons but also some transuranics, utilizing the same approach as [8]. Their findings were in line with what was anticipated.

In this work, the equations of point kinetics considering poisons are deduce from the neutron diffusion equation. The point kinetics model consists of a system of coupled nonlinear equations for neutron density, delayed neutron precursors, and decay chains of poisons from fission products. The system are solved using the decomposition method [1], where the polynomials used to treat non-linearity as a source term are shown. Results are generated for reused elements within the reactor, where poisons are already present from the beginning of operation. The initial conditions used were sufficient poisons in a reactor that operated for time $t = 10^3$ hours.

The results of the investigation carried out by the same author in their preceding thesis [9] are reported in this study. In this new analysis, specific aspects related to the previous findings were addressed, and new discoveries were made based on those earlier results. This continuity of research allowed for a deeper understanding and improved comprehension of the subject matter. Based on the conclusions drawn, it is hoped that this study will contribute to the advancement of knowledge in the field of study and to the development of further research.

2 The proposed point kinetics model from the diffusion equation

2.1 Additional contributions by neutron poisons

The important factors of the two decay sequences are reviewed below, which are needed to determine the poison's impact on the reactivity. The Xenon-135 production, which is driven by the Tellurium-135 decay chain, is simplified by only taking into account the balances of Iodine-135 and Xenon-135 concentrations. This is because the decay of Tellurium-135 is implicitly accounted for in the production of Iodine, which is a reasonable simplification given its significantly shorter half-life compared to the other two nuclides [7, 15]. Thus, we consider a bulk Iodine yield γ_I, the small Xenon yield γ_{Xe}, an effective neutron speed $\bar{v}$ according to one energy group and the physical parameters, the decay constants $\lambda_{I,Xe}$, the macroscopic fission cross section Σ_f, the microscopic absorption cross section σ_{Xe}, that resemble the relations between the concentrations $C_{I,Xe}$ and the neutron density n.

$$\begin{aligned} \frac{dC_I(t)}{dt} &= \gamma_I \Sigma_f \bar{v} n(t) - \lambda_I C_I(t), \\ \frac{dC_{Xe}(t)}{dt} &= \gamma_{Xe} \Sigma_f \bar{v} n(t) + \lambda_I C_I(t) - \lambda_{Xe} C_{Xe}(t) - \sigma_{Xe} C_{Xe}(t) \bar{v} n(t). \end{aligned} \tag{1}$$

The decay chain of Neodymium-149 can also be simplified similarly, assuming bulk production of Promethium-149 (γ_{Pm}) due to the much shorter half-life of Neodymium. The concentration balance of Promethium and Samarium can be described by a similar equation system as the one mentioned before, with the exception of the decay term of Samarium being absent since it is a stable nuclide.

$$\begin{aligned} \frac{dC_{Pm}(t)}{dt} &= \gamma_{Pm} \Sigma_f \bar{v} n(t) - \lambda_{Pm} C_{Pm}(t), \\ \frac{dC_{Sm}(t)}{dt} &= \lambda_{Pm} C_{Pm}(t) - \sigma_{Sm} C_{Sm}(t) \bar{v} n(t). \end{aligned}$$

In the given equation, $C_{Pm,Sm}$ represents the concentrations of Promethium-149 and Samarium-149, respectively, while the other symbols have the same meaning as in the previously mentioned equations (1).

2.2 Point kinetics considering poison from the diffusion equation

To incorporate the impact of neutron poisons in a coherent manner, it is preferable to begin with the time dependent neutron diffusion equation. Additionally, we presume that the system was in equilibrium state until a certain moment $t = t_0$. For the subsequent period of transition, the neutron diffusion is modeled using the familiar equation, along with the concentration of one precursor for the sake of simplicity.

$$\begin{aligned}
&\frac{1}{v(E)}\frac{\partial}{\partial t}\phi(\mathbf{r},E,t) - \boldsymbol{\nabla}\cdot(D(\mathbf{r},E,t)\boldsymbol{\nabla}\phi(\mathbf{r},E,t)) + \Sigma_t(\mathbf{r},E,t)\phi(\mathbf{r},E,t) \\
&\quad = \int_0^\infty \left\{\chi(E)\nu(E')\Sigma_f(\mathbf{r},E',t) + \Sigma_s(\mathbf{r},E'\to E,t)\right\}\phi(\mathbf{r},E',t)\,dE' \\
&\quad +\lambda\chi_p(E)C(\mathbf{r},t) - \int_0^\infty \beta\chi_d(E)\nu(E')\Sigma_f(\mathbf{r},E',t)\phi(\mathbf{r},E',t)\,dE', \\
&\quad \frac{\partial}{\partial t}C(\mathbf{r},t) = \beta\int_0^\infty \nu(E')\Sigma_f(\mathbf{r},E',t)\phi(\mathbf{r},E',t)\,dE' - \lambda C(\mathbf{r},t).
\end{aligned}$$

Here,

$$\chi(E) = (1-\beta)\chi_p(E) + \beta\chi_d(E)$$

and

$$\Sigma_x(\mathbf{r},E,t) = \sum_{i=1}^{I} C_i(\mathbf{r},t)\sigma_x^i\ ,$$

where I is the total number of nuclides, $\chi_p(E)$ and $\chi_d(E)$ are respectively fission spectra of prompt and delayed neutrons.

The process of simplifying the diffusion model into point kinetics involves a traditional approach that utilizes the adjoint flux to solve the stationary problem and results in

$$\begin{aligned}
&\frac{d}{dt}\left(\int_V\int_0^\infty \phi^*(\mathbf{r},E,t_0)\frac{1}{v(E)}\phi(\mathbf{r},E,t)\,dE\,d^3r\right) = \int_V\int_0^\infty \phi^*(\mathbf{r},E,t_0)\times \\
&\quad\times\left(\int_0^\infty \left\{\chi(E)\nu(E')\Sigma_f(\mathbf{r},E',t) + \delta\Sigma_s(\mathbf{r},E'\to E,t)\right\}\phi(\mathbf{r},E',t)\,dE'\right. \\
&\quad\left.+\boldsymbol{\nabla}\cdot(\delta D(\mathbf{r},E,t)\boldsymbol{\nabla}\phi(\mathbf{r},E,t)) - \delta\Sigma_t(\mathbf{r},E,t)\phi(\mathbf{r},E,t)\right)\,dE\,d^3r \\
&\quad -\int_V\int_0^\infty \phi^*(\mathbf{r},E,t_0)\left(\int_0^\infty \beta\chi_d(E)\nu(E')\Sigma_f(\mathbf{r},E',t)\phi(\mathbf{r},E',t)\,dE'\right)\,dE\,d^3r \\
&\quad +\lambda\int_V\int_0^\infty \phi^*(\mathbf{r},E,t_0)\chi_d(E)C(\mathbf{r},t)\,dE\,d^3r\ ,
\end{aligned} \tag{2}$$

where

$$\delta D(\mathbf{r}, E, t) \equiv D(\mathbf{r}, E, t) - D(\mathbf{r}, E, t_0),$$

and

$$\delta\Sigma_x(\mathbf{r}, E, t) \equiv \Sigma_x(\mathbf{r}, E, t) - \Sigma_x(\mathbf{r}, E, t_0) = \sum_{i=1}^{I} \{C_i(\mathbf{r}, t) - C_i(\mathbf{r}, t_0)\}\, \sigma_x^i(E)\ .$$

To emphasize the impact of neutron poisons on kinetics, we simplify equation (2) by disregarding the terms δD, $\delta\Sigma_s$, and $\delta\Sigma_f$. Additionally, we assume that the precursor concentration equation does not involve any spatial differential operator. Furthermore, the initial condition of the precursor concentration follows the same spatial pattern as the scalar flux, which leads to an identical shape of the spatial distributions for both the scalar flux and the precursor concentration, except for a time dependent scale factor ([18]).

$$\begin{aligned} \phi(\mathbf{r}, E, t) &\cong f(\mathbf{r}, E) n(t), \\ C(\mathbf{r}, E, t) &\cong f(\mathbf{r}, E) C(t). \end{aligned}$$

After integrating out energy and spatial degrees of freedom renders equation (2)

$$\frac{d}{dt} n(t) = \frac{\rho(t) - \bar{\beta}(t)}{\Lambda} n(t) + \lambda C(t)\ , \tag{3}$$

where with

$$\begin{aligned} F \equiv & \int_V \left\{ \int_0^\infty \chi(E) \phi^*(\mathbf{r}, E, t_0)\ dE \right\} \times \\ & \times \left\{ \int_0^\infty \nu(E') \Sigma_f(\mathbf{r}, E', t_0) f(\mathbf{r}, E')\ dE' \right\}\ d^3r\ , \end{aligned}$$

the reactivity, the delayed precursor neutron fraction, neutron generation time and precursor concentration are [16]

$$\begin{aligned} \rho(t) \equiv \frac{1}{F} \int_V \int_0^\infty \phi^*(\mathbf{r}, E, t_0) \left(\int_0^\infty \{ \chi(E) \nu(E') \delta\Sigma_f(\mathbf{r}, E', t) \right. \\ \left. + \delta\Sigma_s(\mathbf{r}, E' \to E, t) \}\, f(\mathbf{r}, E')\ dE' + \boldsymbol{\nabla} \left(\delta D(\mathbf{r}, E, t) \boldsymbol{\nabla} f(\mathbf{r}, E) \right) \right. \\ \left. - \delta\Sigma_t(\mathbf{r}, E, t) f(\mathbf{r}, E) \right)\ dE\, d^3r\ , \end{aligned} \tag{4a}$$

$$\begin{aligned} \bar{\beta}(t) \equiv & \beta \frac{1}{F} \int_V \left\{ \int_0^\infty \chi_d(E) \phi^*(\mathbf{r}, E, t_0)\ dE \right\} \times \\ & \times \left\{ \int_0^\infty \nu(E') \Sigma_f(\mathbf{r}, E', t) f(\mathbf{r}, E')\ dE' \right\}\ d^3r \end{aligned} \tag{4b}$$

$$\Lambda \equiv \frac{1}{F}\int_V \int_0^\infty \phi^*(\mathbf{r}, E, t_0)\frac{1}{v(E)} f(\mathbf{r}, E)\; dE\, d^3r = \frac{1}{F\bar{v}}\;, \tag{4c}$$

$$C(t) \equiv \frac{1}{\Lambda F}\int_V \int_0^\infty \phi^*(\mathbf{r}, E, t_0)\chi_d(E) C(\mathbf{r}, t)\; dE\, d^3r\;. \tag{4d}$$

To derive the equation for neutron density that takes into account the impact of fission products, particularly the substances that absorb neutrons and affect the reactor's reactivity, we simplify the algebraic expressions used in (4a):

- Changes in the fission cross section are not taken into account as the duration of the analysis is relatively short compared to the time required for fuel burning to cause significant alterations in the cross section;
- The alterations in the scattering cross section are disregarded because the modifications in the reactor core's chemical composition are minimal;
- The changes in the diffusion parameter, which is inversely related to the transport cross section, are not considered and assumed to remain constant.

Consequently, the changes in the absorption cross section caused by the dominant neutron poisons, such as Xenon and Samarium, are taken into account.

$$\Sigma_a(\mathbf{r}, E, t_0) = \Sigma_a^{Xe}(\mathbf{r}, E, t_0) + \Sigma_a^{Sm}(\mathbf{r}, E, t_0) = \sigma_{Xe} C_{Xe} + \sigma_{Sm} C_{Sm}\;.$$

We are now breaking down the terms in equation (4a) into two parts: the first is a reactivity contribution resulting from control operations performed on the reactor, which is represented by time-dependent functions that are typically of short duration (around 10^1 seconds). The second part is the contribution from the creation and removal of the primary neutron poisons (Xenon-135 and Samarium-149). This latter contribution to reactivity is influenced by changes in the macroscopic absorption cross sections of these two isotopes, which are described in the references [8], [17].

$$\rho(t) = \rho_s(t) + \rho_l(t)\;.$$

Here ρ_s refers to the short time scale (control rod) and ρ_l to the long time scale (poisons). Upon substituting the reactivity in equation (3) yields,

$$\frac{d}{dt}n(t) = \frac{\rho_s(t) - \bar{\beta}(t)}{\Lambda} + \lambda C(t) + \frac{\displaystyle\int_V \int_0^\infty \phi^*(\mathbf{r}, E, t_0)\left(-\delta\Sigma_a^{Xe}(\mathbf{r}, E, t_0) - \delta\Sigma_a^{Sm}(\mathbf{r}, E, t_0)\right)\; dE\, d^3r}{\displaystyle\int_V \int_0^\infty \phi^*(\mathbf{r}, E, t_0)\frac{1}{v(E)} f(\mathbf{r}, E)\; dE\, d^3r} n(t)\;.$$

Note, when examining reactivity on a long timescale, the neutron generation time Λ becomes insignificant and has no impact on the observed changes. Rather, the production and elimination of Xenon-135 and Samarium-149 are the primary drivers of changes. Once the two timescales in the reactivity have been separated and the integrals that make up $\rho_l(t)$ have been identified, the resulting non-linear model equation for neutron density, which explicitly accounts for the presence of the neutron poison products, consists of the usual coupled neutron density and delayed neutron precursor equations, as well as the equations governing the decay chains that regulate the concentration of neutron poisons. Thus, we have the following non-linear system

$$\begin{aligned}
\frac{d}{dt}n(t) &= \frac{\rho_s(t)-\bar{\beta}}{\Lambda}n(t) + \lambda C(t) - \bar{v}\left(\sigma_{Xe}C_{Xe}(t) + \sigma_{Sm}C_{Sm}(t)\right)n(t),\\
\frac{d}{dt}C(t) &= \frac{\bar{\beta}}{\Lambda}n(t) - \lambda C(t),\\
\frac{d}{dt}C_I(t) &= \bar{v}\gamma_I\Sigma_f n(t) - \lambda_I C_I(t),\\
\frac{d}{dt}C_{Xe}(t) &= \bar{v}\gamma_{Xe}\Sigma_f n(t) + \lambda_I C_I(t) - \lambda_{Xe}C_{Xe}(t) - \bar{v}\sigma_{Xe}C_{Xe}(t)n(t),\\
\frac{d}{dt}C_{Pm}(t) &= \bar{v}\gamma_{Pm}\Sigma_f n(t) - \lambda_{Pm}C_{Pm}(t),\\
\frac{d}{dt}C_{Sm}(t) &= \lambda_{Pm}C_{Pm}(t) - \sigma_{Sm}C_{Sm}(t)\bar{v}n(t),
\end{aligned} \tag{5}$$

where $\bar{v}$ given in $[cm/h]$ is the velocity of the neutron, Σ_f given in $[cm^{-1}]$ is the macroscopic cross-section of fission, λ given in $[h^{-1}]$ is the delayed neutron precursor decay constant, $C(t)$ given in $[cm^{-3}]$ is the concentration of delayed neutron precursors at time t, σ_{Xe} given in $[cm^2]$ is the microscopic cross-section of absorption of the element Xenon-135, $C_{Xe}(t)$ given in $[cm^{-3}]$ is the concentration of Xenon-135 in time t, σ_{Sm} given in $[cm^2]$ is the microscopic cross-section of absorption of Samarium-149, $C_{Sm}(t)$ given in is the concentration of Samarium-149 in time t, γ_I is the fission yield of the nuclide Iodine-135, λ_I given in $[h^{-1}]$ is the radioactive decay constant of Iodine-135, $C_I(t)$ given in $[cm^{-3}]$ is the concentration of Iodine-135 at the instant t, γ_{Xe} is the fission yield of the nuclide Xenon-135, λ_{Xe} given in $[h^{-1}]$ is the radioactive decay constant of Xenon-135, γ_{Pm} is the fission yield of the nuclide Promethium-149, $C_{Pm}(t)$ given in $[cm^{-3}]$ is the concentration of Promethium-149 in time t and λ_{Pm} given in $[h^{-1}]$ is the radioactive decay constant of Promethium-149.

3 Applied methodology

For convenience, the non-linear equation system (5) is cast in matrix form, where the solution is

$$\mathbf{Y} = (n(t), C(t), C_I(t), C_{Xe}(t), C_{Pm}(t), C_{Sm}(t))^T = \sum_{i=0}^{\infty} \mathbf{Y}_i \,,$$

which, we expand into an infinite series. The methodology used consists of two parts: the first is solve the linear system with non-homogeneous initial conditions, while the second part one finds approximations to the solution of the system with source term using conditions homogeneous initials, treating the source term as a non-linearity treated through the Adomian decomposition method. present in references [8, 13, 11, 5]. The first part of recursive scheme has a solution given by [4]

$$\mathbf{Y}_0(t) = \exp(\mathbf{A}t)\mathbf{Y}_I \ .$$

All subsequent recursion steps are then set-up by a linear differential equation system, where the non-linearity is present as a source term $\mathbf{F}_k$, which is composed from the solutions of the previous recursion steps, with solution:

$$\mathbf{Y}_k(t) = \int_0^t \exp(\mathbf{A}(t-\tau))\mathbf{F}_k(\mathbf{Y}_0, \mathbf{Y}_1, ..., \mathbf{Y}_{k-1})\, d\tau \ ,$$

where the Adomian polynomials are [2]

$$\mathbf{F}_k(\mathbf{Y}_0, \mathbf{Y}_1, ..., \mathbf{Y}_{k-1}) = -\left(\mathbf{N}_{k-1}\sum_{j=0}^{k-1}\mathbf{Y}_j + \left(\sum_{j=0}^{k-2}\mathbf{N}_j\right)\mathbf{Y}_{k-1}\right) \ . \tag{6}$$

Once $\mathbf{Y}_k$ is convergent one may truncate the series at a finite k such that the solution is within a prescribed precision. This way, $\mathbf{F}_k(\mathbf{Y}_0, \mathbf{Y}_1, ..., \mathbf{Y}_{k-1})$ can be cast in vector form

$$\mathbf{F}_k(\mathbf{Y}_0, \mathbf{Y}_1, ..., \mathbf{Y}_{k-1}) = \begin{pmatrix} -\sigma_{Xe}\bar{v}A_{k-1}^{Xe} - \sigma_{Sm}\bar{v}A_{k-1}^{Sm} \\ 0 \\ 0 \\ -\sigma_{Xe}\bar{v}A_{k-1}^{Xe} \\ 0 \\ -\sigma_{Sm}\bar{v}A_{k-1}^{Sm} \end{pmatrix} ,$$

where A_{k-1}^{Xe} and A_{k-1}^{Sm} are the Adomian polynomials of Xenon and Samarium for recursion step $k-1$. Consequently, the Adomian polynomials for the non-linearities (equation (6)) are of the form [3], [8]

$$\begin{aligned}
A_0^i &= N(C_0^i, n_0) = C_0^i n_0 \ , \\
A_1^i &= \frac{d}{d\omega}((C_i^0 + C_i^1\omega)(n_0 + n_1\omega))|_{\omega=0} = C_i^0 n_1 + C_i^1 n0 \ , \\
A_2^i &= \frac{1}{2}\frac{d}{d\omega}((C_i^1 + 2C_i^2\omega)(n_0 + n_1\omega) + (C_i^0 + C_i^1\omega)(n_1 + 2n_2\omega))|_{\omega=0} \\
&= C_i^0 n_2 + C_i^1 n_1 + C_i^0 n_2 \ , \\
A_3^i &= \frac{1}{3!}\frac{d}{d\omega}((C_i^2 + 3C_i^3\omega)(n_0 + n_1\omega) + (C_i^1 + 2C_i^2\omega)(n_1 + 2n_2\omega) + \\
&\quad +(C_i^0 + C_i^1\omega)(n_2 + 3n_3\omega))|_{\omega=0} = C_i^3 n_0 + C_i^2 n_1 + C_i^1 n_2 + C_i^0 n_3 \ , \\
&\vdots \\
A_j^i &= \sum_{k=0}^{j} C_i^{j-k} n_k \ .
\end{aligned}$$

4 Numerical simulations considering burned-up fuel

We present results for refill condition operation, where neutron poisons are present. To this end we use as initial condition results from the previous case for $t = 10^3 h$.
The initial condition are

$$\begin{aligned}
n(0) &= 10^8 \ [cm^{-3}]; \\
C(0) &= \frac{\beta - \rho(t)}{\lambda\Lambda} n(0) \ [cm^{-3}]; \\
C_I(0) &= \frac{\gamma_I \Sigma_f n(0)}{\lambda_I} \ [cm^{-3}]; \\
C_{Xe}(0) &= \frac{\gamma_{Xe} + \gamma_I}{\lambda_{Xe} + \sigma_{Xe} n(0)} \ [cm^{-3}]; \\
C_{Pm}(x,0) &= \frac{\gamma_{Pm}\Sigma_f + n(0)}{\lambda_{Pm}} \ [cm^{-3}] \quad e, \\
C_{Sm}(x,0) &= \frac{\gamma_{Sm}\Sigma_f}{\sigma_{Sm}} \ [cm^{-3}].
\end{aligned}$$

Two cases for different short term reactivity parametrisations ρ_s are presented, a case

with oscillating positive reactivity and a case with a step wise reactivity function.

$$\begin{aligned} \rho_s(t) &= \rho_1 \sin(at) \\ \rho_s(t) &= \begin{cases} \rho_1 < 0, & mod(t,12) = 0,1,2,3 \\ \rho_2 = 0, & mod(t,12) = 4,5,6,7 \\ \rho_3 > 0, & mod(t,12) = 8,9,10,11 \end{cases} \end{aligned}$$

where ρ_1, ρ_2, ρ_3 and a are constants and mod is the modulus operation, i.e. the remainder after division of the first by the second number. We show the solutions for $n(t)$, $C(t)$, $C_{Xe}(t)$ and $C_{Sm}(t)$.

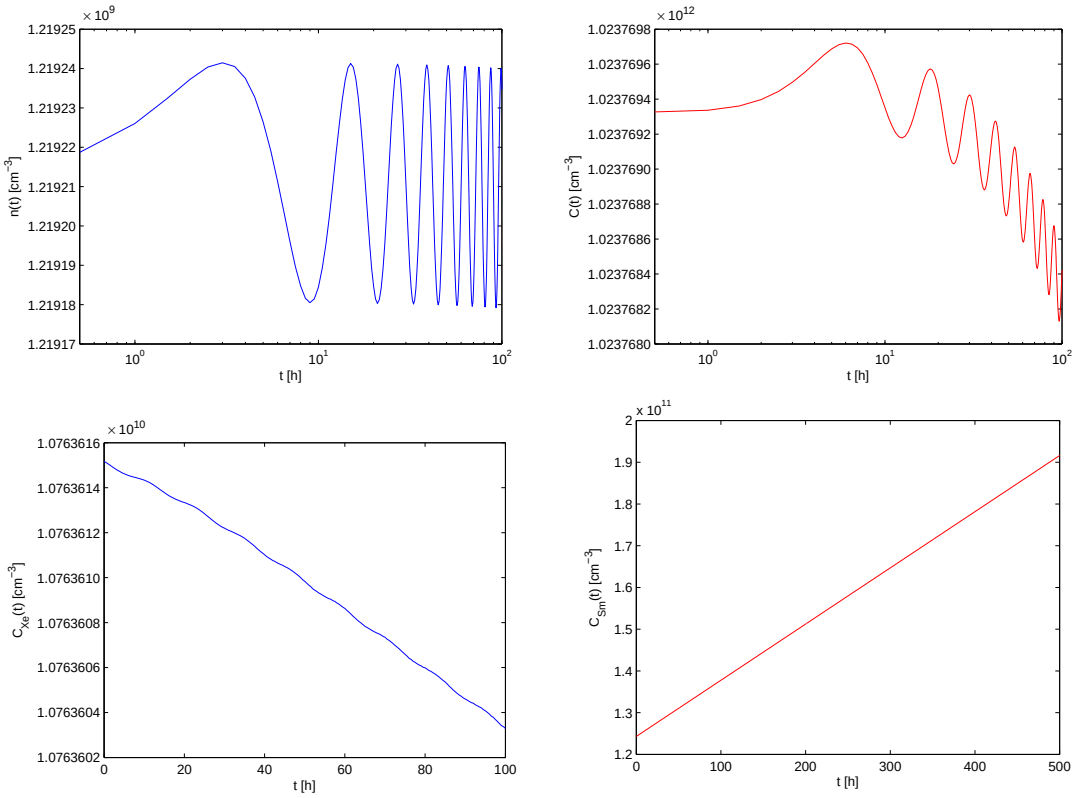

Figure 1: Time dependence of neutron density $n(t)$ (upper left), the precursor concentration $C(t)$ (upper right), the short term concentration of Xenon $C_{Xe}(t)$ (lower left) and Samarium $C_{Sm}(t)$ (lower right) with short therm reactivity $\rho_s = 0.0001 \sin\left(\frac{2\pi}{12}t\right)$.

In the case with oscillatory reactivity (Fig. 1), the neutron density as well as the precursor concentration follow also in this case the imposed time signature comparable with the new fuel case in [8]. Differently, than the new fuel case the Xenon concentration decreases with slight ondulations after passing a maximum at $\sim 10^1 h$ until obtaining an asymptotic value, whereas the Samarium concentration increases linearly.

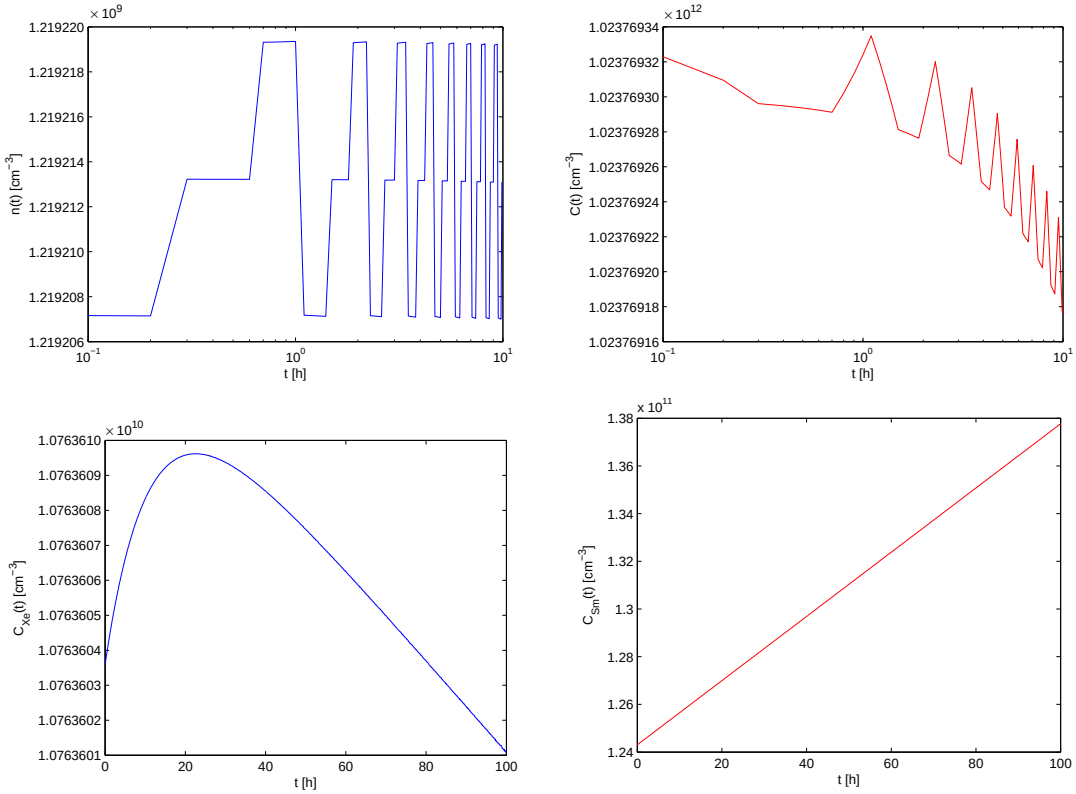

Figure 2: Time dependence of neutron density $n(t)$ (upper left), the precursor concentration $C(t)$ (upper right), the short term concentration of Xenon $C_{Xe}(t)$ (lower left) and Samarium $C_{Sm}(t)$ (lower right) with short therm reactivity given by the step wise function.

In the case with step wise reactivity and fuel elements with burn-up (Fig. 2), neutron density and precursor concentrations show the characteristic time signature whereas Xenon passes through a local maximum at $t \approx 25h$ and decreasing with an almost vanishing slope. The Samarium concentration shows again a linearly rising tendency.

5 Conclusions

In the present work we deduce a model for nuclear reactor point kinetics from the neutron diffusion equation. It should be noted that the usual point kinetics model is linear, but when accounting for neutron poisons, the model becomes nonlinear as their presence is dependent on the neutron population. To solve this coupled system of equations, we utilize an analytical representation based on a decomposition method similar to that found in previous literature [2, 8, 11, 13, 14]. The polynomials used in the source term that depend on previous steps were shown.

In the cases examined in this study, a limited number of 9 recursion steps were nec-

essary to achieve a satisfactory level of precision, as noted in [13]. This solution allows for the calculation of various transient behaviors of nuclear reactor point kinetics, for new reactor fuel as well as fuel compositions containing fuel elements with burn-up.

The study presented results for cases in which fuel elements were reused within the reactor. One to represent scenarios with oscillatory behavior and the other to model stepwise reactivity. The results demonstrated that the obtained solutions were consistent with expected physical behavior. It is expected that this study will contribute to the development of knowledge in this specific area and pave the way for future research.

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