

# PERFORMANCE OF CAD MODEL RECOVERING METHOD BASED ON CMA MEASUREMENT AND NURBS MODELING APPLIED TO SMALL FREE FORM SURFACES

DESEMPENHO DE MÉTODO DE RECUPERAÇÃO DE MODELOS CAD BASEADOS NA MEDIÇÃO COM CMA E MODELAGEM NURBS APLICADO À SUPERFÍCIES DE FORMA LIVRE DE DIMENSÕES REDUZIDAS

Antonio Piratelli-Filho\* and José Maurício S. T. da Motta

Universidade de Brasília, Faculdade de Tecnologia, Departamento de Engenharia Mecânica  
70910-900, Brasília, DF, Brasil

\*E-mail: pirateli@unb.br

## ABSTRACT

This paper presents the description and the performance evaluation of an alternative approach to reconstruct the Computer-aided Design (CAD) model of small free form surfaces. This approach required the measurement of a small size sample using a Coordinate Measuring Arm (CMA) with contact probe and Non-Uniform Rational B-Splines (NURBS) fitting techniques. A measurement strategy was developed on a tooth prosthesis sample and the measurement was carried out with a needle type stylus along predefined paths over the tooth surface. The obtained data points were used to reconstruct the tooth surface using NURBS curves and surfaces modeling techniques. The analysis of the CAD model errors was implemented by the difference between data points and the respective CAD model point and it was observed that the total amount of determined points resulted in a suitable CAD model of the small free form surface. The use of reduced cost equipment in relation to the digitization ones as laser scanning proved suitable to perform the reverse engineering of small parts having free form surfaces.

**Keywords:** reverse engineering, free form surfaces, coordinate measurement, NURBS modelling

## RESUMO

Este trabalho apresenta a descrição e a verificação do desempenho de uma abordagem alternativa para reconstruir o modelo de Projeto Auxiliado por Computador (CAD) de superfícies de forma livre. Esta abordagem foi aplicada através da medição de uma amostra de dimensões reduzidas usando uma Máquina de Medir por Coordenadas modelo Braço de Medição (CMA) com sonda de contato e técnicas de modelagem usando B-Splines não-uniformes racionais (NURBS). Uma estratégia de medição foi desenvolvida com uma amostra de prótese dentária e a medição foi conduzida segundo direções pré-definidas sobre a superfície. Os pontos determinados foram usados para reconstruir a superfície do dente ajustando curvas e superfícies NURBS. A análise dos erros do modelo CAD foi feita calculando a diferença entre os pontos determinados pela medição e os respectivos pontos no modelo e foi observado que o total de pontos determinados foi suficiente para obter um modelo CAD adequado da pequena amostra de superfície de forma livre. O emprego de equipamentos de custo reduzido em relação aos de digitalização, como o escaner laser, mostrou-se adequado para executar operações de engenharia reversa de peças de dimensões reduzidas e com superfícies de forma livre.

**Palavras-chave:** engenharia reversa, superfícies de forma livre, medição por coordenadas, modelagem NURBS

## 1 – INTRODUCTION

Many cases in restorative dental practice may require the substitution of the original tooth by prosthesis fabricated with metal alloys or ceramic materials. Processes involving Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM) technologies to design and produce dental restorations were developed along the past two decades and some commercial systems reported in the literature are named CEREC, DURET, Minnesota, DCS, Procera and CICERO, among others. These different CAD/CAM systems are used to produce crowns and dentures and they differ in production speed, accuracy, used materials, esthetics and implementation [1].

The implementation of these CAD/CAM systems is generally carried out by reverse engineering techniques. This process begins by measuring the part to be manufactured, followed by the development of a CAD model, the generation of a CAM program and finally the manufacturing of a new piece to be submitted to a measurement process [2].

The measurement of sample prosthesis is generally based on digitization methods. Optical scanning instruments are commonly used to obtain the information about the tooth to be manufactured. A charged coupled device (CCD) camera is used in the CICERO system to the digitization procedure of the cast model and an accuracy of 0.01 mm was reported [1]. The use of a coordinate measuring machine (CMM) or a coordinate measuring arm

to capture data was suggested and the drawbacks related to the use of contact probe were reported as the low speed in data acquisition and the accuracy and resolution. Besides, the probe tip radius is a restriction imposed in Measurement as it must be smaller than the curvature of the real surface [3]. However, the use of CMM can reduce the cost of the measurement task once the accuracy is suitable to the manufacturing method that is being suggested.

The surface of a dental prosthesis may be classified as a free form surface as there is no simple mathematical equation that describes its geometry [4]. These surfaces have no regular forms like circles, planes and cylinders and its representation may be accomplished by fitting polynomial equations. Besides, its small dimension is another characteristic that makes difficult the measurement process involved in recovering the tooth form to apply CAD/CAM fabrication by reverse engineering techniques. Some examples of small free form parts are the blades of a helicopter turbine runner and the aspherical lenses used in optical devices.

Feng and Pandey [3] carried out an experimental study using design of experiments and found that the dimension of the measured part on a CMM significantly affects the digitization uncertainty. The shape or profile tolerance of this free form surface is small and the measurement instrument can be selected considering the accuracy, data acquisition process and CAD processing characteristics.

The measurement of small free form surfaces may be performed by Coordinate Measuring Machines (CMM) using contact or non-contact probes. The use of non-contact probes on a CMM was reported by Zou et al [5] to control the profile quality of a molar tooth surface. Digitization procedures were employed to carry out this task and laser scanning was reported as a suitable technique that generates large samples of points (cloud of points) of the surface. The use of contact probes restrict the amount of data generated and increase the time spent in measurement; however they are more accurate and present good repeatability [2, 6].

CAD models are created using the measured data and software based on non-uniform rational B-splines (NURBS), and are mostly used to generate free-form surfaces starting from point coordinates [6, 7, 8]. Two different approaches are referred in literature to address the development of CAD models: the curve-net-based method and the polygon-based method. The curve-net-based method requires the determination of some organized groups of points (points of curves determined on a CMM) that are fitted to generate the CAD surface. The Polygon-based method involves the determination of large samples of points (cloud of points obtained by digitization procedures) that are manipulated and adjusted to generate surfaces. Both approaches involve the development of a NURBS surface. According to Zhongwei [9], the polygon-based methods are fast and efficient but the curve-net-based approach results in a more accurate CAD model that can still be modified after conclusion.

Várady et al. [2] presented a discussion about the sources of errors in CAD modeling processes, from the

calibration of CMM probe to CMM accuracy, surface accessibility, part fixture and surface finish, noise, measurement strategy, CAD surface adjustment, among others. Feng and Pandey [3] presented a study to investigate the impact of CMM parameters on the digitization uncertainty using design of experiments. Huang, Gu and Zernicke [10] presented an approach to compare two free-form surfaces based on a reference surface model. A direct verification of the CAD model was carried out comparing the measured points with the adjusted model and displaying graphically the errors or deviations over the created surface.

A review of free form surface inspection techniques was recently published and in there are discussed contact and non-contact measurement methods and geometric processing steps [11]. According to Li and Gu [11], future research on free form surfaces will focus on the increase of accuracy and in the cost reduction that plays an important role.

This work investigates the measurement of small free form surfaces using a CMA with a contact probe. The free form surface of a plaster made tooth prosthesis was used to the development of a measurement strategy. The prosthesis was measured on a CMM type articulated arm (Coordinate Measuring Arm – CMA) using a needle type stylus and a rigid probe. The implemented approach involved the measurement of small amounts of points along selected paths on the surface. The samples of determined points were exported in IGES (Initial Graphics Exchange Specification) format file and used to reconstruct the tooth model using commercial CAD software. Analysis of the CAD model adjustment errors was performed to compare the data points with the respective CAD point and to performance characterization.

## 2 – MEASUREMENT OF SMALL FREE FORM SURFACES AND CAD MODELING

The experimental work was carried out on a Coordinate Measuring Machine, type articulated arm (Coordinate Measuring Arm - CMA), manufactured by Romer, model ARM 100. This CMA has three articulated tubular segments manufactured in carbon fiber reinforced polymer (CFRP) and encoders positioned in each joint to determine the position of the probe tip.

This CMA had a volumetric reach of 2.5 m and the performance was estimated by manufacturer calibration report, giving an uncertainty of 0.087 mm when measuring length and an accuracy of 0.07 mm to point coordinate. These values indicate that any given point captured by the machine has a maximum error of 0.07 mm in space. A rigid probe with a needle type stylus was connected to the arm extremity and the software G-Pad was used to acquire the point coordinates on the part surface. Figure 1 shows the experimental setup used to measure a part having medium dimension (diameter of 300 mm).

The measurement was performed on a plaster made dental prosthesis with dimensions of approximately 10 x 7 x 8 mm, fixed on the surface of a nylon plate and clamped on a cast iron measuring table, as shown in figure 2. The

measurement strategy was conceived as to allow the CMA stylus to touch the prosthesis surface always at an angle of 90°.

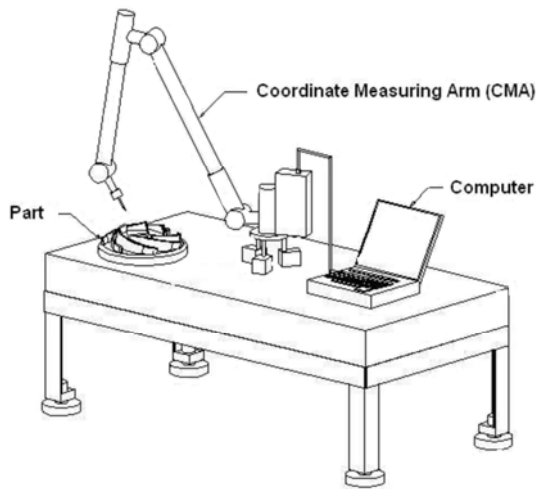


Figure 1 – Experimental setup used to measure a part with CMA

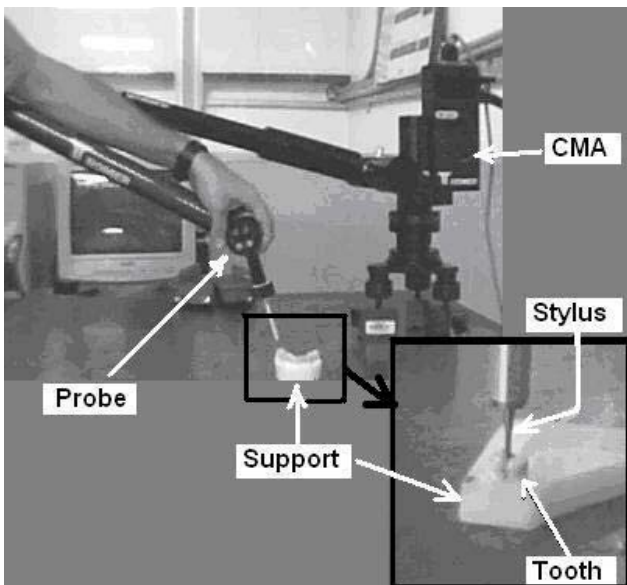


Figure 2 – Experimental setup used to measure the dental prosthesis

The point-to-point measurement approach was carried out designing a small grid of lines on the surface of the prosthesis. The grid was designed on the surface having parallel lines. It was established that regions with steep curvature required a finely divided grid to increase the accuracy of the developed model. Using the CMA G-Pad software, groups of points were determined according to a given direction in order to define the curves after adjustment. There were measured 12 groups of points with 23 points and 3 groups of points with 22 points, resulting in 342 measured points.

The increase in accuracy demanded a precise control of the probe stylus in order to reach contact during all the measurement process. The determined points were stored in an IGES format file.

The curve-net-based method was applied to fit data to curves and after to surface. Thus, each group of points was fitted to a NURBS curve according to the measurement sequence and then a surface was fitted to a NURBS surface. In this approach, Rhinoceros software was used to generate the CAD models.

NURBS may be considered as vector-valued piecewise polynomial functions and the equations associated to the curves and surfaces are presented by expressions 1 and 2 respectively [12].

$$C(u) = \frac{\sum_{i=0}^n P_i \cdot w_i \cdot N_{ik}(u)}{\sum_{i=0}^n w_i \cdot N_{ik}(u)} \quad (1)$$

$$S(u, v) = \frac{\sum_{j=0}^m \sum_{i=0}^n P_{ij} \cdot w_{ij} \cdot N_{ik}(u) \cdot N_{jl}(v)}{\sum_{j=0}^m \sum_{i=0}^n w_{ij} \cdot N_{ik}(u) \cdot N_{jl}(v)} \quad (2)$$

In these equations,  $P_i$  and  $P_{ij}$  are the control points,  $w_i$  and  $w_{ij}$  are the weights attributed to the control points,  $k$  and  $l$  are the degrees of the curves in the directions  $u$  and  $v$ ,  $n$  and  $m$  are the number of control points in the directions  $u$  and  $v$ . The function  $N_{ik}(u)$  is the basis function defined by B-spline function and it is determined by equations 3 and 4. The values  $u_i$  are the  $i$ th element of the knot vector  $U$ , defined by equation 5. The function  $N_{jl}(v)$  is determined by the same equations, changing  $i, k$  and  $u$  by  $j, l$  and  $v$ . The same considerations with respect to knot vector  $U$  applies to knot vector  $V$ , as defined by equation 6.

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \leq u \leq u_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

$$N_{i,k}(u) = \left( \frac{u - u_i}{u_{i+k} - u_i} \right) \cdot N_{i,k-1}(u) + \quad (4)$$

$$\left( \frac{u_{i+k+1} - u}{u_{i+k+1} - u_{i+1}} \right) \cdot N_{i+1,k-1}(u)$$

$$U = \left\{ \underbrace{a, \dots, a}_{k+1}, u_{k+1}, \dots, u_{m-k-1}, \underbrace{b, \dots, b}_{k+1} \right\} \quad (5)$$

$$V = \left\{ \underbrace{c, \dots, c}_{l+1}, v_{l+1}, \dots, v_{n-l-1}, \underbrace{d, \dots, d}_{l+1} \right\} \quad (6)$$

As points were fitted to NURBS curves and surface by approximation, the found curves do not cross exactly over the data points and the errors related to the adjustment procedure must be determined. Least squares method was used to fit the data, minimizing these fitting errors. Error

analysis involved the determination of the distances between data points and the respective points on surface model.

Results were compared with a second measurement approach that was carried out without pre-established sequence of measurement, taking points on the surface using an automatic measurement option of CMA software. This way, groups of 200 points were captured each time at any random placement and sequence over the sample's surface. The beginning and the end of the measurement process was activated pushing the gray button at the CMA probe and again a careful positioning of the needle on the surface was significant for the procedure success. One 2000 points group on the prosthesis surface was determined. The polygon-based method was executed with data from automatic measurement. The generated clouds of points were transformed into polygonal surfaces that were adjusted and improved to finally generate a NURBS model in Rhinoceros software. Other software like Catia or SolidWorks may be used to perform this task, or calculation routines may also be implemented in C++ programming language of MatLab software.

### 3 – RESULTS

The small free form dental prosthesis was measured and the curve-net-based approach was used to adjust data and fit a CAD model. This approach was evaluated using the error analysis procedure. Since all points used to construct the models were determined at the same coordinate reference system, it was not necessary to perform the localization step as explained by Li and Gu (11). It must be noted that the CMA uncertainty estimation was 0.087 mm, performed by the manufacturer performance test and carried out with gage blocks. This verification may be executed according to the standard ANSI/ASME B89 recommended to carry out CMA performance verification [13].

The determined groups of points were imported by Rhinoceros software and Figure 3 shows the curves and points on the software screen. These curves were processed to generate the CAD models, fitting a 3<sup>rd</sup> degree NURBS curves with 10 control points each. Figure 4 shows the NURBS surface and the measured points. Figure 5 shows the shaded NURBS surfaces created.

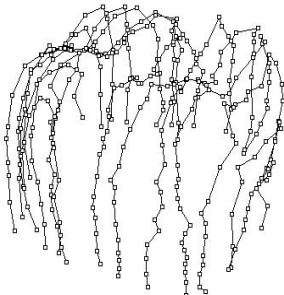


Figure 3 – Determined points and fitted curves, curve-net based measurement with CMA.

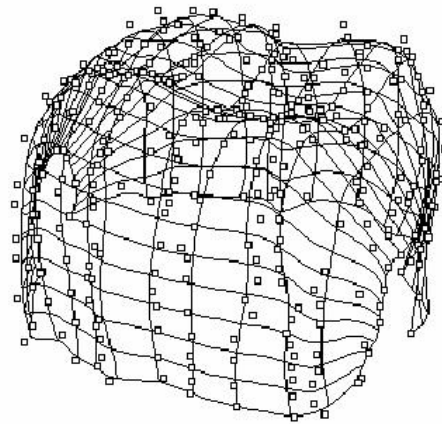


Figure 4 - NURBS surface adjusted with measured points, curve-net based measurement with CMA.



Figure 5 – Shaded NURBS surface fitted, curve-net based measurement with CMA.

Quantitative error analysis was performed and it is shown in figure 6. As noted, the average distance was 0.08 mm and the standard deviation was 0.08 mm. Observing this figure, it can be seen that the CAD model presented a large amount of deviations smaller than 0.1 mm (denoted by blue color), and only a small amount of deviations between 0.1 and 0.5 mm (other colors). It must be observed that standard deviation of points in relation to CAD surfaces were of the same magnitude order of the CMA uncertainty stated by manufacturer.

The IGS file with a cloud of 2000 points was opened using Geomagic CAD software and the operations to create a polygon mesh were carried out. The polygon model was relaxed and spikes were eliminated to construct patches and grids. NURBS modeling to generate CAD model was performed in sequence and the CAD surface had 6 control points and tension 0.25. The generated CAD models were saved in IGES format file. The adjustment errors of these surfaces were verified using Rhinoceros software and figure 7 shows the results of error analysis.



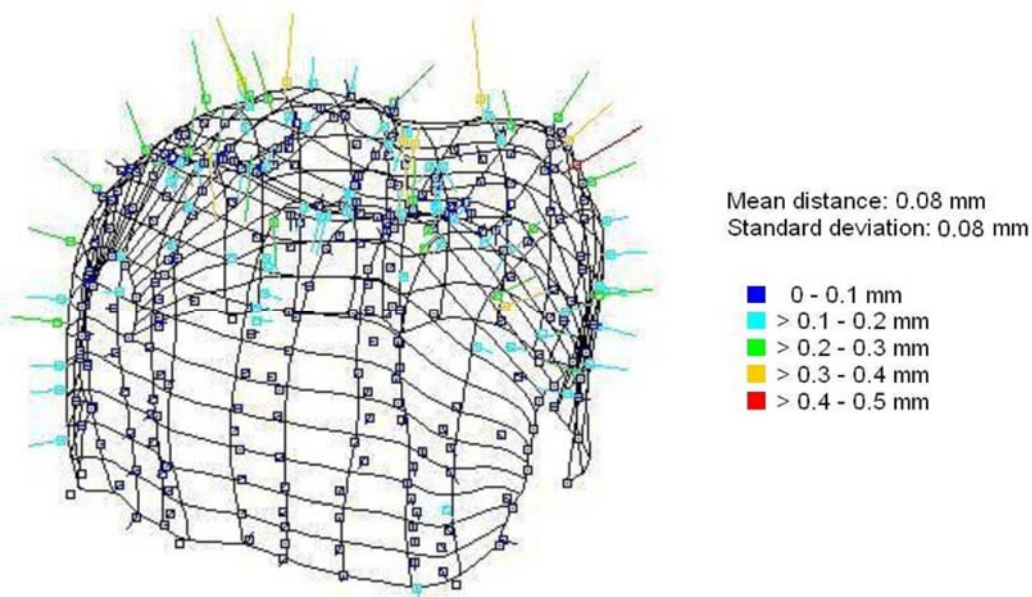


Figure 6 – Error analysis of NURBS surface with 342 points, curve-net based measurement with CMA

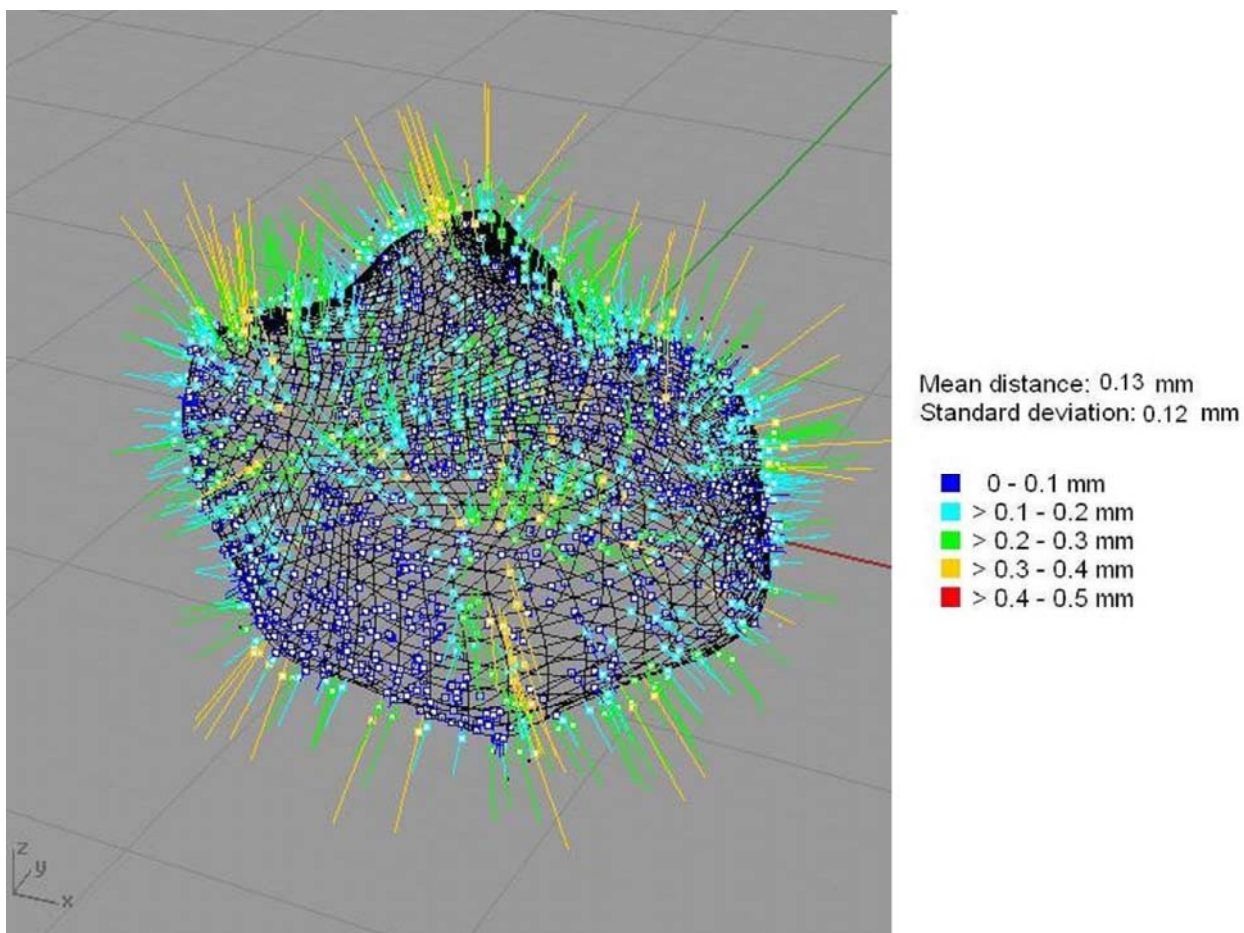


Figure 7 – Error analysis of NURBS surface with 2000 points, automatic measurement with CMA

Comparing the errors obtained in both approaches with CMA measurements, it was observed that the use of curve-net based approach produced average and standard deviation (0.08 mm and 0.08 mm) almost equal to the automatic measurement approach (0.13 mm and 0.14 mm). These results are similar to the accuracy obtained by optical digitization methods.

#### 4 – CONCLUSION

This work has focused on the development and performance characterization of CAD models of small free form surfaces. The methodology involved the use of a Coordinate Measuring Arm (CMA) and the Non-Uniform Rational B-Splines (NURBS) modeling technique. A contact measurement strategy was implemented to carry out the measurements of a small free-form surface of a dental prosthesis and point-to-point measurements were carried out to fit the surface by using the curve-net-based approach.

It was observed that this point-to-point measurement routine proved suitable to create a CAD model of a small free form surface as it demands a lower cost equipment to perform the measurements and provides a CAD model with enough accuracy to CAD/CAM manufacturing processes.

Measurements of small free form surfaces presented some difficulties when using contact measurement approaches. Accessibility of the surface by the CMA stylus is restricted to small dimension parts and the operator skill is an important variable to achieve representative samples of points. It was observed that the increase in the amount of points determined by contact measurements produced larger dispersion of points in relation to the generated CAD model. But the contribution of the NURBS modeling technique to the accuracy of the model was very important.

The CAD model developed was useful to build and implement CAM programs in prototype manufacturing and CAD/CAM manufacturing systems. Besides, the analysis of the measurement uncertainty associated to the CMA and to CAD models is an interesting research topic.

#### ACKNOWLEDGEMENTS

The authors would like to acknowledge the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, CAPES, Brazil, for the grant that made this paper concluded.

#### REFERENCES

[1] Van Der Zel, J.M., Vlaar, S., De Ruiters, W.J., Davidson, C. The CICERO system for CAD/CAM fabrication of full-ceramic crowns. *The Journal of Prosthetic Dentistry*, 85 (3): 261-267. 2001.  
[2] Várady, T., Martin, R.R., Cox, J. Reverse engineering of geometric models – an introduction. *Computer-Aided Design*, 29 (4): 255-268. 1997.  
[3] Feng, C.X., Pandey, V. Experimental study of the effect of digitizing parameters on digitizing uncertainty with a CMM. *International Journal of Production Research*, 40 (3): 683-697. 2002.

[4] ABNT NBR 6409 - Tolerâncias geométricas -Tolerâncias de forma, orientação, posição e batimento - Generalidades, símbolos, definições e indicações em desenho. 1997.  
[5] Zou, L., Samarawickrama, D., Seymour, K., Stout, K. Free Form Surface Measurement using Non-Contact Measurement Methodology. *Proceedings of the XVII IMEKO World Congress*, June 22-27, Dubrovnik, Croatia, 2003.  
[6] Menq, C., Chen, F.L. Curve and surface approximation from CMM measurement data. *Computers and Industrial Engineering*, 30 (2): 211-225, 1996.  
[7] Song, Y-L., Li, J., Yin, L., Huang, T., Gao, P. The feature-based posterior crown design in a dental CAD/CAM system. *International Journal of Advanced Manufacturing Technology*, 31: 1058-1065. 2007.  
[8] Sansoni, G., Docchio, F. In-field performance of an optical digitizer for the reverse engineering of the free-form surfaces. *International Journal of Advanced Manufacturing Technology*, 26: 1353-1361. 2005.  
[9] Zhongwei, Y. Direct integration of reverse engineering and rapid prototyping based on the properties of NURBS or B-spline. *Precision Engineering*, 28: 293-301. 2004.  
[10] Huang, X., Gu, P., Zernicke, R. Localization and comparison of two free-form surfaces. *Computer-Aided Design*, 28 (12): 1017-1022. 1996.  
[11] Li, Y., Gu, P. Free form surface inspection techniques state of the art review. *Computer-Aided Design*, 36: 1395-1417. 2004.  
[12] Piegl, L. and Tiller, W. *The NURBS Book*. Berlin, Springer, 2<sup>nd</sup> Edition. 1997.  
[13] Caskey, G.W., Phillips, S.D., Borchardt, B.R., Eaton, H., Smith, J. Application of the ASME/ANSI B89.1.12M Standard to an Articulating Arm Coordinate Measuring Machine. NIST Report, December 1994.